An Introductory Course on Mathematical Game Theory

Graduate Studies in Mathematics, volume 115.

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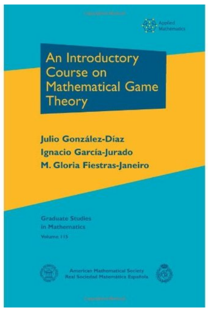

We are grateful to the readers for contributing to improve this book by submitting errata, corrections, comments, and any other form of feedback.

Errata

Page 3, line -8: Replace "Moreover, if $x, y \in \mathbb{R}^n$" with "Moreover, if $x, y \in \mathbb{R}$".

Page 4, line -14: Replace "the sets of superior and inferior gap extremes" with "the sets of inferior and superior gap extremes".

Page 22, line 5: Replace "Quasi-concavity implies concavity" with "Quasi-concavity is implied by concavity".

Page 22, line 10: The assumption on the A_i sets in Proposition 2.2.2 should read:

i) A_i is a nonempty, convex, and compact subset of $\mathbb{R}^{m_i}$.

Page 33, line -1 and Page 34, line 1: Replace "$(2,1)$, $(1/2,1/2)$, and $(2,1)$" with "$(2,1)$, $(1/2,1/2)$, and $(1,2)$".

Page 52, footnote 20: In the definition of P1, replace "$x \in \mathbb{R}$" with "$x \in \mathbb{R}^l$".

Page 64, line -11: Replace "$= 0 \geq 0$" with "≥ 0".

Page 88, line 5: Replace "$\{r^i\}_{i\in I} \subset \mathbb{R}^n$" with "$\{r_i\}_{i\in I} \subset \mathbb{R}$".

Page 89, line 4: Replace "either R is not contained in" with "either R is contained in".

Page 114, Figure 3.3.5 b): The entry in the position $(2,1)$ of the table, corresponding to the strategy profile (MH, MM), should be $(65,75)$ instead of $(75,65)$.

Page 145, lines 6-7: Replace "a continuum of actions" with "an infinite number of actions".

Page 181, lines 9-10: The sentence after "i.e.," in Proposition 4.4.1 should read "for each $\hat{a}_{-i} \in \hat{A}_{-i}$ and each $\hat{a}_i \in \hat{A}_i$, $\hat{u}_i^{II}(\hat{a}_{-i}, \hat{a}_i^{II}) \geq \hat{u}_i^{II}(\hat{a}_{-i}, \hat{a}_i)$, with strict inequality for some $\hat{a}_{-i} \in \hat{A}_{-i}$".

Page 195, Example 4.6.1. At the end of the first paragraph, add the sentence: "Take the strategy profile in which, at period t, player 1 plays l if his type is $\tilde{\theta}_1$ and r if his type is $\bar{\theta}_1$."

Page 205, lines 16-18: Replace "which would imply, in particular, that the players" with "which allows to model, in particular, situations where the players" and remove "that" in line 18.

Page 205, Example 5.2.1. Throughout the example, replace all the instances of "v" with "V".

Page 206, Definition 5.3.1: Replace "compact" with "closed" in the properties of F and, further, introduce F_d and impose its compactness (F_d is currently introduced on page 207). This is to avoid a domain inconsistency in the proof of Theorem 5.3.3 on page 209 (with the sets U and $f^A(U)$). Even if the above changes are not made, the result as stated is true, but then a (minor) modification of the proof would be needed to avoid the domain problem.

Pages 210-211, proof of Proposition 5.3.4. The paragraphs "**Remove CAT**" and "**Remove IIA**" should be replaced with:

Remove CAT: Let φ be the allocation rule that, for each bargaining problem (F,d), selects the allocation $\varphi(F,d) := d + \bar{t}(1,\ldots,1)$, where $\bar{t} := \max\{t \in \mathbb{R} : d + t(1,\ldots,1) \in F_d\}$ (the compactness of F_d ensures that $\bar{t}$ is well defined). This allocation rule, known as the *egalitarian solution* (Kalai 1977), satisfies SYM and IIA, but, although φ always selects points in the boundary of F, it does not satisfy EFF. Consider now an allocation rule $\hat{\varphi}$ that, for each two-player bargaining problem (F,d), selects the Pareto efficient allocation closest to $\varphi(F,d)$. It is not difficult to check that $\hat{\varphi}$ satisfies EFF, SYM, and IIA for two-player bargaining problems. Thus, Theorem 5.3.3 is not true if we drop CAT.

Remove IIA: The *Kalai-Smorodinsky solution*, which we present below, satisfies EFF, SYM, and CAT for two-player bargaining problems. Thus, Theorem 5.3.3 is not true if we drop IIA.

