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- p. 23, l. -10 says “Borel measure”, should say “complex Borel measure”.
- p. 55, l. -11 says “ $P_t(x) = \frac{1}{\pi} \frac{x}{t^2+x^2}$ ”, should say “ $P_t(x) = \frac{1}{\pi} \frac{t}{x^2+t^2}$ ”; l. -9 says “ $P(x) = \frac{1}{\pi} \frac{x}{t^2+x^2}$ ”, should say “ $P(x) = \frac{1}{\pi} \frac{1}{x^2+1}$ ”.
- p. 56, l. 7 says “ $e^{2\pi it/T}$ ”, should say “ $e^{2\pi ikt/T}$ ”; l. -10 says “ $\lim_{\lambda \rightarrow \infty} \int_{\delta}^{T-\delta} |K_{\lambda}(x)| dx = 0$ for all $0 < \delta < T$ ”, should say $\lim_{\lambda \rightarrow \infty} \int_{\delta < |x| < T/2} |K_{\lambda}(x)| dx = 0$ for every $0 < \delta < T/2$ ”.
- p. 63, l. 5, 6, 7 and 8 should be “ $h = f - g$, and $s_k^2 = s_k - s_k^1$. By dominated convergence, $s_k \rightarrow f$, $s_k^1 \rightarrow g$, and $s_k^2 \rightarrow h$ in in L^p , L^{p_0} and L^{p_1} respectively.
- Then $Ts_k^1 \rightarrow Tg$ in L^{p_0} and $Ts_k^2 \rightarrow Th$ in L^{p_1} , and, by taking subsequences if necessary, also $Ts_k \rightarrow Tf$ a.e.”.
- p. 80, l. 8 says “ $\sum_{n=1}^{k+1} 2^{-n}$ ”, should say “ $\sum_{n=1}^{k+1}$ ”.
- p. 95, l. -6 says “ $z(H)$ ”, should say “ $T(H)$ ”.
- p. 98, l. 10 says “ $= \frac{S}{4}$ ”, should say “ $= \frac{S}{2}$ ”.
- p. 103, l. 2 to 5, better: “If $f = \text{sgn}(h)\chi_A$, then $v(f) \geq (\|v\|_{(L^1)'} + \varepsilon)\mu(A)$ and, since also $v(f) \leq \|v\|_{(L^1)'} \|f\|_1 = \|v\|_{(L^1)'} \mu(A)$, we arrive to a contradiction.”.
- p. 106, l. -8 and -7 say “ $(1/|t|)v(|t|z \pm y) \leq (1/|t|)q(|t|z \pm y)$ ”, should say “ $|t|v(|t|^{-1}z \pm y) \leq |t|q(|t|^{-1}z \pm y)$ ”.
- p. 110, l. -6 says “ $E \setminus F$ ”, should say “ $E \setminus \bar{F}$ ”.
- p. 111, l. -5 says “Theorem 4.20 and Theorem 4.21,”, should say “Theorem 4.20,”.
- p. 112, l. -5 says “closed subspace”, should say “subspace”.
- p. 113 Theorem 4.24 says
 “Suppose E is a locally convex space. The closed linear span \overline{A} of a subset A of E coincides with A^{oo} , the annihilator in E of $A^o \subset E'$, so that A is total in E if and only if $A^o = \{0\}$.
 Thus, a vector subspace F of E is closed if and only if $F^{oo} = F$.”
 Better say:
 “Suppose E is a locally convex space. The closed linear span \overline{A} of a subset A of E coincides with A^{oo} , the annihilator in E of $A^o \subset E'$. Thus, a vector subspace F of E is closed if and only if $F^{oo} = F$, and A is total in E if and only if $A^o = \{0\}$.”.
- p.128, l. 4 says “We have a similar situation with”, should say “Note that $E' \subset \mathbf{K}^E$ and the weak* convergence of a sequence in E' is the restriction of”.
- p.128, l. 5, 8 and 10 say “ \mathbf{C}^X ”, should say “ \mathbf{K}^E .”; suppress lines 6,7 and 8; before Theorem 5.1, add the following paragraph:
 “By defining $\hat{x}(u) := u(x)$, we obtain an injection $x \in E \hookrightarrow \hat{x} \in \mathcal{E}$ that allows us to also consider (\mathcal{E}, E) as a dual couple. In this way, if E is a locally convex space, then (E', E) is also a dual couple.”
- p. 133, l. -4 says “ $b^2 + 2xb \leq 0$ ”, should say: “ $b^2 + 2xb \leq 1$ ”.
- p. 146, l. 6 says “ $f * \rho \in \mathcal{E}(\mathbf{R}^n)$ and $\text{supp}(f * \rho) \subset K + \bar{B}(0, \delta) = K(\delta)$ ”, should say: “ $\varphi * \rho \in \mathcal{E}(\mathbf{R}^n)$ and $\text{supp}(\varphi * \rho) \subset K + \bar{B}(0, 2\delta) = K(2\delta)$ ”.
- p. 149, l. 16 says “ Ru is a distribution on Ω_1 , and $R :$ ”, should say “ $T'u$ is a distribution on Ω_1 , and $T' :$ ”.
- p. 151, l. 10 says “adjoint”, should say “transpose”; l. -3 says “ $f \in \mathcal{E}^1(\Omega)$ ”, should say “ $f \in \mathcal{E}(\Omega)$ ”.
- p. 160, l. 4 says “ $u(\varphi) =$ ”, should say “ $u(\tilde{\varphi}) =$ ”.
- p. 165, l. 6, 10 and 13 say “ Δ^* ”, should say “ E_n^* ”; l. -10 says “ LE_n ”, should say “ ΔE_n ”.
- p. 170, l. 8, 10 and 11 say “ $\int_a^{\xi^-} - \int_{\xi^+}^b$ ”, should say “ $\int_a^{\xi^-} + \int_{\xi^+}^b$ ”. Also, write $-qG_{\xi}$ instead of qG_{ξ}
- p. 192, l. 7 and 9 say $e^{2k\pi ia \cdot x}$, should say $e^{2\pi ia \cdot x}$; suppress l. -6 and l. -5, and translate the label of footnote 4; before subsection 7.3.2, add **Remark 7.13**. If $\varphi \in \mathcal{S}(\mathbf{R}^n)$ and $u \in \mathcal{S}'(\mathbf{R}^n)$, then the convolution $(u * \varphi)(x) := \langle \tau_x \tilde{\varphi}, u \rangle$ is a well-defined function $u * \varphi \in \mathcal{S}'(\mathbf{R}^n)$ and $\widehat{u * \varphi} = \widehat{\varphi} \widehat{u}$. Cf. Exercise 7.26
- p. 201, l. 8 says “ $P_t(x) = \frac{1}{\pi} \frac{x}{t^2+x^2}$ ”, should say “ $P_t(x) = \frac{1}{\pi} \frac{t}{x^2+t^2}$ ”.

