Errata & Addenda
for
Alexandru Scorpan’s
THE WILD WORLD OF 4–MANIFOLDS
AMS 2005

So far, only a few corrections, an update and an extra reference. Readers of the book are again encouraged to report any mistakes, comments, updates, etc. that have not yet been caught in the current version of the present Errata. We thank Andrew Ranicki and Greg Friedman for pointing out a few such items.

CORRECTIONS

Preview

Page ix, sixth paragraph, just before Travel guide: Delete “arXiv” from “Errata... will be maintained on the arXiv and also on...”

The arXiv only accepts self-contained submissions and has rejected this Errata. While it already contains several items named “errata” or “corrigenda”, the arXiv now seems to have become more strict about it.

Chapter 4. Intersection forms and topology

4.1 Whitehead’s theorem and homotopy type.

Pontryagin–Thom argument

Page 147, Figure 4.6: In case it is not clear or ambiguous: the picture attempts to represent the Seifert surface made from most of the sphere, with a twisted clover-like hole in it. The shaded region represents a hole, not the surface.
4.3 Intersection forms and characteristic classes.

Third Stiefel–Whitney class

Page 165, first line of the subsection: Instead of “\( w_2(T_M) \in H^3(M; \mathbb{Z}_2) \)”, one should read “\( w_3(T_M) \in H^3(M; \mathbb{Z}_2) \)”.

That’s it, the bundle is done

Page 167, last displayed equation: Instead of “\( p_1(T_M) = b_2^+ (M) - b_2^- (M) \)”, it should have been “\( \frac{1}{3} p_1(T_M) = b_2^+ (M) - b_2^- (M) \)”.

Chapter 5. Classifications and counterclassifications

5.2 Serre’s algebraic classification of forms.

Indefinite forms / Example

Page 238, last line on page: There’s a missing \( \mathbb{CP}^2 \): instead of “we have a diffeomorphism \( M \# \mathbb{CP}^2 \# kS^2 \times S^2 \cong \ldots \)”, it should have been “we have a diffeomorphism \( M \# \mathbb{CP}^2 \# \mathbb{CP}^2 \# kS^2 \times S^2 \cong \ldots \)”.

Chapter 10. The Seiberg–Witten invariants

10.5 Invariants of symplectic manifolds.

Seiberg–Witten and \( J \)–holomorphic curves

Page 412, end of second paragraph: “…represented by at least one \( J \)–holomorphic curve.” Add: “(which might be disconnected).”

The fact that \( SW_M \) counts curves that might be disconnected is mentioned in the end-note on page 471 (The Gromov–Taubes invariants of symplectic 4–manifolds), but it is better if it also appears in the main text.

Page 412, third paragraph, before Corollary: Delete \( K^* \) from “In particular, we notice that both \( K^* \) and \( -K^* \) can always be represented by \( J \)–holomorphic curves.”

From \( SW_M(-K^*) \neq 0 \) and by writing \( -K^* = K^* + 2\varepsilon \) with \( \varepsilon = -K^* \), it follows that the canonical class \( K_M = -K^* \) can be represented by a \( J \)–holomorphic curve. However, for the representability of \( K^* \) we would need information about \( SW_M(3K^*) \). Since a \( J \)–holomorphic curve always carries a natural orientation (from its complex structure), the distinction between the representability of \( K^* \) and of \( -K^* \) is important.

Notice also that \( SW_M(K^*) \neq 0 \) implies that the class \( \varepsilon = 0 \) can be represented by a \( J \)–holomorphic curve. This should only be understood as referring to the empty curve, as there can be no homologically-trivial non-trivial \( J \)–holomorphic curves.
10.7 Notes.

Note: Lefschetz pencils and fibrations

Page 417, last line on page: “The picture of this crossing is the one from figure 10.8 on the following page.” Add: “or its twisted version”.

The twisted version is the one in which, as one travels along $S^1$, the hyperboloids undergo a rotation of $\pi$; in other words, their axis describes a non-orientable real-line bundle over $S^1$.

Note: The Seiberg–Witten moduli space


Chapter 11. The minimum genus of embedded surfaces

11.3 Digression: the happy case of 3–manifolds.

Page 493, Gabai’s theorem, footnote 16: “An $n$–manifold is called irreducible if it does not split as a connected sum of simpler manifolds (homotopy spheres do not count).” Add or replace with: “A 3–manifold is called irreducible if every embedded 2–sphere bounds a 3–ball. In particular, $S^1 \times S^2$ is not irreducible.”

A 3–manifold that does not split as a non-trivial connected sum is called prime. Connected sum splittings with fake 3–sphere terms are excluded by the Poincaré conjecture. Further, the only 3–manifold that is prime but not irreducible is $S^1 \times S^2$. Therefore the difference between the high-dimensional definition of “irreducible” and its 3–dimensional namesake is the exclusion of $S^1 \times S^2$.

ADDITIONAL BIBLIOGRAPHY

For the gap between topological and PL (nicely triangulated) manifolds in high-dimensions, an important reference is A. Ranicki, A. Casson, D. Sullivan, M. Armstrong, C.P. Rourke and G. Cooke’s The Hauptvermutung book [Haupt96], containing mainly papers written in the 1960s.

UPDATE

Along the trend outlined at the end of the volume (Vast geographies, p. 553f), J. Park’s recent preprint Exotic smooth structures on $3\mathbb{CP}^2 \# 8\mathbb{CP}^2$ [Par05] adds the finishing touch (proves simple-connectedness) to a construction from A. Stipsicz and Z. Szabó’s Small exotic 4–manifolds with $b^+_2 = 3$ [SS05], and hence shows that $# 3\mathbb{CP}^2 \# 8\mathbb{CP}^2$ admits infinitely-many distinct smooth structures.
REFERENCES