Page 220, Figure 5.5.1: The coordinates of the two lower points in the figure, $(1500,0,500)$ and $(0,1500,500)$, should be replaced with $(1500,500,0)$ and $(500,1500,0)$, respectively.

Page 248, line 10: Replace "the more patient" with "the more impatient".

Page 256, line above Eq. (5.10.1): Replace "c" with "v".

Page 265, line -4: Replace "$f^{\text{TR}}(E,d) \leq d_i/2$" with "$f_i^{\text{TR}}(E,d) \leq d_i/2$".

Pages 265-267, proof of Theorem 5.11.2. Replace the three instances of "$f^{TR}(\hat{E},c)$" with "$f^{TR}(\hat{E},d)$".

Page 266, proof of Theorem 5.11.2. The proof of Case 1.2 is incomplete. Refer to the end of this document for the complete argument.

Page 267, line 11: Replace "If $\hat{x} \in I(v)$" with "If $\hat{x} \in I(\hat{v})$".

Page 278, line 7: Replace "$\bar{x} = (\bar{x}_1, \bar{x}_2)$" with "$\bar{x} = (\bar{x}_1, \bar{x}_2, \bar{x}_3)$".

Page 278, line 10: Replace "$C(v) = \text{conv}\{(2,6,3),(0,6,5),(0,13/2,9/2),(2,13/2,5/2)\}$" with "$C(v) = \text{conv}\{(2,6,3),(0,6,5),(0,13/2,9/2),(5/2,13/2,2),(2,7,2)\}$".

Page 280, line 20: Replace "minimun" with "minimum".

Pages 282-289, Subsection 5.13.3. Inventory games: All instances of the following form: "$c^2(i), c^2(S), d^2(j) \ldots$", should be replaced, respectively, with "$c(i)^2, c(S)^2, d(j)^2 \ldots$".

Page 283, line -9: Replace "optimal ordering cost" with "optimal ordering size".

Page 285, line 18: Replace "$m \in \mathbb{R}^m$" with "$m \in \mathbb{R}^N$".

Page 285, line 20: Replace "$S \subset \mathbb{N}$" with "$S \subset N$".

Page 293, line 6: Replace "ecole" with "école" and "Enonometrica" with "Econometrica".

Page 296, line 21: Replace "Mathematizues" with "Mathematiques" and "Theorie" with "Théorie".

Page 299, line -12: Replace "Brower" with "Brouwer".

Page 307, line 1: S. Tijs is missing in the reference. The list of authors should read "van Gellekom, J. R. G., Potters, J. A. M., Reijnierse, J. H., Engel, M. C. and Tijs, S." Accordingly, on page 313, the number 277 should be added to Tijs's entries.

Proof of Case 1.2 in Theorem 5.11.2.

Case 1.2: there is $i \in N$, such that $f_i^{\mathrm{TR}}(E,d) = d_i/2$. If $|N| = 2$ and $d_1 = E$, we can proceed as in Case 1.1. Now, we consider the remaining cases. Let $T = \{j \in N : f_j^{\mathrm{TR}}(E,d) = d_j/2\}$. If $T = N$, then, for each $j \in T \backslash \{n\}$,

$$\begin{array}{rcl} E - d_j & = & \sum_{i \in T} \frac{d_i}{2} - d_j \\ & = & \sum_{i \in T \backslash \{j,n\}} \frac{d_i}{2} + \frac{d_n}{2} - \frac{d_j}{2} > 0. \end{array}$$

On the other hand, if $T \subsetneq N$, then $n \notin T$ and, for each $j \in T$,

$$\begin{array}{rcl} E - d_j & > & \sum_{i \in T} \frac{d_i}{2} + (n - |T|)\frac{d_{|T|}}{2} - d_j \\ & = & \sum_{i \in T \backslash \{j\}} \frac{d_i}{2} + (n - |T| - 1)\frac{d_{|T|}}{2} + \frac{d_{|T|}}{2} - \frac{d_j}{2} \geq 0. \end{array}$$

Thus, no matter whether $T = N$ or $T \subsetneq N$, we have that, for each $j \in T \backslash \{n\}$, $E - d_j > 0$; hence, $v(N \backslash \{j\}) = E - d_j$ and $e(N \backslash \{j\}, f^{\mathrm{TR}}(E,d)) = -f_j^{\mathrm{TR}}(E,d) = e(\{j\}, f^{\mathrm{TR}}(E,d))$.

