FOURIER ANALYSIS
ERRATA AND ADDITIONS

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Updated: February 8, 2008

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Errata and Additions:

All corrections and additions are given by page and line number, with negative line numbers being counted from the bottom of the page.

Error: p. xviii, l. 13 The measures ν and μ in Minkowski's inequality must be positive.

Error: p. 5, l. 3 This line should read

$$= (ge^{\pi i \cdot})^\frown(-N) - (ge^{-\pi i \cdot})^\frown(N).$$

Error: p. 22, l. 9 The Hermite functions are

$$h_n(x) = \frac{(-1)^n}{n!} \exp(\pi x^2) \frac{d^n}{dx^n} \exp(-2\pi x^2).$$

Error: p. 31, l. 11 The intervals must be open.

Error: p. 39, l. -7 The left-hand side of (2.12) should be $|E(\lambda)|$.

Addition: p. 43, l. -1 The sharp constant for the centered maximal operator when $n = 1$ is $(11 + \sqrt{61})/12$. This was shown by A. Melas (*The best constant for the centered Hardy-Littlewood maximal inequality*, Ann. of Math. **157** (2003), 647-688).

Error: p. 58, l. 4 Interchange the order of $\|b_j\|_1$ and $\sum_j$.

Error: p. 64, l. 17 The name should be "I. V. Havin" (replace "," by ".").

Error: p. 70, l. 10 The denominator needs a square root, that is,

$$u(x_1, x_2, x_3) = \int_{\mathbb{R}^2} \frac{f(y_1, y_2)}{[(x_1 - y_1)^2 + (x_2 - y_2)^2 + x_3^2]^{1/2}} \, dy_1 dy_2.$$

Error: p. 90, l. 6 Reference should read "D. R. Adams (*A note on Riesz potentials*, Duke Math)."

Error: p. 97, l. 11 Replace b^{it} with b^{-it}.

Error: p. 99, l. 14 Capitalize "Condition".

Error: p. 100, l. 10 The text should read "If $\|A'\|_\infty$ is finite, then ...".

Error: p. 104, l. 11 Delete the exponent ν on the left-hand side.

Error: p. 114, l. 22-33 The discussion of pseudo-differential operators with symbols in $S^0_{\rho,\delta}$ is incorrect and should be replaced by the following.

If T is a pseudo-differential operator with symbol in $S^m_{\rho,\delta}$, then it can again be written in the form (5.24). Operators with symbols in $S^0_{\rho,\rho}$ exhibit markedly different behavior depending on whether $\rho = 1$ or $0 < \rho < 1$. If $\sigma \in S^0_{1,1}$, then the kernel K satisfies the standard estimates (5.12), (5.13) and (5.14), but the associated operator need not be bounded on L^2. On the other hand, by a remarkable result due to A. P. Calderón and R. Vaillancourt (*A class of bounded pseudo-differential operators*, Proc. Nat. Acad. Sci. U.S.A. **69** (1972), 1185–1187) pseudo-differential operators with symbols in $S^0_{\rho,\rho}$ for some $0 \leq \rho < 1$ are bounded on L^2, but are not necessarily bounded on L^p, $p \neq 2$. Finally, if $\sigma \in S^0_{1,\delta}$, $0 < \delta < 1$, then the associated pseudo-differential operator is a Calderón-Zygmund operator and is bounded on L^p, $1 < p < \infty$. For these and related results, See Journé [**8**, Chapter 5] and Stein [**17**, Chapter 7]

Addition: p. 116, l. -8 It should be stressed that the hypotheses in Corollary 6.3 on the operator T are necessary. It is not true in general that if an operator T is uniformly bounded on all atoms in H^1 then the operator is bounded on H^1. For a counter-example and further discussion, see the article by M. Bownik (*Boundedness of operators on Hardy spaces via atomic decompositions*, Proc. Amer. Math. Soc. **133** (2005), 3535–3542).

Error: p. 129, l. 20 Delete the "," after Cruz-Uribe.

Addition: p. 130, l. 18 After the reference to the paper by Cianchi and Pick, add "and the paper by A. Fiorenza (*A summability condition on the gradient ensuring BMO*, Rev. Mat. Complut. **11** (1998), no. 2, 313–324)."

Error: p. 137, l. 2 In inequality (7.7), the exponent on the second integral should be $p-1$.

Addition: p. 154, l. 5 Change last sentence to read "For further details see the article by Jawerth cited above and C. Pérez (*Weighted norm inequalities for general maximal operators*, Conference on Mathematical Analysis (El Escorial, 1989). Publ. Mat. 35 (1991), 169–186)."

