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September 15, 2005

# Errata & Addenda

for

Alexandru Scorpan's

**THE WILD WORLD OF 4-MANIFOLDS**

AMS 2005

So far, only a few corrections, an update and an extra reference. Readers of the book are again encouraged to report any mistakes, comments, updates, *etc.* that have not yet been caught in the current version of the present Errata. We thank Andrew Ranicki and Greg Friedman for pointing out a few such items.

## CORRECTIONS

### Preview

Page ix, sixth paragraph, just before *Travel guide*: Delete "arXiv" from "Errata... will be maintained on the arXiv and also on..."

*The arXiv only accepts self-contained submissions and has rejected this Errata. While it already contains several items named "errata" or "corrigenda", the arXiv now seems to have become more strict about it.*

## Chapter 4. Intersection forms and topology

### 4.1 Whitehead's theorem and homotopy type.

*Pontryagin-Thom argument*

Page 147, Figure 4.6: In case it is not clear or ambiguous: the picture attempts to represent the Seifert surface made from most of the sphere, with a twisted clover-like hole in it. The shaded region represents a hole, not the surface.

### 4.3 Intersection forms and characteristic classes.

#### Third Stiefel–Whitney class

Page 165, first line of the subsection: Instead of “ $w_2(T_M) \in H^3(M; \mathbb{Z}_2)$ ”, one should read “ $w_3(T_M) \in H^3(M; \mathbb{Z}_2)$ ”.

#### That’s it, the bundle is done

Page 167, last displayed equation: Instead of “ $p_1(T_M) = b_2^+(M) - b_2^-(M)$ ”, it should have been “ $\frac{1}{3}p_1(T_M) = b_2^+(M) - b_2^-(M)$ ”.

## Chapter 5. Classifications and counterclassifications

### 5.2 Serre’s algebraic classification of forms.

#### Indefinite forms / Example

Page 238, last line on page: There’s a missing  $\overline{\mathbb{C}\mathbb{P}^2}$ : instead of “we have a diffeomorphism  $M \# \mathbb{C}\mathbb{P}^2 \# k\mathbb{S}^2 \times \mathbb{S}^2 \cong \dots$ ”, it should have been “we have a diffeomorphism  $M \# \mathbb{C}\mathbb{P}^2 \# \overline{\mathbb{C}\mathbb{P}^2} \# k\mathbb{S}^2 \times \mathbb{S}^2 \cong \dots$ ”

## Chapter 10. The Seiberg–Witten invariants

### 10.5 Invariants of symplectic manifolds.

#### Seiberg–Witten and J–holomorphic curves

Page 412, end of second paragraph: “... represented by at least one J–holomorphic curve.” Add: “(which might be disconnected).”

*The fact that  $SW_M$  counts curves that might be disconnected is mentioned in the end-note on page 471 (The Gromov–Taubes invariants of symplectic 4–manifolds), but it is better if it also appears in the main text.*

Page 412, third paragraph, before Corollary: Delete  $K^*$  from “In particular, we notice that both  $K^*$  and  $-K^*$  can always be represented by J–holomorphic curves.”

*From  $SW_M(-K^*) \neq 0$  and by writing  $-K^* = K^* + 2\varepsilon$  with  $\varepsilon = -K^*$ , it follows that the canonical class  $K_M = -K^*$  can be represented by a J–holomorphic curve. However, for the representability of  $K^*$  we would need information about  $SW_M(3K^*)$ . Since a J–holomorphic curve always carries a natural orientation (from its complex structure), the distinction between the representability of  $K^*$  and of  $-K^*$  is important.*

*Notice also that  $SW_M(K^*) \neq 0$  implies that the class  $\varepsilon = 0$  can be represented by a J–holomorphic curve. This should only be understood as referring to the empty curve, as there can be no homologically-trivial non-trivial J–holomorphic curves.*

### 10.7 Notes.

Note: *Lefschetz pencils and fibrations*

Page 417, last line on page: “The picture of this crossing is the one from figure 10.8 on the following page.” Add: “or its twisted version”.

*The twisted version is the one in which, as one travels along  $S^1$ , the hyperboloids undergo a rotation of  $\pi$ ; in other words, their axis describes a non-orientable real-line bundle over  $S^1$ .*

Note: *The Seiberg–Witten moduli space*

Page 440, second paragraph: Ignore first sentence, “One possibility... all derivatives.”, as well as its footnote 20.

## Chapter 11. The minimum genus of embedded surfaces

### 11.3 Digression: the happy case of 3-manifolds.

Page 493, Gabai’s theorem, footnote 16: “An  $n$ -manifold is called **irreducible** if it does not split as a connected sum of simpler manifolds (homotopy spheres do not count).” Add or replace with: “A 3-manifold is called **irreducible** if every embedded 2-sphere bounds a 3-ball. In particular,  $S^1 \times S^2$  is not irreducible.”

*A 3-manifold that does not split as a non-trivial connected sum is called **prime**. Connected sum splittings with fake 3-sphere terms are excluded by the Poincaré conjecture. Further, the only 3-manifold that is prime but not irreducible is  $S^1 \times S^2$ . Therefore the difference between the high-dimensional definition of “irreducible” and its 3-dimensional namesake is the exclusion of  $S^1 \times S^2$ .*

## ADDITIONAL BIBLIOGRAPHY

For the gap between topological and PL (nicely triangulated) manifolds in high-dimensions, an important reference is **A. Ranicki, A. Casson, D. Sullivan, M. Armstrong, C.P. Rourke and G. Cooke’s *The Hauptvermutung book* [Haupt96]**, containing mainly papers written in the 1960s.

## UPDATE

Along the trend outlined at the end of the volume (*Vast geographies*, p. 553f), **J. Park’s** recent preprint *Exotic smooth structures on  $3\mathbb{C}P^2 \# 8\overline{\mathbb{C}P^2}$*  [Par05] adds the finishing touch (proves simple-connectedness) to a construction from **A. Stipsicz and Z. Szabó’s *Small exotic 4-manifolds with  $b_2^+ = 3$***  [SS05], and hence shows that  $3\mathbb{C}P^2 \# 8\overline{\mathbb{C}P^2}$  admits infinitely-many distinct smooth structures.

## REFERENCES

- [Par05] Jongil Park, *Exotic smooth structures on  $3\mathbb{C}P^2 \# 8\overline{\mathbb{C}P^2}$* , 2005. [arXiv:math.GT/0507085](#).
- [Haupt96] A.A. Ranicki, A.J. Casson, D.P. Sullivan, M.A. Armstrong, C.P. Rourke, and G.E. Cooke, *The Hauptvermutung book*, *K-Monographs in Mathematics*, vol. 1, Kluwer Academic Publishers, Dordrecht, 1996, A collection of papers on the topology of manifolds. MR MR1434100 (98c:57024).
- [SS05] András I. Stipsicz and Zoltán Szabó, *Small exotic 4-manifolds with  $b_2^+ = 3$* , 2005. [arXiv:math.GT/0501273](#).