ERRATA FOR *J-HOLOMORPHIC CURVES AND SYMPLECTIC TOPOLOGY*

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The most substantive change here is to the proof of Theorem 6.2.6 (ii): the previous argument did not handle transversality quite correctly. We have listed some other smaller corrections. We intend to update this file from time to time and so welcome further comments.

pp 102/103: Simplify the proof of Step 3 (the old proof was correct but this one is shorter and more elegant):

STEP 3. *We prove the identity*

$$\lim_{R\to\infty}\lim_{\nu\to\infty} E(u^\nu; B_{R\delta^\nu}) = m_0. \tag{1}$$

By definition of m_0 there is a sequence ε^ν such that

$$\lim_{\nu\to\infty} E(u^\nu; B_{\varepsilon^\nu}) = m_0, \qquad \lim_{\nu\to\infty} \varepsilon^\nu = 0. \tag{2}$$

More precisely, for every $\ell \in \mathbb{N}$, there exists $\varepsilon_\ell \in (0, 1/\ell)$ and $\nu_\ell \in \mathbb{N}$ such that

$$|E(u^\nu; B_{\varepsilon_\ell}) - m_0| \le 1/\ell$$

for $\nu \ge \nu_\ell$. Suppose, without loss of generality, that $\varepsilon_{\ell+1} < \varepsilon_\ell$ and $\nu_{\ell+1} > \nu_\ell$ for every ℓ. Then the sequence ε^ν, defined by $\varepsilon^\nu := \varepsilon_\ell$ for $\nu_\ell \le \nu < \nu_{\ell+1}$, satisfies (2). (Note that we may be unable to choose ε^ν so that $E(u^\nu; B_{\varepsilon^\nu})$ is precisely m_0 because the sequence $u^\nu : B_r \to M$ may consist of maps with energy less than m_0 converging away from 0 to a constant.) Since $R\varepsilon^\nu \to 0$ for every $R \ge 1$, we also have $\lim_{\nu\to\infty} E(u^\nu; B_{R\varepsilon^\nu}) = m_0$ and hence

$$\lim_{\nu\to\infty} E\big(u^\nu, A(\delta^\nu, R\varepsilon^\nu)\big) = \delta/2 \tag{3}$$

for every $R \ge 1$. We consider two cases.

If $\delta^\nu/\varepsilon^\nu$ is bounded away from zero then there is a constant $R > 0$ such that $R\delta^\nu \ge \varepsilon^\nu$ for ν sufficiently large. Hence $E\big(u^\nu; B_{R\delta^\nu}\big) \ge E\big(u^\nu; B_{\varepsilon^\nu}\big)$ for large ν and so (1) follows from (2).

If $\delta^\nu/\varepsilon^\nu \to 0$ then, by (3) and Lemma 4.7.3 with $e^T = R \ge 2$, we have

$$\lim_{\nu\to\infty} E\big(u^\nu; A(R\delta^\nu, \varepsilon^\nu)\big) \le \frac{c}{R^{2\mu}} \lim_{\nu\to\infty} E\big(u^\nu; A(\delta^\nu, R\varepsilon^\nu)\big) \le \frac{c\delta}{2R^{2\mu}}.$$

Hence, by (2),

$$\lim_{\nu\to\infty} E(u^\nu; B_{R\delta^\nu}) \ge m_0 - \frac{c\delta}{2R^{2\mu}}$$

and (1) follows by taking the limit $R \to \infty$. This proves Step 3.

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p 119, line 15: Replace $du^\nu(z):\mathbb{C}\to T_{u(z)}M$ by $du^\nu(z):\mathbb{C}\to T_{u^\nu(z)}M$.

p 128: The last sentence before Step 4 should read: "Now (5.4.5) follows by taking the limit $\varepsilon\to 0$. This proves Step 3."

p 151/152: Change the proof of Theorem 6.2.6 (ii), starting with line -6 on page 151, as follows.

Now consider the projections

$$p^\ell:\mathcal{M}^*(\{A_\alpha\};\mathcal{J}^\ell)\to\mathcal{J}^\ell,\qquad \pi^\ell:\widetilde{\mathcal{M}}^*_{0,T}(\{A_\alpha\};\mathcal{J}^\ell)\to\mathcal{J}^\ell.$$

These are Fredholm maps of indices

$$\mathrm{index}(p^\ell)=2n(1+e(T))+2c_1(A),\qquad \mathrm{index}(\pi^\ell)=\mu(A,T)+\dim\mathrm{G}_T.$$

Hence, by the Sard–Smale theorem A.5.1, the set $\mathcal{J}^\ell_{\mathrm{reg}}(T,\{A_\alpha\})$ of common regular values of p^ℓ and π^ℓ is of the second category in $\mathcal{J}^\ell$ for ℓ sufficiently large. Moreover, an almost complex structure $J\in\mathcal{J}^\ell$ is a common regular value of p^ℓ and π^ℓ if and only if it satisfies the conditions of Definition 6.2.1

