

## Errata, as of September, 2016<sup>1</sup>

- p. 9, 8dn: Change “ $F \supseteq$ ” to “ $F_\epsilon \subseteq$ ”
- p. 14, 10dn: Change “isometry” to “bijection”
- p. 17, 7up: Change “any  $\alpha \in \mathbb{R}$ ” to “any  $\alpha \in [0, \infty)$ ”
- p. 18, 4up: Change “ $C_R = 4R^3$ ” to “ $C_R = 8R^3$ ”
- p. 21, 2up: Change “a subset” to “a  $k$ -element subset”
- p. 23, 12up: Change “ $W_{2M}(2M)$ ” to “ $W_{2M}(\omega)$ ”
- p. 25, 10dn: Change “ $B_{2(M-r)}^{2(M-r)}$ ” to “ $B_{2n}^{2(M-r)}$ ”
- p. 28, eq. (1.2.29): Change “ $P(B)$ ” to “ $\mathbb{P}(B)$ ”
- p. 32, 5dn: Change “ $\prod_{n=1}^N$ ” to “ $\prod_{n=m}^N$ ”
- p. 32, 6dn: Change “(1.1.11)” to “(1.1.12)”
- p. 33, 15up: Change “ $k \in \mathbb{Z}$ ” to “ $k \in \mathbb{Z}^+$ ”
- p. 34, 2up: Change “ $\mathbb{P}_q(A) = \mathbb{P}_q$ ” to “ $\mathbb{P}_q(A) = \mathbb{P}_p$ ”
- p. 35, 12up: Change “Exercise 1.2.38” to “Exercise 1.2.40”
- p. 35, 4up: Change “ $\alpha^m \left(1 - \frac{\alpha}{n}\right)^{n-m}$ ” to “ $\left(1 - \frac{\alpha}{n}\right)^{n-m}$ ”
- p. 38, 3dn: Change “ $\zeta_k^{(1)}$ ” to “ $\zeta_{N-n}^{\{1\}}$ ”
- p. 38, 3 & 2up: Change “ $\rho^{(m-1)}$ ” to “ $\rho_N^{(m-1)}$ ”
- p. 39, 2up: Change “ $\sum_{r=0}^\infty$ ” to “ $\sum_{r=1}^\infty$ ”
- p. 40, 2dn & 4dn: Change “ $\frac{1-\sqrt{1-4pqx}}{2q}$ ” to “ $\frac{1-\sqrt{1-4pqx^2}}{2qx}$ ”
- p. 45, 11dn: Change “(1.1.6)” to “(1.1.5)”
- p. 45, 13dn: Change “(1.1.5)” to “(1.1.6)”
- p. 45, 15 & 16dn: Change “ $X \geq \alpha - \frac{1}{n}$ ” to “ $X \leq \alpha - \frac{1}{n}$ ”
- p. 46, 5dn: Change “(1.4.3)” to “(1.4.7)”
- p. 50, 2up: Change “any  $h$ ” to “any non-zero  $h$ ”

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<sup>1</sup>Most of these errors were found by Peter Landweber, to whom I am deeply indebted.

- p. 52, 12dn: Change “[ $m, m + 1$ ]” to “( $m, m + 1$ )”
- pp. 55 & 56, 1up & 3dn: Change “§ 1.1.1” to “§ 1.1.2”
- p. 57, 10up: Change “ $\mu(B) = \mu(B)$ ” to “ $\mu(B) = \mu(A)$ ”
- p. 57, 8up: Change “ $\mu(A \setminus B)$ ” to “ $\mu(B \setminus A)$ ”
- p. 58, 16up: Change “ $A \cup B = (A \cap B) \cup (B \setminus (A \cap B))$ ” to “ $A \cup B = A \cup (B \setminus (A \cap B))$ ”
- p. 61, 3up: Change “ $\sum_{A \in \mathcal{C}}$ ” to “ $\sum_{C \in \mathcal{C}}$ ”
- p. 64, 2dn: Change “let  $\epsilon > 0$ ” to “let  $F \in \mathfrak{F}(\Omega)$  and  $\epsilon > 0$ ”
- p. 64, 10 & 11dn: Change “ $\mathbb{P}(B)$ ” to “ $\tilde{\mathbb{P}}(B)$ ”
- p. 65, 10dn: Change “ $n \geq 1$ ” to “ $m \geq 1$ ”
- p. 67, 11up: Change “ $T_x^{[0,1)}\mathbb{R}$ ” to “ $T_x^{[0,1)} : \mathbb{R}$ ”
- p. 69, 14up: Change “ $\Gamma \in \mathcal{B}$ ” to “ $\Gamma \in \mathcal{B}_{\mathbb{R}}$ ”
- p. 69, 3up: Change “Borel measure” to “Borel measure on”
- p. 71, 7dn: Change “ $\mathcal{B}_R$ ” to “ $\mathcal{B}_{\mathbb{R}}$ ”
- p. 71, 15up: Change “ $\lim_{s \nearrow u}$ ” to “ $\lim_{v \nearrow u}$ ”
- p. 72, 5dn: Change “(1.4.3)” to “(1.4.6)”
- p. 72, 17up: Change display to

