ERRATA TO "SEMICLASSICAL ANALYSIS" BY M ZWORSKI

Many thanks to **Plamen Stefanov, Fréderic Klopp, Long Jin and Minjae Lee** for pointing out errors and misprints, and for suggesting solutions.

- page 43, the displayed formula of step 4 of the proof should read
$$h^k J(0, P^k u) = 2\pi P^k u(0) = 2\pi (h\epsilon/2i)^k u^{(2k)}(0).$$
- page 47, Lemma 3.14: The constant $C$ depends also on $\varphi$.
- page 57, EXAMPLES: quantization is applied formally here as the definitions have been so far given only for symbols in $\mathscr{S}$; Theorem 4.1 below justifies the use of more general class of symbols.
- page 58, step 2 should read: "The kernel of $\mathrm{Op}_t(a)^*$ is $K_t^*(x,y) := \overline{K}_t(y,x)$, which is the kernel of $\mathrm{Op}_{1-t}(\bar{a})$"
- page 60, line 5 from the bottom: $c_j(\frac{x+y}{2}, \xi)$ should be $c_j(\frac{x+y}{2})$ ($c_j$ is independent of $\xi$).
- page 78: in Step 2 the sentence should be "... the method of stationary phase and Theorem 4.8 give..."
- page 80, (4.4.18) should be
$$a\#b = ab + \frac{h}{2i}\{a, b\} + O_{S_\delta(m_1 m_2)}(h^{1-2\delta}),$$
that is, $i/2h$ should be $h/2i$.
- page 111: the last line in the second displayed formula in Step 2 should be
$$\geq \frac{1}{2}(\operatorname{Im}\tau)^2 \|u\|_{L^2}^2.$$
(a square is missing in the book)
- page 116: the estimate (5.3.31) is incorrect as stated. To obtain the correct version, we return to (5.3.30) and note that pole of $P(\tau)^{-1}$ at 0 is simple and the image of the residue, $A$, is $\mathbb{C}$ (constant functions). Hence,
$$\check{u}_1(\tau) = \check{u}_0(\tau) + \check{u}_2(\tau),$$
$$\check{u}_0(\tau) := c_1 \int_{\mathbb{T}^n} \check{g}_1(\tau, x)\,dx/\tau, \quad \check{u}_2(\tau) := (P(\tau)^{-1} - A/\tau)\check{g}_1(\tau),$$

where $Q(\tau) := P(\tau)^{-1} - A/\tau$ satisfies the same estimates as $P(\tau)^{-1}$ but is holomorphic near 0. We note that $u_0 = c_0$ for $t > 0$.

We can now replace $u_1$ with $u_2$ as the constant term does not affect the energy estimate. Another way to look at this is changing the initial conditions from $u|_{t=0} = 0$, $\partial_t u_{t=0} = f$, to $u|_{t=0} = -c_0$, $\partial_t u_{t=0} = f$. This does not change energy $E(t)$.

In Step 3 we now have, for $u_2$, which has the same energy of $u_1$,

$$\begin{aligned} \|e^{\beta t} u_2\|_{L^2(\mathbb{R}_+;H^1)} &= (2\pi)^{-\frac{1}{2}} \| \widehat{e^{\beta t} u_2} \|_{L^2(\mathbb{R};H^1)} \\ &= (2\pi)^{-\frac{1}{2}} \|\check{u}_2(\cdot - i\beta)\|_{L^2(\mathbb{R};H^1)} \\ &= (2\pi)^{-\frac{1}{2}} \|Q(\cdot - i\beta)^{-1} \check{g}_1(\cdot - i\beta)\|_{L^2(\mathbb{R};H^1)} \\ &\leq C\|\check{g}_1(\cdot - i\beta)\|_{L^2(\mathbb{R};L^2)}. \end{aligned}$$

The remainder of the proof is the same, with $u - c_0$ in place of $u$.

