Errata & Addenda

for

Alexandru Scorpan's
THE WILD WORLD OF 4–MANIFOLDS
AMS 2005

So far, only a few corrections, an update and an extra reference. Readers of the book are again encouraged to report any mistakes, comments, updates, *etc.* that have not yet been caught in the current version of the present Errata. We thank Andrew Ranicki and Greg Friedman for pointing out a few such items.

CORRECTIONS

Preview

Page **ix,** sixth paragraph, just before *Travel guide*: Delete "`arXiv`" from "*Errata...will be maintained on the* `arXiv` *and also on...*"

The `arXiv` *only accepts self-contained submissions and has rejected this Errata. While it already contains several items named "errata" or "corrigenda", the* `arXiv` *now seems to have become more strict about it.*

Chapter 4. Intersection forms and topology

4.1 Whitehead's theorem and homotopy type.

Pontryagin–Thom argument

Page **147,** Figure 4.6: In case it is not clear or ambiguous: the picture attempts to represent the Seifert surface made from most of the sphere, with a twisted clover-like hole in it. The shaded region represents a hole, not the surface.

4.3 Intersection forms and characteristic classes.

Third Stiefel–Whitney class

Page **165,** first line of the subsection: Instead of "$w_2(T_M) \in H^3(M;\mathbb{Z}_2)$", one should read "$w_{\mathbf{3}}(T_M) \in H^3(M;\mathbb{Z}_2)$".

That's it, the bundle is done

Page **167,** last displayed equation: Instead of "$p_1(T_M) = b_2^+(M) - b_2^-(M)$", it should have been "$\frac{\mathbf{1}}{\mathbf{3}}\, p_1(T_M) = b_2^+(M) - b_2^-(M)$".

Chapter 5. Classifications and counterclassifications

5.2 Serre's algebraic classification of forms.

Indefinite forms / Example

Page **238,** last line on page: There's a missing $\overline{\mathbb{CP}}^2$: instead of "*we have a diffeomorphism* $M \mathbin{\#} \mathbb{CP}^2 \mathbin{\#} k\, \mathbb{S}^2 \times \mathbb{S}^2 \cong \ldots$", it should have been "*we have a diffeomorphism* $M \mathbin{\#} \mathbb{CP}^2 \mathbin{\#} \overline{\mathbb{CP}}^2 \mathbin{\#} k\, \mathbb{S}^2 \times \mathbb{S}^2 \cong \ldots$"

Chapter 10. The Seiberg–Witten invariants

10.5 Invariants of symplectic manifolds.

Seiberg–Witten and J–holomorphic curves

Page **412,** end of second paragraph: "*. . . represented by at least one J–holomorphic curve.*" Add: "*(which might be disconnected).*"

The fact that $\mathcal{SW}_M$ counts curves that might be disconnected is mentioned in the end-note on page 471 (The Gromov–Taubes invariants of symplectic 4–manifolds), but it is better if it also appears in the main text.

Page **412,** third paragraph, before Corollary: Delete K^* from "*In particular, we notice that both K^* and $-K^*$ can always be represented by J–holomorphic curves.*"

From $\mathcal{SW}_M(-K^*) \neq 0$ and by writing $-K^* = K^* + 2\varepsilon$ with $\varepsilon = -K^*$, it follows that the canonical class $K_M = -K^*$ can be represented by a J–holomorphic curve. However, for the representability of K^* we would need information about $\mathcal{SW}_M(3K^*)$. Since a J–holomorphic curve always carries a natural orientation (from its complex structure), the distinction between the representability of K^* and of $-K^*$ is important.

Notice also that $\mathcal{SW}_M(K^*) \neq 0$ implies that the class $\varepsilon = 0$ can be represented by a J–holomorphic curve. This should only be understood as referring to the empty curve, *as there can be no homologically-trivial non-trivial J–holomorphic curves.*

10.7 Notes.

Note: Lefschetz pencils and fibrations

Page **417,** last line on page: *"The picture of this crossing is the one from figure 10.8 on the following page."* Add: *"or its twisted version"*.

The twisted version is the one in which, as one travels along $\mathbb{S}^1$, the hyperboloids undergo a rotation of π; in other words, their axis describes a non-orientable real-line bundle over $\mathbb{S}^1$.

Note: The Seiberg–Witten moduli space

Page **440,** second paragraph: Ignore first sentence, *"One possibility... all derivatives."*, as well as its footnote 20.

Chapter 11. The minimum genus of embedded surfaces

11.3 Digression: the happy case of 3–manifolds.

Page **493,** Gabai's theorem, footnote 16: *"An n–manifold is called* **irreducible** *if it does not split as a connected sum of simpler manifolds (homotopy spheres do not count)."* Add or replace with: *"A 3–manifold is called* **irreducible** *if every embedded 2–sphere bounds a 3–ball. In particular, $\mathbb{S}^1 \times \mathbb{S}^2$ is not irreducible."*

A 3–manifold that does not split as a non-trivial connected sum is called **prime**. *Connected sum splittings with fake 3–sphere terms are excluded by the Poincaré conjecture. Further, the only 3–manifold that is prime but not irreducible is $\mathbb{S}^1 \times \mathbb{S}^2$. Therefore the difference between the high-dimensional definition of "irreducible" and its 3–dimensional namesake is the exclusion of $\mathbb{S}^1 \times \mathbb{S}^2$.*

ADDITIONAL BIBLIOGRAPHY

For the gap between topological and PL (nicely triangulated) manifolds in high-dimensions, an important reference is **A. Ranicki, A. Casson, D. Sullivan, M. Armstrong, C.P. Rourke** and **G. Cooke**'s ***The Hauptvermutung book*** **[Haupt96]**, containing mainly papers written in the 1960s.

UPDATE

Along the trend outlined at the end of the volume (*Vast geographies*, p. 553f), **J. Park**'s recent preprint ***Exotic smooth structures on*** $\mathbf{3\,\mathbb{CP}^2\,\#\,8\,\overline{\mathbb{CP}}^2}$ **[Par05]** adds the finishing touch (proves simple-connectedness) to a construction from **A. Stipsicz** and **Z. Szabó**'s ***Small exotic 4–manifolds with*** $\boldsymbol{b_2^+ = 3}$ **[SS05]**, and hence shows that $\#\,3\,\mathbb{CP}^2\,\#\,8\,\overline{\mathbb{CP}}^2$ admits infinitely-many distinct smooth structures.

REFERENCES

[Par05] Jongil Park, *Exotic smooth structures on* $3\mathbb{CP}^2 \# 8\overline{\mathbb{CP}}^2$, 2005. `arXiv:math.GT/0507085`.

[Haupt96] A.A. Ranicki, A.J. Casson, D.P. Sullivan, M.A. Armstrong, C.P. Rourke, and G.E. Cooke, *The Hauptvermutung book*, K–Monographs in Mathematics, vol. 1, Kluwer Academic Publishers, Dordrecht, 1996, A collection of papers on the topology of manifolds. MR MR1434100 (98c:57024).

[SS05] András I. Stipsicz and Zoltán Szabó,*Small exotic* 4*–manifolds with* $b_2^+ = 3$, 2005. `arXiv:math.GT/0501273`.