Let $x \in I(v)$ be such that $x \neq f^{\mathrm{TR}}(E,d)$. Let $j \in N$ be the smallest index with $x_j \neq f_j^{\mathrm{TR}}(E,d)$. Since $\sum_{i \in N} x_i = \sum_{i \in N} f_i^{\mathrm{TR}}(E,d) = E$, we have $j < n$. Take a nonempty coalition $S \subsetneq N$. We show now that, if $e(S, f^{\mathrm{TR}}(E,d)) > e(\{j\}, f^{\mathrm{TR}}(E,d))$, then $e(S, f^{\mathrm{TR}}(E,d)) = e(S,x)$. Suppose that $e(S, f^{\mathrm{TR}}(E,d)) > e(\{j\}, f^{\mathrm{TR}}(E,d))$. If $v(S) = 0$, then

$$-f_j^{\mathrm{TR}}(E,d) = e(\{j\}, f^{\mathrm{TR}}(E,d)) < e(S, f^{\mathrm{TR}}(E,d)) = -\sum_{i \in S} f_i^{\mathrm{TR}}(E,d).$$

Now, given $i \in S$ we have $f_i^{\mathrm{TR}}(E,d) < f_j^{\mathrm{TR}}(E,d)$ and, by the definition of the Talmud rule, $i < j$. Thus, for each $i \in S$, $x_i = f_i^{\mathrm{TR}}(E,d)$. Therefore, $e(S, f^{\mathrm{TR}}(E,d)) = e(S,x)$. If $v(S) > 0$, then

$$\begin{array}{rcl} -f_j^{\mathrm{TR}}(E,d) & = & e(\{j\}, f^{\mathrm{TR}}(E,d)) < e(S, f^{\mathrm{TR}}(E,d)) \\ & = & -\sum_{i \notin S}(d_i - f_i^{\mathrm{TR}}(E,d)) \leq -\sum_{i \notin S} f_i^{\mathrm{TR}}(E,d), \end{array}$$

where the last inequality holds because, for each $i \in N$, $f_i^{\mathrm{TR}}(E,d) \leq d_i/2$. Arguing as above we have that, for each $i \notin S$, $i < j$. Thus, for each $i \notin S$, $x_i = f_i^{\mathrm{TR}}(E,d)$ and so $e(S, f^{\mathrm{TR}}(E,d)) = e(S,x)$.

Since $e(S, f^{\mathrm{TR}}(E,d)) > e(\{j\}, f^{\mathrm{TR}}(E,d))$ implies that $e(S, f^{\mathrm{TR}}(E,d)) = e(S,x)$, to show that $x \succ_L f^{\mathrm{TR}}(E,d)$ it suffices to find a coalition $\bar{S}$ such that $e(\bar{S},x) > e(\bar{S}, f^{\mathrm{TR}}(E,d)) = e(\{j\}, f^{\mathrm{TR}}(E,d))$. We distinguish several cases. First, if $x_j < f_j^{\mathrm{TR}}(E,d)$, then

$$e(\{j\},x) = -x_j > -f_j^{\mathrm{TR}}(E,d) = e(\{j\}, f^{\mathrm{TR}}(E,d)).$$

Second, if $x_j > f_j^{\mathrm{TR}}(E,d)$ and $j \in T$, then

$$\begin{aligned} e(N\backslash\{j\},x) = -(d_j - x_j) &> -(d_j - f_j^{\mathrm{TR}}(E,d)) = e(N\backslash\{j\}, f^{\mathrm{TR}}(E,d)) \\ &= -f_j^{\mathrm{TR}}(E,d) = e(\{j\}, f^{\mathrm{TR}}(E,d)). \end{aligned}$$

Finally, if $x_j > f_j^{\mathrm{TR}}(E,d)$ and $j \in N\backslash T$, then $|N\backslash T| \geq 2$ and there is some $l \in N\backslash T$ with $x_l < f_l^{\mathrm{TR}}(E,d) = f_j^{\mathrm{TR}}(E,d)$; in particular, $l \geq j$. Now, by the definition of T and using that, for each $i \notin T$, $d_i \geq d_{|T|+1}$, we have

$$\begin{aligned} v(n) &= E - \sum_{i \in N\backslash\{n\}} d_i \leq \sum_{i \in T} \frac{d_i}{2} + (n - |T|)\frac{d_{|T|+1}}{2} - \sum_{i \in N\backslash\{n\}} d_i \\ &= -\sum_{i \in T} \frac{d_i}{2} - \sum_{i \in N\backslash(T \cup \{n-1,n\})} (d_i - \frac{d_{|T|+1}}{2}) - (d_{n-1} - d_{|T|+1}) \leq 0. \end{aligned}$$

Thus, $v(n) = 0$. Then, also $v(l) = 0$ and we have

$$e(\{l\},x) = -x_l > -f_l^{\mathrm{TR}}(E,d) = -f_j^{\mathrm{TR}}(E,d) = e(\{j\}, f^{\mathrm{TR}}(E,d)).$$

Therefore, we have shown that $\eta(v) = f^{\mathrm{TR}}(E,d)$. $\square$