

Toric Varieties

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Errata for the first printing as of November 20, 2011

Page 70, line –1: Delete this line and replace it with the following:

We also have the following result about the Hilbert basis of $C(P) \cap (M \times \mathbb{Z})$.

Page 71, lines 1–7: Delete these lines and replace them with the following:

Lemma 2.2.16. Let $P \subseteq M_{\mathbb{R}} \simeq \mathbb{R}^n$ be a lattice polytope of dimension $n \geq 2$ and let k_0 be the maximum height of an element of the Hilbert basis of $C(P)$. Then

$$k_0 \leq n - 1.$$

Proof. This follows easily from (2.2.3). □

The Hilbert basis of the simplex P of Example 2.2.15 has maximum height 2 by Lemma 2.2.16. The paper [187] gives a version of Lemma 2.2.16 that applies to Hilbert bases of more general cones. See also Exercise 2.2.9.

Page 74, Exercise 2.2.9: Delete the entire exercise and replace it with the following:

2.2.9. Let $\text{Cone}(\mathcal{A}) \subseteq M_{\mathbb{R}} \simeq \mathbb{R}^n$ be a full dimensional strongly convex cone and let \mathcal{H} be the Hilbert basis of $\text{Cone}(\mathcal{A}) \cap M$. Write $\mathcal{A} = \{m_1, \dots, m_s\} \subset M$, where each m_i is nonzero and primitive. Following [187], the *height* of $m \in \text{Cone}(\mathcal{A}) \cap M$ is defined to be

$$h(m) = \max \left(\sum_{j=1}^n \lambda_j \mid m = \sum_{j=1}^n \lambda_j m_{i_j}, m_{i_1}, \dots, m_{i_n} \text{ linearly independent} \right).$$

The main result of [187] states that if $n \geq 3$, then $h(m) < n - 1$ for all $m \in \mathcal{H}$.

(a) Let $C(P) \subseteq M_{\mathbb{R}} \times \mathbb{R}$ be the cone from Lemma 2.2.16. Prove that $h((m, k)) = k$ for all $(m, k) \in \text{Cone}(P) \cap M \times \mathbb{Z}$.

(b) Explain how Lemma 2.2.16 follows from the result of [187].

Page 113, line –3: “(that is, X_{Σ} has” should be “(i.e., X_{Σ} has”

Page 113, line –2: Add the following new sentence at the end of the existing line: “We say that X_{Σ} is *simplicial* in this case.”

Page 115, line –4: “the *relative interior* of” should be “the relative interior of”

Page 136, line 2 of Exercise 3.3.5: “property that” should be “such that $\overline{\phi}(\hat{\sigma} \cap N) = \sigma' \cap N'$ and”

Page 137, line 3 of part (b) of Exercise 3.3.7: “ $\overline{N''}$ ” should be “ $\overline{\phi}^{-1}(N'')$ ”

Page 138, line 1 of Exercise 3.3.12: “Consider the fan” should be “Consider the complete fan”

Page 184, line –10: “ $-\sum_{\rho} \varphi_D(u_{\rho})D_{\rho}$ ” should be “ $-\sum_{\rho} \varphi(u_{\rho})D_{\rho}$ ”

Page 187, part (a) of Exercise 4.2.7: “ $O(\sigma) =$ ” should be “ $V(\sigma) = \overline{O(\sigma)} =$ ”

Page 188, line 2 of Exercise 4.2.9: “instead use” should be “use”

Page 188, line 3 of Exercise 4.2.9: “with $a > 1$ ” should be “with a odd”

Page 224, line –1: “where $\sigma_1, \sigma_2 \in \Sigma$ are as in Example 5.1.16” should be “where $\sigma_i = \text{Cone}(u_0, u_i)$ for u_0, u_1, u_2 as in Example 5.1.16”

Page 226, line 2 of 5.2.6: Add the new sentence: “Let σ_1, σ_2 be as in Example 5.2.11.”

Page 228, line 2 of proof of Lemma 5.3.5: “for all ρ ” should be “for all $\rho \in \sigma(1)$ ”

Page 249, third display: “ $\text{Hom}_{\mathcal{O}_X(U)}(\mathcal{F}(U), \mathcal{G}(U))$ ” should be “ $\text{Hom}_{\mathcal{O}_U}(\mathcal{F}|_U, \mathcal{G}|_U)$ ”

Page 276, line 1 of Exercise 6.1.2: “(b) \Rightarrow (d)” should be “(b) \Rightarrow (f)”

Page 309, Exercise 6.4.2: In two places, “Example 6.4.2” should be “Example 6.4.6”

Page 331, line –1: “ $k \geq n$ ” should be “ $k \geq n - 1$ ”

Page 343, line 1: “rays in P ” should be “rays in Q ”

Page 379, lines 2 and 3 of Exercise 8.2.14: Replace the sentence beginning “In the discussion” with “Let $P \subseteq M_{\mathbb{R}}$ be a lattice polytope containing the origin as an interior point. In the discussion of the algebra (8.2.5), we saw that P gives the cone”

Page 382, part (a) of Lemma 8.3.6: “common edge” should be “common facet”

Page 384, line –14: “most more recent” should be “most recent”

Page 385, line 4: “contains exactly two vertices” should be “contains exactly two lattice points”

Page 398, line 3 of §9.1: “torus-invariant \mathbb{Q} -Cartier divisors” should be “torus-invariant Weil divisors”

Page 398, line –13: “write these as σ_i and” should be “write these as $\sigma_1, \dots, \sigma_\ell$ and”

Page 398, line –11: “torus-invariant Cartier divisor” should be “torus-invariant Weil divisor”

Page 410, line 10: “ $\langle m, v \rangle < \varphi_D(u)$ ” should be “ $\langle m, v \rangle < \varphi_D(v)$ ”

Page 420, line 1 of part (b) of Exercise 9.2.12: “ σ is contained” should be “ Δ_σ is contained”

Page 450, line –7: “ $\text{Ext}_S^{p+1}(S/B(\Sigma)^{[k]}, S)$ ” should be “ $\text{Ext}_S^{p+1}(S/B(\Sigma)^{[k]}, S)_\alpha$ ”

Page 477, part (c) of Theorem 10.2.12: “(resp. Θ_0)” and “(resp. even)” should be “(resp. Θ_0)” and “(resp. even)”

Page 496, line –10: “ $b_1 > 2$ ” should be “ $b_1 \geq 2$ ”

Page 496, line –9: “ $b_1 < 2$ ” should be “ $b_1 \leq 2$ ”

Page 501, line 2 of Exercise 10.4.2: “ $b_1 < 2$ ” should be “ $b_1 \leq 2$ ”

Page 529, line –5: “dimension ≥ 3 ” should be “dimension ≥ 4 ”

Page 705, fourth bullet of Example 14.3.8: “ $(\mathbb{C}^3)^{\text{ss}}$ ” and “ $(\mathbb{C}^3)^{\text{s}}$ ” should be “ $(\mathbb{C}^3)_X^{\text{ss}}$ ” and “ $(\mathbb{C}^3)_X^{\text{s}}$ ”

Page 797, bottom: Add the following new paragraph:

New packages and updates to old packages (including new URLs) can be found at <http://www.cs.amherst.edu/~dac/toric.html>.

Page 817, reference [2]: “K. Matsuk” should be “K. Matsuki”

Page 819, reference [40]: “vectors bundles” should be “vector bundles”

Page 819, reference [56]: “W. Brun” should be “W. Bruns”

Page 839, index entry for *simplicial toric variety*: “180” should be “113, 180”