p. 204, l. -10 says “ $\varphi \in \mathcal{S}(\mathbf{R})$ ”, better say “ $\varphi \in \mathcal{D}(\mathbf{R})$ ”.

p. 205, after l. 14, add “This identity follows by applying the inversion theorem to the identity

$$\widehat{H\varphi} * \widehat{H\varphi} = \widehat{\varphi} * \widehat{\varphi} + 2m(\xi)\widehat{\varphi} * \widehat{H\varphi},$$

where $m(\xi) := -i \operatorname{sgn}(\xi)$, which is proved by noticing that the right hand side $(\widehat{\varphi} * \widehat{\varphi})(\xi) + 2m(\xi)(\widehat{\varphi} * \widehat{H\varphi})(\xi)$ is

$$\int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta) d\eta + 2m(\xi) \int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta)m(\eta) d\eta$$

or

$$\int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta) d\eta + 2m(\xi) \int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta)m(\xi - \eta) d\eta.$$

The average of these expressions is

$$(\widehat{\varphi} * \widehat{\varphi})(\xi) + 2m(\xi)(\widehat{\varphi} * \widehat{H\varphi})(\xi) = \int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta)[1 + m(\xi))(m(\eta) + m(\xi - \eta))] d\eta,$$

and $1 + m(\xi)(m(\eta) + m(\xi - \eta)) = m(\eta)m(\xi - \eta)$. So, $\int_{\mathbf{R}} \varphi(\eta)\varphi(\xi - \eta)m(\eta)m(\xi - \eta) d\eta = (\widehat{H\varphi} * \widehat{H\varphi})(\xi)$. ”

p. 208, l. -14 and l. -13 say “ 4π ”, should say “ $4\pi^2$ ”.

