Notes on my book:

Large Networks and Graph Limits

American Mathematical Society
Providence, Rhode Island, 2012

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December 16, 2012

Section 1.3.1 Property testing. Talking about "property testing" I should have used the phrase "graph property testing". In a different context, testing whether a polynomial is linear or multilinear has been considered earlier (M. Blum, M. Luby and R. Rubinfeld: Self-Testing/Correcting with Applications to Numerical Problems, Proc. 22nd ACM STOC (1990), 73-83; L. Babai, L. Fortnow and C. Lund: Non-Deterministic Exponential Time has Two-Prover Interactive Protocols, 31st IEEE FOCS (1990), 16-25.)

Section 6.4.1: On the dimension of graph algebras for homomorphism functions. Martin Dyer has kindly pointed out that the proof of Theorems 6.36–6.40 only works if we allow loops. Luckily, the theorem remains true without the loops; the proof can be corrected using the results of the preceding section (6.3. Contractors and connectors). Dyer also suggests a connection with the computational complexity theory of homomorphism numbers. For details, see http://www.cs.elte.hu/~lovasz/book/homnotes-6-4-1.pdf.

Section 9.5: Weak uniqueness of weak regularity partitions. We have seen that weak regularity partitions do not give unique template graphs in general. But if we consider weak regularity partitions of a graphon, then the the template graphs become more and more similar to each other as the error decreases. For details, see http://www.cs.elte.hu/~lovasz/book/homnotes-9-5.pdf.

Section 13.5: Compactness of the automorphism group of a pure graphon. For a proof of this fact, see http://www.cs.elte.hu/~lovasz/book/homnotes-13-5.pdf.

Section 17.2: On the convergence of multigraphs with unbounded edge-multiplicities. One can extend the notions of regularity partitions and the cut distance to multigraphs with unbounded edge-multiplicities, and prove extensions of the Regularity and Counting lemmas. Based on these results, one can construct a limit object reflecting the limits of subgraph densities of simple graphs. A complete result (extending also the results in Chapter 17) is not yet available. For details, see http://www.cs.elte.hu/~lovasz/book/homnotes-17-2.pdf.

Section 23.4: Regularity Lemmas for categories. There is a quite natural generalization of the Regularity Lemma (in various forms, weak and stronger) to categories. The main shortcoming is that there is no Counting Lemma to go with it. For details, see http://www.cs.elte.hu/~lovasz/book/homnotes-23-4.pdf.

Section 20.1.1: Existence of a stationary distribution for the Markov chain on weighting pairs of a graph. For a proof of Lemma 20.2, see http://www.cs.elte.hu/~lovasz/book/homnotes-20-1-1.pdf.

Appendix A.3.4: Characterizing convergence of coupling measures. For a proof of Proposition A.6, see http://www.cs.elte.hu/~lovasz/book/homnotes-A-3-4a.pdf.

Appendix A.3.4: Coupling measures concentrated on a specified set. For a proof and generalization of Proposition A.7, see http://www.cs.elte.hu/~lovasz/book/homnotes-A-3-4b.pdf.