Addition: p. 156, l. -1 Add the following subsection:

5.11. More on extrapolation.

The theory of extrapolation in Section 3 can be extended in several ways. First, there is a theory of extrapolation for the A_∞ weights defined in Section 5.3. If S and T are operators such that for some $p_0 > 0$ and every weight $w \in A_\infty$,

$$\int_{\mathbb{R}^n} |Sf(x)|^{p_0} w(x)\,dx \le C \int_{\mathbb{R}^n} |Tf(x)|^{p_0} w(x)\,dx,$$

then the same inequality holds with p_0 replaced by any p, $0 < p < \infty$. The motivation for such inequalities is the Coifman-Fefferman inequality discussed in Section 5.7 or the Fefferman-Stein inequality in Lemma 7.10.

Second, given the above inequality, with w in either A_{p_0} or A_∞, extrapolation can be used to prove vector-valued inequalities of the form

$$\left(\int_{\mathbb{R}^n}\left(\sum_{k=1}^{\infty}|Sf|^q\right)^{p/q} w\right)^{1/p} \leq C\left(\int_{\mathbb{R}^n}\left(\sum_{k=1}^{\infty}|Tf|^q\right)^{p/q} w\right)^{1/p},$$

where w is in A_p or in A_∞, respectively, and $1 < q < \infty$.

Finally, extrapolation can be used to deduce norm inequalities in a variety of Banach functions spaces, such as weighted Orlicz spaces, starting from inequalities in weighted Lebesgue spaces. This extends a remark attributed to A. Córdoba that there are no L^p spaces, only weighted L^2.

For all of these results, see the papers by D. Cruz-Uribe, J. M. Martell and C. Pérez (*Extrapolation from A_∞ weights and applications*, J. Funct. Anal. 213 (2004), 412-439; *Extensions of Rubio de Francia's extrapolation theorem*, Proceedings of El Escorial, 2004, Collect. Math. (2006) 1-37), D. Cruz-Uribe, A. Fiorenza, J. M. Martell and C. Pérez (*The boundedness of classical operators on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math. 31 (2006), 239–264), and G. P. Curbera, J. García-Cuerva, J. M. Martell and C. Pérez (*Extrapolation with weights to Rearrangement Invariant Function Spaces and modular inequalities, with applications to Singular Integrals*, Adv. Math. 203 (2006), 256-318).

Error: p. 186, l. -7--3 The first part of the discussion of weighted Littlewood-Paley theory gives an incorrect reference and should be replaced by the following.

There are also weighted versions of the Littlewood-Paley inequalities. Theorem 8.6 is true in $L^p(w)$ when $w \in A_p$. This has not appeared explicitly in the literature, but is implicit in our treatment of Littlewood-Paley operators as vector-valued singular integrals. Let $g(f)$ denote the operator on the left-hand side of inequality (8.6). The proof of Theorem 8.6 shows that the vector-valued kernel of this operator satisfies a regularity condition, which can then be used to prove that $M^\#(g(f))(x) \leq C_s M(|f|^s)(x)^{1/s}$ for any $s > 1$. The estimates in $L^p(w)$ then follow from the results in Chapter 7.

D. Kurtz (*Littlewood-Paley and multiplier theorems on weighted L^p spaces*, Trans. Amer. Math. Soc. **259** (1980), 235–254) proved a weighted version of Theorem 8.7, with the stronger assumption that the weights w are in strong A_p (see Section 5.8 in Chapter 7).

Error: p. 208, l. 1 Equation should be $T_j 1 = 0$ (the subscript j is missing).

Error: p. 208, l. 13 Change this line to read: "Formally, this identity is equivalent to $T_j^* 1 = 0$, and we will use $T^* 1 = 0$ to prove it."

Addition: p. 215, l. 3 The proof by L. Grafakos and X. Li appeared in *Uniform bounds for the bilinear Hilbert transforms I*, Ann. of Math. **159** (2004), 889-933.

Addition: p. 217, l. 0 Additions to the bibliography:

J. Arias de Reyna, *Pointwise convergence of Fourier series*, Lecture Notes in Mathematics, 1785, Springer-Verlag, Berlin, 2002.

L. Grafakos, *Classical and Modern Fourier Analysis*, Pearson Education, Upper Saddle River, NJ, 2004.

S. G. Krantz, *A panorama of harmonic analysis*, Mathematical Association of America, Washington, DC, 1999.

M. A. Pinsky, *Introduction to Fourier Analysis and Wavelets*, The Brooks/Cole Series in Advanced Mathematics, Brooks/Cole, Pacific Grove, CA, 2001.

T. H. Wolff, *Lectures on harmonic analysis*, University Lecture Series, 29, American Mathematical Society, Providence, RI, 2003.

Error: p. 221, l. -22 Add reference "Lorentz spaces, 41."