ADDITIONS AND CORRECTIONS FOR
CLASSICAL METHODS IN ORDINARY DIFFERENTIAL EQUATIONS
BY S. P. HASTINGS AND J. B. MCLEOD

1 Chapter 1

In her book *Differential Equations: Introduction and Qualitative Theory,* Professor Jane Cronin shows how a short proof of the existence theorem for initial value problems when there is no uniqueness can be given using Schauder's fixed point theorem. This is a place where the functional analytic approach is very useful in proving a basic result. Pedagogically however, this method has the drawback that most people learn ode's before functional analysis, and certainly before seeing a proof of Schauder's theorem. Perhaps presentation of the existence result assuming this theorem would encourage students to study nonlinear functional analysis.

On the other hand, the usual proof of this result uses Euler's method for numerical solutions, also an important technique. As Cronin points out, the constructive nature of the classical proof leads to an approximation to a solution, at least in a theoretical sense, while Schauder's theorem is non-constructive.

2 Chapter 2

page 8, 7th line after (2.1): insert "Recall that $x(t,\alpha)$ is continuous in both variables wherever it is defined."

page 10, just before section 2.2.1: An alternative formulation of what we call the shooting method was given in the text "Differential Equations, a Dynamical Systems Approach", by J. Hubbard and B. West (Springer, 1991). Their technique of "funnels" and "anti-funnels" leads to a similar proof of the result we gave for equation (2.1), and probably could be applied to some other problems discussed in this book as well. We thank Professor Hubbard for calling this to our attention.

page 17. The proof in section 2.3.3 is incomplete. In the last paragraph the assertion in the second sentence is not justified a priori from the definitions of A and B. We **can** assert a priori that $y \geq 0$ and $y' \leq 0$. But $y = 0$ is not possible because of the argument after (2.17), while if $y' = 0$ and $y > 0$ then $y'' > 0$ and immediately thereafter, $y' > 0$, so $\alpha \in B$.

page 19. 5th line below (2.23) This equation should be

$$w' = \frac{\varepsilon}{c}(u - \gamma w).$$

Fortunately, we do not consider this equation further in this chapter, and when it becomes important, in Chapter 6, it is given correctly.

page 26. In the statement of Proposition 2.5, there was a change in notation from earlier in this section which was not sufficiently highlighted. The notation (u_c, v_c) is defined in part (C) as the unique solution on $\mathcal{U}_c^+$ such that $u(0) = a$. It was shown earlier that for $c \geq 0$ every solution on $\mathcal{U}_c^+$ crosses the line $u = a$, so this is a shift of the independent variable in this autonomous system. This notation is also used in Chapter 6.

3 Chapter 3

4 Chapter 4

5 Chapter 5

page 65, beginning of 5.2. Although probably not related to any of the problems studied in this book, an interesting result in which uniqueness is shown to imply existence was pointed out to us by H. Gebran. This is Theorem 1 in the paper "On the existence and uniqueness of solutions of a boundary value problem for an ordinary second-order differential equation", by A. Lasota and Z Opial, *Colloquium Mathematicum* **XVIII** (1967), 1-5.

page 69, three lines above (5.14): insert ", where $F_i(x) = \int_0^x f_i(s)\,ds$," before the word "for" on this line.

6 Chapter 6

pg. 80. At the bottom of the page, it should have been emphasized that the definition of $(u_{c,\varepsilon}, v_{c,\varepsilon}, w_{c,\varepsilon})$ is different from the usage in most of Chapter 2, until the statement of Proposition 2.5. See the correction above for page 26.

7 Chapter 7

Section 7.8, pgs. 138-139. It has been pointed out by S. Schechter that we are in error in suggesting that the method will work whenever an equilibrium point is non-hyperbolic, and in particular, that it will work on example 7.91, with enough repetitions of blow-up. It was stated in the first paragraph of this section that the method is sometimes used in conjunction with center manifold analysis, and indeed, this example requires the center manifold. When we arrive at a system with exactly one zero eigenvalue, and no other with zero real part, the center manifold theorem then supplies the details not found from linearization. A comprehensive discussion can be found in the book "Qualitative behavior of planar systems" by Dumortier, Libre, and Artis, Springer, 2006.

Also, though the modern use of the method appears to have been started by Takens (cited in the book), it goes back at least to Bendixson, Acta Mathematica **25** (1901), 1-88.

8 Chapter 8

9 Chapter 9

10 Chapter 10

11 Chapter 11

12 Chapter 12

13 Chapter 13

14 Chapter 14

page 248. Add condition $u = 0$ on $\partial\Omega$ to (14.34) (as in (14.36)).

pages 255-256 (Section 14.6) Add a citation to Littlewood:

Littlewood, J. E., Unbounded solutions of an equation $\ddot{y}+g(y) = p(t)$, with $p(t)$ periodic and bounded and $\frac{g(y)}{y} \to \infty$ as $y \to \pm\infty$, *J. Lond. Math. Soc.* **41 (1966), 497-507.**

In this paper Littlewood shows that if p can be discontinuous then there can be unbounded solutions, so some regularity of p is required for the result ascribed to Levi on page 256. The error is not Levi's – see his paper [148] in our bibliography for further references and details.

15 Chapter 15

16 Chapter 16

pg. 290, fifth line from bottom, change "equation" to "equations"

17 Chapter 17

18 Chapter 18

pg 326. Change caption of Figure 18.5 to "$x_0 < x^* \leq 1$"

19 Chapter 19

pg. 347. A partial solution to the suspension bridge problem was published by S. Santra and J. Wei in 2009, in the addition to the bibliography below. We did not discover this reference until after the book appeared. Existence is proved for the range $0 < \sqrt{q} < .7427...$ using modern methods (the "mountain pass" lemma and "Morse Index"). The authors believe that the method could be refined to include the full range $0 < q < 2$, but as far as we know this has not yet been done. We are unaware of progress using classical methods.

pg. 352. Troy's transformation can be motivated by considering a standard Euler equation

$$w'' + \frac{a}{s}w' + \frac{b}{s^2}w'' = 0.$$

It is common to set $w(s) = u(\log s)$, getting

$$u'' + (a-1)\,y' + bu = 0.$$

Applying the same transformation to (19.15) gives

$$u'' + u' + e^{2s}u^3 = 0.$$

If we then try $u(t) = e^{\alpha t}\gamma(t)$, we find that setting $\alpha = -1$ gives the autonomous equation (19.17).

20 Bibliography

Add:the following:

S. Santra and J. Wei, Homoclinic solutions for fourth order traveling wave equations, *Siam J. Math. Anal.* **41** (2009), 2038-2056.

A. Lasota and Z. Opial, On the existence and uniqueness of solutions of a boundary value problem for an ordinary second-order differential equation, *Colloquium Mathematicum* **XVIII** (1967), 1-5.

J. E. Littlewood, Unbounded solutions of an equation $\ddot{y} + g(y) = p(t)$, with $p(t)$ periodic and bounded and $\frac{g(y)}{y} \to \infty$ as $y \to \pm\infty$, *J. Lond. Math. Soc.* **41 (1966), 497-507.**

21 Index

Add page 351 to index entries for Troy

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