$$\rho(\xi, \eta) = \frac{1}{\pi} \left| \int_{\xi}^{\eta} \frac{1}{1+t^2} dt \right| = \frac{|\arctan \eta - \arctan \xi|}{\pi}$$

- p. 79, 3dn: Change “every” to “everywhere”
- p. 85, 1dn: Change “function  $f$ ” to “function  $g$ ”
- p. 88, 15dn: Change “( $E_1 \times E_2$ )” to “( $E_1 \times E_2, \mathcal{F}_1 \times \mathcal{F}_2$ )”
- p. 93, 5dn: Change “ $\mathbb{R}$ -valued functions” to “ $\mathbb{R}$ -valued  $\mu$ -integrable functions”
- p. 93, 7–8dn: Change “show ... with respect to  $\mu$ .” to “show that  $\{\varphi_n : n \geq 1\}$  is uniformly integrable with respect to  $\mu$  if and only if  $\varphi$  and all the  $\varphi_n$ ’s are  $\mu$ -integrable and  $\int |\varphi_n - \varphi| d\mu \rightarrow 0$ .”
- p. 93, 8up: Change “measures” to “measure”
- p. 93, 7up: Change “ $\int g d\nu$ ” to “ $\int f d\nu$ ”
- p. 93, 4up: Delete “unless  $\mu(E) < \infty$ ”