p. 211, l. 8 and 11, three places, say “ L^p ”, should say “ L^2 ”; l. -11 says “for functions”, should say “for real functions”.

p. 219, l. 14, says “ $((-\Delta)^{-1}\varphi, \psi)_2 = (\nabla\varphi, \nabla\psi)_2 = (\varphi, \psi)_D = (\varphi, (-\Delta)^{-1}\psi)_2$ ”, should say “ $(\varphi, (-\Delta)^{-1}\psi)_2 = ((-\Delta)^{-1}\varphi, (-\Delta)^{-1}\psi)_D = (((-\Delta)^{-1}\psi, (-\Delta)^{-1}\varphi)_D = ((-\Delta)^{-1}\varphi, \psi)_2$ ”; l. -7 says “since $\Delta : H_0^1(\Omega) \rightarrow L^2(\Omega)$ is injective”, better say “since Δ is injective on $H_0^1(\Omega)$ ”.

p. 221, l. -6, -5, -3 and -2, four places, say “ H^{t+1} ”, should say “ H^{t-1} ”; l. -2 and -1, say “ H^{t-1} ”, should say “ H^{t+1} ”.

p. 224, add

“**Exercise 7.26** Suppose that $x \in \mathbf{R}^n$, $\varphi \in \mathcal{S}(\mathbf{R}^n)$ and $u \in \mathcal{S}'(\mathbf{R}^n)$, and denote $(u * \varphi)(x) := \langle \tau_x \tilde{\varphi}, u \rangle$. Prove the following facts:

(a) $(u * \varphi)(x)$ is continuous with respect to u in $\mathcal{S}'(\mathbf{R}^n)$, with respect to φ in $\mathcal{S}(\mathbf{R}^n)$, and with respect to x in \mathbf{R}^n .

(b) $u * \varphi$ is a tempered distribution.

(c) If $\psi \in \mathcal{D}(\mathbf{R}^n)$, $\langle \psi, u * \varphi \rangle = u\left(\int_{\mathbf{R}^n} \psi(x)(\tau_x \tilde{\varphi})(\cdot) dx\right)$. Extend this identity to the case $\psi \in \mathcal{S}(\mathbf{R}^n)$.

(d) The mapping $(u, \varphi) \in \mathcal{S}'(\mathbf{R}^n) \times \mathcal{S}(\mathbf{R}^n) \mapsto u * \varphi \in \mathcal{S}'(\mathbf{R}^n)$ is continuous with respect to u and φ .

(e) $\widehat{u * \varphi} = \widehat{\varphi} \widehat{u}$.

(f) The identities of Theorem 7.23 hold for every $\varphi \in \mathcal{S}(\mathbf{R}^n)$.

p. 300, l. -1 says “first part.”, should say “first part if $\lambda_k \geq 0$; if not, consider $\lambda_k = \lambda_k^+ - \lambda_k^-$ ”.

p. 304, Exercise 3.5 says “Use the open mapping theorem”, should say “Show that $A\|f^{(n)}\|_1 \leq \|f^{(n)}\|_p \leq B\|f^{(n)}\|_\infty$ and $|f^{(n)}(x)| \leq |\int_a^x f^{(n+1)}| \leq \|f^{(n+1)}\|_1$ ”.

p. 316, after l. 7, add

“Exercise 7.26. To prove (b), check that $|(u * \varphi)(x)| \leq C(1 + |x|^2)^N$ ” by proving an estimate $q_N(\tau_x \varphi) \leq 2^N(1 + |x|^2)^N q_N(\varphi)$. To prove (e), show that, for any $\widehat{\psi} \in \mathcal{D}(\mathbf{R}^n)$, $\langle \widehat{\psi}, \widehat{u * \varphi} \rangle = \langle \widehat{\psi}, \widehat{\varphi} \widehat{u} \rangle$.