

*Representations of Semisimple Lie Algebras
in the BGG Category \mathcal{O}*
(Revisions)

On the dedication page, the list of names should be: Rowan Leland Gerlis,
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- xvi** In last paragraph of Preface, read “Stroppel” in place of “Stroppele”.
- 8** In line –11, delete the minus sign.
- 10** In line –9, replace $L(3)$ by $L(2)$.
- 14** In line 7, replace 0.7 by 0.8.
- 25** In line 3, replace μ by $w\mu$.
- 27–28** The proofs of (b) and (c) in 1.10 are out of focus at several points and should be revised as follows:
- 27** In line –6, replace $W \cdot \lambda$ by $W(\lambda + \rho)$ and $W \cdot \mu$ by $W(\mu + \rho)$.
- 27** In line –4, replace “dot orbits” by “ W -orbits”.
- 27** In lines –4, –3, replace “Using the assumption about ψ , take any pre-image ...” by “Using part (a), take the pre-image ...”.
- 27** In line –2, the expression should be $\chi_\lambda(z) = (\lambda + \rho)(\psi(z))$ and similarly for μ .
- 28** In lines 11–12, replace the three occurrences of λ by $\lambda + \rho$, and similarly for the first occurrence of λ in line 13.
- 29** In line –19, replace “for $M \in \mathcal{O}$ ” by “for isomorphism class representatives $M \in \mathcal{O}$ ”.
- 35** In line –17, replace “ $\lambda \geq 0$ ” by “ $\lambda \in \mathbb{Z}^+$ ”.
- 35** In line –6, the last symbol in the Exercise should be $[M(\lambda)]$.
- 55** In line –6, replace $w' \cdot (\lambda - s_\alpha \cdot \lambda)$ by $(w' \cdot \lambda - w \cdot \lambda)$. Then in line –4, replace $w' \cdot (\lambda - s_\alpha \cdot \lambda) = w(s_\alpha \cdot \lambda - \lambda)$ by $w' \cdot \lambda - w \cdot \lambda = ws_\alpha \cdot \lambda - w \cdot \lambda$.
- 60** In line –18, replace “the right exact functor ... is also left exact.” by “the contravariant functor ... is exact.”

- 60** In lines $-11, -10$, read “is dominant if $-\lambda$ is antidominant.”
- 64** In the proof of Corollary 3.10, delete the sentence “Use induction on the length ...” at the end of the first paragraph. Then replace the last line of the proof by “After discarding these summands, induction on $\sum c_\mu$ takes over for the remaining summand of P .”
- 65** Reword Exercise 3.11: “If $\lambda \in \Lambda$, prove that $M(\lambda)$ is projective only when λ is dominant. [Using results of Chapter 4, this can be proved for all $\lambda \in \mathfrak{h}^*$.]”
- 80** In line -16 , replace “at least” by “an”.
- 91** In the paragraph of the Notes starting with “Following ...”, replace the second sentence by: “Lutsyuk [204] independently develops a recursive formula for the elements of $U(\mathfrak{n}^-)$ inducing such embeddings.”
- 95** In line -18 , replace the occurrences of the symbol $<$ by \uparrow and \leq respectively.
- 97** In line -6 , replace “cases $w = s_\beta s_\alpha$ and $w = w_\circ$ ” by “case $w = s_\beta s_\alpha$, while the case $w = w_\circ$ follows by comparing the alternating sum formula for $\text{ch } L(w_\circ \cdot \lambda)$ in 2.4”
- 98** In lines $9-10$, replace the long sentence “By induction, ...” by the shorter “By induction, $\mu \uparrow s_\alpha \cdot \lambda$.”
- 101** In lines $-9, -8$, reword the sentence: “Since $M(\lambda)$ has finite length, it follows that ...”
- 101** In line -5 ,. replace “3.14” by “Exercise 3.14”.
- 118** In the last two lines, replace “case $k = m$ ” by “case $k = m - 1$ ” and “let $k < m$ ” by “let $k < m - 1$ ”.
- 121** In line 15, the second $M(w \cdot 0)$ should be $M(w' \cdot 0)$.
- 121** The ideas sketched in the last paragraph for the proof of the theorem stated at the top of page 122 are probably not adequate for the purpose. It is safer to rely on the arguments given by Mazorchuk [214], though it would be interesting to minimize the prerequisites for the theorem.
- 122** In line -3 , delete the extra right parenthesis.