- p. 97, 6dn: Change “ $T_A \circ T_x = T_x \circ T_A$ ” to “ $T_A \circ T_x = T_{Ax} \circ T_A$ ”
- p. 99, 10dn: Change “ $\int_{\mathbb{S}^{N-1}} (\int_{\mathbb{S}^{N-1}} \cdots$ ” to “ $\int_{\mathbb{S}^{N-1}} (\int_{(0,\infty)} \cdots$ ”
- p. 103, 5dn: Change “ $(b_i^+ \wedge 1 - a_i^+)$ ” to “ $(b_i^+ \wedge 1 - a_i^+ \wedge 1)$ ”
- p. 103, 10dn: Change “ $[0, 1]^N$ ” to “ $\mathbb{R}^N$ ”
- p. 103, 7up: Change “the unit ball” to “Lebesgue measure of the unit ball”
- p. 104, 2dn: Change “ $dx$ ” to “ $ds$ ”
- p. 108, 8up–6up: Change “isomorphism” to “bijection.” Also, the example of lexicographic ordering should be deleted because it does not give a bijection. However, it is well known that  $\mathbb{N} \times \mathbb{N}$  is countable and therefore in one-to-one correspondence with  $\mathbb{N}$ .
- p. 110, 8dn: Change “ $\mathbb{E}^{\mathbb{P}}[Y, A_m] \mathbb{P}(A_m) = \mathbb{E}^{\mathbb{P}}[X, A_m]$ ” to “ $Y(\omega) \mathbb{P}(A_m) = \mathbb{E}^{\mathbb{P}}[X, A_m]$  for  $\omega \in A_m$ ”
- p. 110, 15up & 14up: Change “ $L^1(\mathbb{P}, \mathbb{R})$ ” to “ $L^1(\mathbb{P}; \mathbb{R})$ ” on 15up and “ $\Sigma$  measurable” to “ $\Sigma$ -measurable” on 14up
- p. 112, 10dn: Change “ $\mathbb{E}^{\mathbb{P}}[X|\Sigma]$ ” to “ $X_{\Sigma}$ ”
- p. 112, 16dn: Change “bounded” to “bounded by  $C$ ”
- p. 114, 5dn: Change “a.e.” to “a.s.”
- p. 115, 2dn: Change “ $\mathbb{E}^{\mathbb{P}}[X^p] \leq \mathbb{E}[X^p]^{1-\frac{1}{p}} \mathbb{E}^{\mathbb{P}}[Y^p]^{\frac{1}{p}}$ ” to “ $\mathbb{E}^{\mathbb{P}}[X^p] \leq \frac{p}{p-1} \mathbb{E}[X^p]^{1-\frac{1}{p}} \mathbb{E}^{\mathbb{P}}[Y^p]^{\frac{1}{p}}$ ”
- p. 115, 13dn: Change “ $\mathbb{P}(B_{k_{\ell}, \ell})$ ” to “ $\mathbb{P}(X \in B_{k_{\ell}, \ell})$ ”
- p. 119, 1dn & p. 120, 2up: Change “Exercise 1.4.19” to “Exercise 1.4.20”
- p. 121, 11 & 12dn: Change “ $f(\lambda)$ ” to “ $f_p(\lambda)$ ” once on each line.
- p. 121, 12dn: Change “ $e^{\lambda p}$ ” to “ $e^{-\lambda p}$ ”
- p. 121, 13dn: Change “ $a, b \geq 0$ ” to “ $a, b > 0$ ”
- p. 122, 10dn: Change “ $\sum_{x \in \text{Image}(X)}$ ” to “ $\sum_{y \in \text{Image}(Y)}$ ”
- p. 123, 6dn & 7dn: Change “if  $[b + c, a + d]$ ” to “if  $z \in [b + c, a + d]$ ” on 6dn and “ $[b + c, b + d]$ ” to “ $[a + d, b + d]$ ” on 7dn
- p. 124, 1up: Change “ $\sup_{n \geq 0}$ ” to “ $\sup_{n \geq 1}$ ”
- p. 125, 1up: Change “ $n^{\frac{1}{8}}$ ” to “ $n^{-\frac{1}{8}}$ ” and “ $n^{-\frac{7}{2}}$ ” to “ $n^{\frac{7}{2}}$ ”
- p. 131, 8up: Change “ever point” to “every point”
- p. 132, 8dn: Change “ $\frac{2\|f\|_{\mathfrak{u}}}{n}$ ” to “ $\frac{4\|f\|_{\mathfrak{u}}}{n}$ ”

p. 132, 15dn: This integral can be defined either as a Riemann-Stieltjes integral or as  $\int_{[0,\infty)} f d\mu_F$  where  $\mu_F$  is the measure determined by  $F$  as in §2.3.3.

p. 132, 16dn: Change “ $F : \mathbb{R} \rightarrow [0, \infty)$ ” to “ $F : [0, \infty) \rightarrow [0, \infty)$ ”

p. 133, 10up: Change “ $n^{-\alpha}$ ” to “ $n^{-\alpha^2}$ ”

p. 134, 2dn: Change “ $|S_n|$ ” to “ $S_n$ ”

p. 134, 7up: Change “§3.2” to “§3.3”

p. 136, 6dn: Change “§3.2.2” to “§3.3.2”

p. 136, 1up: Change “ $\sum_{k=1}^n$ ” to “ $\sum_{k=1}^\ell$ ” twice

p. 138, 2dn: Change “ $\mathcal{F} \times \mathcal{F}_{\mathbb{R}^n}$ ” to “ $\mathcal{F} \times \mathcal{B}_{\mathbb{R}^n}$ ”

p. 138, 4dn: Change “ $T_n = \sum_{k=1}^n X_k$ ” to “ $T_n = \sum_{k=1}^n Y_k$ ”

p. 140, 5dn: Change “ $C_b(R; \mathbb{R})$ ” to “ $C_b(\mathbb{R}; \mathbb{R})$ ”

