# A Modern Theory of Integration

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## Errata

The following errata and corrections have been submitted by readers. We are particularly grateful to David M. Bradley, of the University of Maine. To submit comments, please email bookpages@ams.org.

### Theorem 6.6(b)

The theorem is stated as an "if and only if", but the reverse implication is false. For a counter-example, let $I = [0, 1]$, let $E$ be a non-measurable subset of $I$, let $f$ be the characteristic function of $E$ and let $g = 1 - f$, the characteristic function of $I \setminus E$. Then $f$ and $g$ are not measurable functions, but $\max(f, g) = 1$ and $\min(f, g) = 0$ are both constant functions on $I$, and hence are measurable.

### Proof of Fatou's Lemma

In the second last line of the proof of Fatou's Lemma (Lemma 8.7) on page 122, you need to know that $\phi$ is real-valued (almost) everywhere on $I$ to invoke the Monotone Convergence theorem. Conceivably $\phi$, being the limit of an increasing sequence of functions $\phi_k$, could be infinite on a set of positive measure.

Looking ahead at Theorem 9.3, we know that can't happen in this case because the sequence of integrals $\{\int \phi_k : k \geq 1\}$ is bounded. Unfortunately, the proof of Theorem 9.3 uses Theorem 9.1b, whose proof depends on the Dominated Convergence Theorem (Theorem 8.8), whose proof in turn depends on Fatou's Lemma. So it appears that the book proves only that these results imply each other. What is missing is an independent proof that any one of them is true.

## Small changes

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|---|---|
| p.21 ex. 1.W a) | Since Definition 1.7 assumes $C$ is real, it would be better to state the problem as "Show that the definition of the integral applies *mutatis mutandis* to a complex-valued function defined on $I$." |
| p.59, line-3 | Replace $I$ by $[0, 1]$. |
| p.61, line 1 | Insert "necessarily adjacent" between "two" and "subintervals" for clarity. |
| p.76, Lemma 5.3 | Replace $[a, b]$ by $I$. |
| p.83 line 1 | To be consistent with the statement being proved, replace $c$ as a subscript to $\Psi$ in both places by $a$, and replace $c$ by $a$ in the lower limit of the integral. |
| p.90, Theorem 6.3d) | The hypothesis could be weakened to $\phi : f(I) \to \mathbf{R}$ is continuous on the open interval $(\inf f(I), \sup f(I))$. This weaker hypothesis is needed in the Measurable Limit Theorem 9.2 on page 136, where $\phi = \tan$ is used, which is continuous only on $(-\pi/2, \pi/2)$, not all of $\mathbf{R}$. |
| p.111 ex. 7.G | Delete "Let $\phi \in BV[a, b]$ and" |
| p.111 ex. 7.Ja) | Remove "and 7.H" from the hint: 7.H isn't needed. |
| p.119 Theorem 8.5 | "$f(x) := \lim f_k(x)$" the proof assumes implicitly that $f(x)$ is real for each $x$. Being the limit of a monotone sequence of real numbers, it could be infinite, so perhaps it should be emphasized that the existence of the limit is in the stronger sense of being finite. |
| p.122 $(8.\theta)$ | The outer two inequalities are redundant. |
| p.131 ex. 8.Cb) | The reference to $(8.\kappa)$ is almost certainly intended to be $(8.\theta)$. |
| p.131 ex. 8.Eb) | The word "conclusion" is missing an "l". |
| p.136 Theorem 9.1b) | The wording here might be confusing. What is meant is that $f \in L(I)$ if and only if there exist $\alpha \in \mathbf{R}^*(I)$ and $\omega \in \mathbf{R}^*(I)$ such that $\alpha \leq f \leq \omega$ a.e. on I <u>and</u> at least one of $\alpha, \omega$ is in $\mathcal{L}(I)$. |
| p.137 line-4 | Replace "Thus" with "By 5.10e)". Insert "so" after "and" at the end of this line. |
| p.159 line 2 | Replace $I$ by $E$. |

p.161, Theorem 10.11 Insert "a.e." after "bounded on" and before "$I$".

p.163, Theorem 10.13 The seventh line of the proof assumes additivity of the RS-integral over intervals, but the remarks at the bottom of page 397 and the top of page 398 suggest that it is not additive over intervals.

p.166, exer. 10.P a) Replace 10.2 by 10.4.

p.173, last line Theorem 20.18 should be 19.18.

p.178 Theorem 11.11 states as one conclusion that $f \in \mathbf{R}^*(I)$, but the proof omits a demonstration of this. This is easily rectified, since Fatou's lemma also gives that $|f - f_n| \in \mathbf{R}^*(I)$. Since $-|f - f_n| \le f - f_n \le |f - f_n|$, it follows that $f - f_n \in \mathbf{R}^*(I)$ by Theorem 9.1a). Thus $f = (f - f_n) + f_n \in \mathbf{R}^*(I)$ by Theorem 3.1a).

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