Now, for every $K>0$, consider the subset $\mathcal{M}^*_K(\{A_\alpha\};\mathcal{J}^\ell)\subset\mathcal{M}^*(\{A_\alpha\};\mathcal{J}^\ell)$ of all tuples $(\mathbf{u},J)\in\mathcal{M}^*(\{A_\alpha\};\mathcal{J}^\ell)$ that satisfy

$$\|du_\alpha\|_{L^\infty}\le K$$

and

$$\inf_{\zeta\in S^2\setminus\{z_\alpha\}}\frac{d(u_\alpha(z_\alpha),u_\alpha(\zeta))}{d(z_\alpha,\zeta)}\ge\frac{1}{K},\qquad \inf_{\zeta\in S^2}d(u_\alpha(z_\alpha),u_\beta(\zeta))\ge\frac{1}{K}$$

for every $\alpha\in T$, $\beta\in T\setminus\{\alpha\}$, and some collection of points $\{z_\alpha\}_{\alpha\in T}$ in S^2. Likewise, let $Z_K(T)\subset Z(T)$ be the set of all tuples $\mathbf{z}\in Z(T)$ that satisfy

$$d(z_{\alpha\beta},z_{\alpha\gamma})\ge\frac{1}{K},\qquad d(z_i,z_j)\ge\frac{1}{K},\qquad d(z_{\alpha\beta},z_i)\ge\frac{1}{K}$$

for all α, $\beta\ne\gamma$, $i\ne j$ with $\alpha E\beta$, $\alpha E\gamma$, and $\alpha_i=\alpha_j=\alpha$, and denote

$$\widetilde{\mathcal{M}}^*_{0,T;K}(\{A_\alpha\};J):=\widetilde{\mathcal{M}}^*_{0,T}(\{A_\alpha\};J)\cap\Big(\mathcal{M}^*_K(\{A_\alpha\};\mathcal{J}^\ell)\times Z_K(T)\Big).$$

Then the projections

$$p^\ell_K:\mathcal{M}^*_K(\{A_\alpha\};\mathcal{J}^\ell)\to\mathcal{J}^\ell,\qquad \pi^\ell_K:\widetilde{\mathcal{M}}^*_{0,T;K}(\{A_\alpha\};\mathcal{J}^\ell)\to\mathcal{J}^\ell$$

are proper Fredholm maps and so the set $\mathcal{J}^\ell_{\mathrm{reg,K}}(T,\{A_\alpha\})$ of common regular values of p^ℓ_K and π^ℓ_K is open and dense in $\mathcal{J}^\ell$ for ℓ sufficiently large. By the same reasoning the set

$$\mathcal{J}_{\mathrm{reg,K}}(T,\{A_\alpha\}):=\mathcal{J}^\ell_{\mathrm{reg,K}}(T,\{A_\alpha\})\cap\mathcal{J}_\tau(M,\omega)$$

is open in $\mathcal{J}_\tau(M,\omega)$. Moreover, since $\mathcal{J}^\ell_{\mathrm{reg,K}}(T,\{A_\alpha\})$ is dense in $\mathcal{J}^\ell$ for ℓ sufficiently large it follows as in the proof of Theorem 3.1.5 that $\mathcal{J}_{\mathrm{reg,K}}(T,\{A_\alpha\})$ is dense in $\mathcal{J}_\tau(M,\omega)$. Hence the set

$$\mathcal{J}_{\mathrm{reg}}(T,\{A_\alpha\})=\bigcap_{K>0}\mathcal{J}_{\mathrm{reg,K}}(T,\{A_\alpha\})$$

is a countable intersection of open and dense sets in $\mathcal{J}_\tau(M,\omega)$. This proves (ii).

p 160: The condition "$\overline{F(W)}$ is compact" is needed in the definition of bordant pseudocycles.

p 161, before Lemma 6.5.5: Replace "dim M" by "dim X".

p 340: The last line in the proof of Proposition 9.7.2 should read:

Hence the loop $t \mapsto \phi^{-1} \circ \psi_t \circ \phi$ is smoothly isotopic to $t \mapsto \psi_t$ and preserves the symplectic form ω_λ. This proves the proposition.

p 356: The constant c_0 in Proposition 10.5.1 depends not only on p but also on c.

p 373: The assertion of Step 2 should read: *"For every $\varepsilon > 0$ there are positive constants δ_2 and ε_2 such that, for every $(\delta, R) \in \mathcal{A}(\delta_2)$, the following holds..."*

p 378/9: The proof of Step 4 should read: "First choose $\varepsilon_1 > 0$ such that the assertion of Step 1 holds. Then choose $\varepsilon_2 > 0$ and $\delta_2 < \delta_0(c)$ such that the assertion of Step 2 holds with this constant ε_1. Finally, choose $\varepsilon_3 > 0$ and $\delta_3 < \delta_2$ such that the assertion of Step 3 holds with this constant ε_2. Now..."

p 379/381: Replace S_0 by S^0 in Theorem 10.8.1 (twice), Remark 10.8.2 (once), and in equation (10.8.1) (once).

p 421: In line 4 from below it should read $\deg(u) = \langle x_1, L\rangle = 1$.

p 454, Rmk 12.1.1: Replace "Cohen–James–Segal" by "Cohen–Jones–Segal".

p 457: The discussion uses nonexistence of holomorphic spheres with negative Chern numbers for generic 2-parameter families of almost complex structures (in the proof that $\Phi^{\alpha\beta}$ is independent of the homotopy from J^α to J^β used to define it). This holds only under the strong semipositivity assumption (8.5.1). If one wants to prove the Arnold conjecture in the general semipositive case with the methods described in the book, then one has to fix a generic almost complex structure J once and for all, and then construct Floer homology groups that are independent of H but, apriori, might depend on J. The best way around this subtlety would be to assume (8.5.1) and allow J to depend on t.

p 509: Refer to "Abraham–Robbin, *Transversal Mappings and Flows*, Benjamin, 1970" for the proof of Sard's theorem with sharp differentiability hypotheses. This doesn't follow from the proof in Milnor's book.

p 587, line 12: Replace "$\alpha_i = i$" by "$\alpha_i = \alpha$".

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