124 In line 8 of (1) in the proof, the first Ext term in the exact sequence should be $\text{Ext}_{\mathcal{O}}$.

130-131 Revise the last paragraph of 7.1: At the end of the existing text in line -9, insert the first sentence at the top of page 131 “The rationale ...” In line -8, replace “a couple of” by “some”. Then reword the proposition as follows: “Let $\lambda, \mu \in \mathfrak{h}^*$.

(a) The exact functor T_{λ}^{μ} commutes with the duality functor.

(b) T_{λ}^{μ} takes projective modules to projective modules.

(c) $T_{\lambda}^{\mu}M(\lambda)$ has a standard filtration involving $M(\mu)$ as a subquotient.”

For the proof, follow the current wording for the first two parts and the current wording on page 131 for (c) with obvious modifications (omitting parentheses around the last sentence):

(a) “To see that ... in that section.”

(b) “This follows from ... to a direct summand.”

(c) “When $M = M(\lambda)$, a standard filtration ... This module also has a standard filtration, thanks to Proposition 3.7(b).”

134 In line -15, begin (c) with: “Suppose $\Phi_{[\lambda]} = \Phi_{[\mu]} = \Phi_{[\lambda+\mu]}$.” Then in line -6, begin (c) with: “By assumption, $\Phi_{[\lambda+\mu]} = \Phi_{[\lambda]}$.”

136 In line 14, replace C' by $\overline{C'}$.

136 In line -14, delete “and (7)”.

136 In line -1, read: $\xi = \lambda^{\natural} + \nu = \mu^{\natural}$.

137 Delete the bottom text on the page, starting at line -10: “But it ...”

141 Replace line 5 by “ $\alpha \in w\Phi_{F'}^0$, with F' the facet of μ .” Then at the end of line 6, replace F by F' , and in line 8 replace F by F' .

143 In line 3, replace W_{μ} by W_{μ}° .

144 In line 7, replace “Corollary 7.6” by “Proposition 7.1(c)”.

144 In line -3, replace \mathcal{O} by $\mathcal{O}_{\chi_{\mu}}$.

146 In line 1, replace $P(w_{\lambda} \cdot \lambda)$ by $P(w_{\lambda}w_{\mu^{\circ}} \cdot \lambda)$.

147 In line 7, replace “Corollary 7.6” by “Proposition 7.1(c)”.

- 147** In line -2 expand “ > 0 ” to “ > 0 if $T_\lambda^\mu L(w' \cdot \lambda) \neq 0$ ”.
- 150** In line -1 , replace “bonce” by “once”.
- 151** In line 2, replace “just one” by “no”. Then in lines $-16, -15$, remove the sentence “Using the projective property ...”
- 151-152** In lines 151 : -2 and 152:1, replace $L(\lambda_\circ)$ by $M(\lambda_\circ)$.
- 163** In line 14, replace “which means” by “which implies”. Then in the lemma replace “ $\ell(w) - \ell(x) = 1$ ” by “ $w = rx$ for some reflection r ”.
- 177** In line 3, replace $\ell(w_\circ) + 1$ by $\ell(w_\circ)$.
- 178** In line 9, replace \mathfrak{h} by \mathfrak{h}^* .
- 200** In the third line of 9.15, read “more refined partition”.
- 233** In line 15, replace “Theorem 3.9” by “Theorem 3.7”.
- 243** Revise the third paragraph of 12.7, starting with line 4:
“and $N_w = \mathbb{C}[y_\alpha]$. The algebra N_w is \mathbb{Z} -graded: the standard grading of U by Λ_r induces a \mathbb{Z} -grading on U if all simple root vectors are placed in U_1 . The graded dual N_w^* , with n th graded piece the dual space $(N_w)_{-n}^*$, then becomes a \mathbb{Z} -graded N_w -bimodule. Define $S_w := U \otimes_{N_w} N_w^*$. Somewhat miraculously, ...”
- 265** In line -4 , replace “on W ” by “on V ”.
- 269** In line -15 , replace W_I by W_1 .

(December 2016) Most of these revisions were suggested by Brian Boe, Andreas Glang, Chun-ju Lai, Visnambhara Makam, David Nies, David Savitt.