p. 142, 15up: Change “ $\sup_{x \in \mathbb{R}} (1 + |x|^2) \sup_{\varphi \in S} |\varphi(x)|$ ” to “ $\sup_{x \in \mathbb{R}} (1 + |x|^2)^{-1} \sup_{\varphi \in S} |\varphi(x)|$ ”

p. 142, 2up: Change “ $\check{S}_n$ ” to “ $S_n$ ”

p. 145, 9dn: Change “as the” to “as”

p. 145, 2up: Change “ $\omega_{N-1}$ ” to “ $\omega_{N-2}$ ” in the factor before the final integral

p. 146, 14up: Change “the matrix” to “the”

p. 150, 1up: Change “ $f$ ” to “ $\int_\Gamma$ ” in final expression

p. 152, 9dn: Change “ $= \mathbb{R}^N$ ” to “ $= \mathbb{R}^M$ ”

p. 152, 10dn: Following “ $\tilde{X} = (X_1, \dots, X_M)$ ”, add “where  $X_{N+1}, \dots, X_M$  are mutually independent  $N(0, 1)$  random variables that are independent of  $X = (X_1, \dots, X_N)$ ”

p. 152, 13dn: Change “ $\mathbf{e}_1, \dots, \mathbf{e}_M \perp \text{Ker}(\Sigma)$ ” to “ $\mathbf{e}_1, \dots, \mathbf{e}_M$  whose span contains  $\text{Ker}(\Sigma)^\perp$ ”

p. 156, 1up: Change “non-degenerate” to “non-singular”

p. 160, 2dn: Change to

$$\begin{aligned} \rho^{(1)}(\omega) &= \inf\{n \geq 1 : W_n(\omega) = 0\} \text{ and} \\ \rho^{(m+1)}(\omega) &= \inf\{n > \rho^{(m)}(\omega) : W_n(\omega) = 0\} \text{ for } m \geq 1. \end{aligned}$$

p. 160, 12up: Change “Exercise 3.1.19” to “Exercise 3.3.19”

p. 160, 5up: Change “ $\zeta^{(k)}$ ” to “ $\zeta^{\{k\}}$ ”

- p. 160, 1up: Change “ $(-4)^{n+1}$ ” to “ $-(-4)^{n+1}$ ”
- p. 161, 3dn: Change “ $[0, \frac{1}{4}]$ ” to “ $(0, \frac{1}{4}]$ ”
- p. 161, 7dn: Change “ $0 \leq m \leq n$ ” to “ $0 \leq m \leq n$  and  $k \geq 1$ ”
- p. 161, 7up: Change “ $p < \frac{1}{2}$ ” to “ $p \leq \frac{1}{2}$ ”
- p. 162, 12 & 10up: Change “ $e^{\zeta^{\{1\}}}$ ” to “ $e^{\lambda\zeta^{\{1\}}}$ ” and “ $e^{\zeta^{\{-1\}}}$ ” to “ $e^{\lambda\zeta^{\{-1\}}}$ ”
- p. 162, 10up: Change “ $\zeta^{\{1\}} < \infty$ ” to “ $\zeta^{\{1\}} < \infty$ ”
- p. 163, 5dn: Change “ $(p \wedge q)^{m-1}$ ” to “ $(2(p \wedge q))^{m-1}$ ”
- p. 163, 18up: Change “ $u_{n-1}(m+1)$ ” to “ $u_{n-1}(m)$ ”
- p. 165, 13up: Change “ $2qe^{2\lambda}$ ” to “ $2qe^\lambda$ ” in denominator
- p. 168, 5up: Change “measure” to “measures”
- p. 169, 7up: Change “ $\mathbb{P}(X_1 \in \Gamma_1, \dots, X_n \in \Gamma_n)$ ” to “ $\mathbb{P}(X_0 \in \Gamma_0, \dots, X_n \in \Gamma_n)$ ”
- p. 172, 1dn & 8dn: Change “ $B(E; \mathcal{B})$ ” to “ $B(E, \mathbb{R})$ ”
- p. 172, 12up: Change “ $V(\nu, \nu)$ ” to “ $V(\mu, \nu)$ ”
- p. 175, 2up: Change “that fact” to “the fact”
- p. 177, 3dn: Change “ $B(E; \mathcal{B})$ ” to “ $B(E, \mathbb{R})$ ”
- p. 180, 10dn: Change “ $\sum_{k \in e}$ ” to “ $\sum_{k \in E}$ ”
- p. 181, 4dn: Change “ $E^m$ ” to “ $E^{m+1}$ ”
- p. 181, 13up: Change “written of” to “written as”
- p. 182, 6up: Change “ $\rho^{(1)} = n$ ” to “ $\rho^{(1)}(\omega) = m$ ”
- p. 183, 4, 6, 7, 12dn & 7, 4up: Change “ $\rho_i^{(i)}$ ” to “ $\rho_i^{(1)}$ ”
- p. 183, 6dn & 7, 4up: Change “ $\mathbb{E}^{\mathbb{P}_k}$ ” to “ $\mathbb{E}^{\mathbb{P}_i}$ ” several times
- p. 184, 5dn: Change “ $\sum_{m=0}^\infty$ ” to “ $\sum_{n=0}^\infty$ ”
- p. 188, 4up: Change “ $\rho^{(i)}$ ” to “ $\rho_i^{(1)}$ ”
- p. 194, 16up: Change “ $P : [0, \infty) \times E \rightarrow [0, 1]$ ” to “ $P : [0, \infty) \times E \times \mathcal{B} \rightarrow [0, 1]$ ”
- p. 196, 17up: Change “ $\sum_{m=0}^n$ ” to “ $\sum_{m=1}^n$ ”
- p. 202, 15dn: Change “(a.s. $\mathbb{P}$ )” to “(a.s.,  $\mathbb{P}$ )”
- p. 205, 17dn & p. 206, 9dn: Change “ $\sum_{n=0}^\infty$ ” to “ $\sum_{n=1}^\infty$ ”

