

# Mistakes, Improvements, Additions, and Comments

August 26, 2010

*A First Course in Sobolev Spaces*, First edition  
G. Leoni,  
Graduate Studies in Mathematics, AMS, 2009

For the original text I use the color **Red**, for corrections the color **Green**, and for improvements and additions the color **Blue**. Names in brackets refer to the persons who called the error to my attention (to the best of my recollection) or suggested improvements and additions.<sup>1</sup>

## CHAPTER 1:

- p. 5 In Exercise 1.5 one should assume that  $u$  is bounded from below, since otherwise the series could diverge to  $\infty$ . Alternatively, one should choose a point of  $x_0 \in I^\circ$  at which  $u$  is continuous and then take

$$u_J(x) := \begin{cases} \sum_{y \in I, x_0 \leq y < x} (u_+(y) - u_-(y)) + u(x) - u_-(x) & \text{if } x \geq x_0, \\ -\sum_{y \in I, x < y \leq x_0} (u_+(y) - u_-(y)) + u(x) - u_+(x) & \text{if } x \leq x_0. \end{cases} \quad (1)$$

## CHAPTER 3:

- p. 109 In Corollary 3.74 one should either assume that  $u$  is bounded from below or replace (3.33) with (1) (see the bullet above).

## CHAPTER 10:

- p. 305 In Exercise 10.51,  $p$  should be  $\infty$ . Also the fact that  $W^{1,\infty}(\Omega)$  is a dual space is not obvious. See the file “Lipschitz functions as a dual space”. [M.G. Mora]

## CHAPTER 11:

- p. 305 In line 8, by (1.15) (with  $u$  replaced by  $u_n$ ) should be replaced by reasoning somewhat as in (1.15). [M.G. Mora]

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<sup>1</sup>The style of this file is inspired by <http://www.hss.caltech.edu/kcb/IDA-Errata.pdf>

### CHAPTER 12:

- p. 353 In Exercises 12.6 and 12.7,  $d_{\text{reg}}$  be its regularized distance should be replaced by  $d_{\text{reg}}$  be the regularized distance corresponding to  $\mathbb{R}^N \setminus \bar{\Omega}$ .
- p. 354 In Line 2,  $td_{\text{reg}}(x)$  should be replaced by  $Ctd_{\text{reg}}(x)$ , where  $C$  is the constant given in Exercises 12.6. See also the file “Extension domains for higher order Sobolev spaces” for a complete proof.
- p. 356 In formula (12.7),  $\frac{1}{\varepsilon}$  should be replaced by  $\frac{M}{\varepsilon}$ .
- p. 359 In lines 2-6 and in Remark 12.16,  $\frac{1}{\varepsilon}$  should be replaced by  $\frac{M}{\varepsilon}$ .

### CHAPTER 13:

- p. 402 Thanks to P. Piovano for pointing out that the proof of Theorem 13.30 is too fast. It should be divided in two steps. Assuming first that  $u \in L^{1^*}(\mathbb{R}^N)$ , by mollification it follows that

$$\left( \int_{\mathbb{R}^N} |u(x)|^{1^*} dx \right)^{\frac{1}{1^*}} \leq C |Du|(\mathbb{R}^N).$$

To remove the additional hypothesis that  $u \in L^{1^*}(\mathbb{R}^N)$ , one should truncate  $u$  as in Step 4 of the proof of Theorem 11.2. The problem here is the fact the mollification of a function vanishing at infinity needs not vanish at infinity. See also the file “An extension of the Sobolev–Gagliardo–Nirenberg theorem”.

More to come for sure... :(