

## ERRATA FOR GRADUATE ALGEBRA: NONCOMMUTATIVE VIEW

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Page 29 line -14: The Weyl algebra is left Noetherian (by Exercises 13A5 and 13A7) but not a PLID; Exercise 13A8 is not applicable until one localizes at  $x$ .

### 1. NILPOTENCE OF THE RADICAL OF A LEFT ARTINIAN RING, P. 50

Page 51 Theorem 15.18: The proof of (iv) needs modification, since we do not know yet that  $A$  is finitely generated. Instead, suppose that  $J^k = J^{k+1}$ . We claim that  $J^k = 0$ . Otherwise  $J^k a \neq 0$  for some  $a \in J$ , so take  $0 \neq a' \in J^k a$ . Then  $a' \in J^{k+1} a = J a'$ , so  $R a' = J a' = J(R a')$  implying  $R a' = 0$  by Remark 15.3, a contradiction.

### 2. THE REGULAR REPRESENTATION OF A GROUP, CHAPTERS 19 AND 20

There is a better way to prepare Example 20.7, which deals with  $\chi_{\text{reg}} := \chi(\rho_{\text{reg}})$ . First of all, note that  $\rho_{\text{reg}}$  operates on  $G$  (and thus on the group algebra  $F[G]$ ) by left multiplication, so corresponds to  $F[G]$  itself as an  $F[G]$ -module which, in terms of Remark 19.35, could be viewed as  $\oplus L_i^{(n_i)}$  when  $F$  is a splitting field of  $G$ . Taking traces yields Example 20.7 at once.

### 3. LIE ALGEBRAS, P. 284

After Proposition 21.30 one should add the obvious corollary that  $L$  is solvable iff  $L'$  is solvable, since  $L/L'$  is Abelian.

Then in page 288, line 8, we should first say  $L'$  is solvable, and this implies  $L$  is solvable.

### 4. NICHOLS-ZOELLER THEOREM, P. 559

Page 558 line -3: The tensor products in the statement of Proposition 26.27 should be over the base field  $F$ , and not over  $K$  as written.

This causes a gap in the proof of the Nichols-Zoeller Theorem (Corollary 26.28) which can be filled as follows, using facts about left integrals, cf. Remark 26.31:

**Lemma:** If  $M$  is a f.g. module over a f.d. Hopf algebra  $H$  such that  $M^{(j)}$  is a free  $H$ -module for some  $j$ , then  $M$  is already free.

**Proof:** Write  $H = \oplus P_i$ , a direct sum of indecomposable projective  $H$ -modules. Then any left integral  $t$  of  $H$  decomposes as  $\sum t_i$ . Each  $t_i \in P_i$  is also a left integral and thus a multiple of  $t$ ; hence, we may assume that  $t = t_1 \in P_1$ , and all the other  $P_i$  are not isomorphic to  $P_1$ . By assumption,  $M^{(j)} \cong H^{(u)}$  for some  $u$ . But then the Krull-Schmidt theorem shows that  $M$  must be a direct sum of the  $P_i$ . If  $P_1$  appears  $k$  times in  $M$ , then  $jk = u$ , and consequently  $M \cong H^{(k)}$ .  $\square$

### 5. MISPRINTS AND MINOR CHANGES

- Page xxv, line -8 any  $\alpha_1, \dots, \alpha_{n^2} \in F$ .
- Page 46 line -2: submodule of  $M$ ;
- Page 144 line 11:  $V$  is a vector space of dimension  $n$
- Page 150 line -9:  $\prod L[\lambda]/L[\lambda]g_i$
- Page 157 line 11: by Corollary 5.16'
- Page 169 line 22,23: (remove duplication) for every element  $a$  in a
- Page 186 line -4:  $Q \cap S = 0$
- Page 214 line -14: Exercise 19.6(i)
- Page 240 line 1,2: so are  $Sg$  and  $gS$ , of the same dimension
- Page 245:  $F_0 = \mathbb{Q}$  (throughout Step III of the proof).

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- Page 282 line -8:  $L^{(k)} = [L^{(k-1)}L^{(k-1)}]$ .
- Page 285 line -12:  $Lv \subseteq Fv$ .
- Page 289 line -5:  $[(a_1, b_1)(a_2, b_2)] = ([a_1a_2], [b_1b_2])$ ;
- Page 389 Exercise 33: Define  $U_{a,c} = U_{a+c} - U_a - U_c$ .
- Page 467 line 16: is a prime ring (of dimension  $n^2$ )
- page 472 line 1:  $\bar{F}(\Lambda)$  instead of  $\bar{F}$ .
- Page 630 [Hof] "Goedel, Escher, Bach: an Eternal Golden Braid".

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