- page 129, Theorem 6.7: the statement about the constant $h_0$ should be made before (i) as $0 < h < h_0$ is also required for (ii) and (iii). (The statements are actually true for all values of $h$ but that is not our concern.)
- page 130, line 5 of step 2: $K(-i, h)$ should be $K(i, h)$.
- page 135, line 3 from the bottom should read

$$\|b^{\mathrm{w}}(x, hD)\| \leq \lambda + \frac{3\epsilon}{4},$$

  for $0 < h < h(\epsilon)$. This follows, for instance from Theorem 4.30 applied to $(\lambda + \frac{3\epsilon}{4})^2 - (b^{\mathrm{w}}(x, hD))^* b^{\mathrm{w}}(x, hD)$ (see also Theorem 13.13).
- page 188, Remark: the second sentence should read "According to Theorem 8.10, if $a \in h^k S(m)$ for *some* $k \in \mathbb{R}$ and *some* order function $m$, then $T = a^{\mathrm{w}}(x, hD)$ is tempered.
- page 190: (8.4.7) should be

$$\mathrm{WF}_h\left(\exp\left(i\langle x, \omega\rangle / h^\alpha\right)\right) = \begin{cases} \mathbb{R}^n \times \{0\}, & \alpha < 1, \\ \mathbb{R}^n \times \{\omega\}, & \alpha = 1, \\ \quad \emptyset & \alpha > 1. \end{cases}$$

  that is, the division by $h^\alpha$ should be in the exponent.
- page 214, last line: $\mathrm{spt}\, \tilde{!}\chi_j$ should be $\mathrm{spt}\, \tilde{\chi}_j$.
- page 276: the first two references to **[H2]** should be to **[H3]** and the next two to **[H4]**.
- page 277, line 7: "canonical transformation" means "symplectic transformation".

- page 278, Thm 12.4: The constant $C$ in (12.2.3) depends on $P$ and the neighbourhood containing $\mathrm{WF}(u)$.
- page 279-280: In Theorem 12.5 and in its proof $p$ (which determines the flow) needs to be replaced by $p_0$. Since the proof reduces the general case to the normal form we apply the Jacobi Theorem 2.10 (the flow of $\xi_1$ becomes that of $p_0$) to make the conclusion about the general case.
- page 323: the proof in Step 2 is incorrect. First, replace (13.5.7) with

$$\|b^{\mathrm{w}}(x, hD)\|^2 \geq \langle M_{|q|^2} T_\varphi u, T_\varphi u \rangle_{L^2_\Phi} - C_0 h, \quad \|u\|_{L^2} = 1.$$

and assume that $\Phi = |z|^2/2$. If the supremum of $|q|^2$ is achieved at, say, $z = 0$ then $\partial |q|^2(0) = 0$, and for $v = (2\pi h)^{-n/2}$

$$\begin{aligned} \langle M_{|q|^2} v, v \rangle_{L^2_\Phi} &= \frac{1}{(2\pi h)^n} \int_{\mathbb{C}^n} |q(z)|^2 e^{-|z|^2/h} dm(z) \\ &= \frac{1}{(2\pi h)^n} \int_{\mathbb{C}^n} (|q(0)|^2 + O(|z|^2)) e^{-|z|^2/h} dm(z) = |q(0)|^2 + \mathcal{O}(h). \end{aligned}$$

If the supremum is not attained then $|q(z_n)|^2 \to \sup |q|^2$, where $z_n \to \infty$ . Since $|\partial^2 q| \leq C$, we have $\partial |q(z_n)|^2 \to 0$. We then choose $n$ large enough so that $|q(z_n)|^2 \geq \sup |q|^2 - h$, and $|\partial q(z_n)| \leq h$, translate $z_n$ to 0, and use the previous argument (or, directly, use $v(z) = (2/\pi h)^{n/2} (\det \Phi_{z\bar{z}})^{1/2} e^{\Psi(z, \bar{z}_n)/h}$ as a test function).
- page 347, first displayed formula in §14.2.2: $S(\mathbb{R}^n)$ should be $S(\mathbb{R}^{2n})$.