- p. 208, 1dn: Change “ $m + \frac{\eta}{2} \geq 0$ ” to “ $(m + \frac{\eta}{2}) \wedge (m' + \frac{\eta'}{2}) \geq 0$ ”
- p. 212, 3up: Change “ $(t_1^{-1} \wedge t_j^{-1})$ ” to “ $(t_i^{-1} \wedge t_j^{-1})$ ”
- p. 219, 19up: Change “numbers of” to “number of”
- p. 223, 5dn: Change “ $e^{\alpha(t-\tau)}$ ” to “ $e^{\alpha(\tau-t)}$ ”
- p. 224, 8up: Change “Brownain” to “Brownian”
- p. 225, 2dn: Change “[9]” to “[15]”
- p. 249, 8up: Change “ $n \geq 0$ ” to “ $n \geq 1$ ”
- p. 253, 9up: Part (ii) is misstated. It should be the statement that if  $p \in (\frac{1}{2}, 1)$  and  $\lambda_p = -\log(4pq)$ , then  $\lambda_p > 0$  and  $\mathbb{E}^{\mathbb{P}_p} [e^{\lambda_p \zeta^{\{k\}}}] \leq \left(\frac{p}{q}\right)^{\frac{k}{2}}$ .
- p. 254, 2dn: Change “ $\mathbb{E}^{\mathbb{P}_p} [e^{-\lambda_p(\alpha)\zeta^{\{k\}}}]$ ” to “ $\mathbb{E}^{\mathbb{P}_p} [e^{-\ell_p(\alpha)\zeta^{\{k\}}}]$ ”
- p. 254, 4dn: Change “ $\frac{1-\sqrt{1-2pqe^{2\lambda}}}{2qe^{2\lambda}}$ ” to “ $\frac{1-\sqrt{1-2pqe^{2\lambda}}}{2qe^{\lambda}}$ ”
- p. 255, 14up: Change “a Brownian motion” to “is a Brownian motion”
- p. 265, 9dn: Change “martingale” to “martingales”
- p. 266, 10–11dn: Change “exists” to “exits”
- p. 270, 1up: Change “differential” to “differentiable”
- p. 271, 19dn: Change “ $\zeta(\omega) \leq t \implies \zeta(\omega) = \zeta(\omega^t)$ ” to “ $\zeta(\omega) \leq t \iff \zeta(\omega) = \zeta(\omega^t) \leq t$ ”