GSM 95 QUANTUM MECHANICS FOR MATHEMATICIANS ERRATA

LEON A. TAKHTAJAN

Here I correct the proof of Corollary 1.2 in §1 of Chapter 2 and list some other smaller corrections. I intend to update this file from time to time and so welcome further comments.

Page 71, Line 5. Replace 'finite measures' by ''finite complex measures'.

Page 71. Clarification of the proof of Proposition 1.1. Instead of complex measures μ_n one can use 'ordinary' measures $\tilde{\mu}_n$, defined as follows. Put $M = B^2$, where in accordance with (1.1), non-negative operator B is given by

$$B = \sum_{n=1}^{N} \sqrt{\alpha_n}\, P_{\psi_n}.$$

Then $\mu_A(E) = \operatorname{Tr} \mathsf{P}_A(E)B^2 = \operatorname{Tr} B\mathsf{P}_A(E)B$ and

$$\mu_A(E) = \sum_{n=1}^{\infty}(B\mathsf{P}_A(E)Be_n, e_n) = \sum_{n=1}^{\infty}(\mathsf{P}_A(E)Be_n, Be_n) = \sum_{n=1}^{\infty} \|\mathsf{P}_A(E)Be_n\|^2.$$

Thus we get

$$\mu_A(E) = \sum_{n=1} \tilde{\mu}_n(E),$$

where $\tilde{\mu}_n$ are finite Borel measures on $\mathbb{R}$ defined by $\tilde{\mu}_n(E) = \|\mathsf{P}_A(E)Be_n\|^2$, and the proof of Proposition 1.1 goes without change.

Page 71, Line 17. Change the proof of Corollary 1.2 as follows. Define Borel measures ν_n on $\mathbb{R}$ by $\nu_n(E) = (\mathsf{P}_A(E)Me_n, Me_n) = \|\mathsf{P}_A(E)Me_n\|^2$. Since

$$\sum_{n=1}^{\infty} \nu_n(E) = \operatorname{Tr} M\mathsf{P}_A(E)M = \operatorname{Tr} \mathsf{P}_A(E)M^2 = \sum_{n=1}^{N} \alpha_n^2(\mathsf{P}_A\psi_n, \psi_n)$$
$$\leq \sum_{n=1}^{N} \alpha_n(\mathsf{P}_A\psi_n, \psi_n) = \operatorname{Tr} \mathsf{P}_A(E)M = \mu_A(E),$$

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we get

$$\int_{-\infty}^{\infty} \lambda^2 d\nu_n(\lambda) \leq \int_{-\infty}^{\infty} \lambda^2 d\mu_A(\lambda) = \langle A^2 | M\rangle < \infty.$$

Thus $Me_n \in D(A)$, i.e., $e_n \in D(AM)$, and the result follows from the proof of Proposition 1.1.

Page 71, Line 4 from the bottom. Replace $A_n = f_n(A)$ by $A_n = Af_n(A)$.

Page 89, Lines 14-16. Replace T by B.

Page 95, Last line. Replace $\sqrt{\frac{m}{t}}$ by $\frac{m}{t}$.

Page 140, Line 5. Replace $\mathcal{A}_t = \mathbb{C}[[t]] \otimes_{\mathbb{C}} \mathcal{A}$ by $\mathcal{A}_t = \mathcal{A}[[t]]$.

Page 156, Second line of equations in part (ii) of Theorem 2.1. Replace

$$\lim_{x\to-\infty} e^{ikx} f_1'(x,k) = -ik \quad \text{by} \quad \lim_{x\to-\infty} e^{ikx} f_2'(x,k) = -ik.$$

Page 207, Equation (6.10). Replace $\varphi(q)$ by $\log \varphi(q)$.

Page 207, Equation (6.12). In the left-hand side replace $\left(\frac{\partial}{\partial t} + \frac{\partial S_0}{\partial q}\right)$ by $\left(\frac{\partial}{\partial t} + \frac{1}{m}\frac{\partial S_0}{\partial q}\frac{\partial}{\partial q}\right)$.

Page 210, Line 3. Replace $\sigma'^2 \ll \hbar\sigma''$ by $\hbar|\sigma''| \ll (\sigma')^2$.

Page 225, Line 6 from the bottom. Replace $S_3(S_3 + I)$ by $S_3(S_3 + \hbar I)$.

Page 229, Equation (3.5). Replace $s(s+1)\Phi$ by $\hbar^2 s(s+1)\Phi$.

Page 233, Equation (3.8). Replace $s(s+1)\chi_{mj}$ by $\hbar^2 s(s+1)\chi_{mj}$ and $m\chi_{mj}$ by $\hbar m\chi_{mj}$.

Page 254. Clarification of the periodic boundary conditions in equation (2.16). It immediately follows from the Euclidean version of (2.12) that

$$\frac{1}{\pi\hbar}\int_{\mathbb{C}} U_n(\bar{a}, a; -iT) e^{-\frac{1}{\hbar}\bar{a}a} d^2a$$

$$= \underset{\mathbb{C}^n}{\int\cdots\int} e^{-\frac{1}{\hbar}\sum_{k=1}^{n}(-\bar{a}_k(a_{k-1}-a_k)+H(\bar{a}_k, a_{k-1})\Delta t)} e^{-\frac{1}{\hbar}\bar{a}_n a_n} \prod_{k=1}^{n} \frac{d^2 a_k}{\pi\hbar},$$

where one puts $a_n = a = a_0$ and $\bar{a}_n = \bar{a} = \bar{a}_0$. In the limit $n \to \infty$ one gets equation (2.16).

Page 258, Line 22. Replace

$$B_{1n-1} = B_{n-11} = \frac{1}{a_n} = \frac{\sin n\theta}{\sin\theta}$$

by

$$B_{1n-1} = B_{n-11} = \frac{1}{a_{n-1}} = \frac{\sin n\theta}{\sin\theta}$$

Page 292, Lines 14-15. Replace $(1+\varepsilon^2\alpha_i^2)$ by $(1+2\varepsilon^2\alpha_i^2)$.

Page 297, Line 18. The statement of Theorem 2.1 should read: "… is the conditional Wiener measure with the diffusion coefficient $D = \frac{\hbar}{2m}$."

Page 300, Line 10. Replace 'bounded below' by 'bounded'.

Page 300, Line 12. Replace $L_\hbar(\boldsymbol{q}', t'; \boldsymbol{q}, t)$ by "$\tilde{L}_\hbar(\boldsymbol{q}', t'; \boldsymbol{q}, t)$ for the operator $\tilde{H} = H_0 - V$".

Page 300, Equation (2.9). Replace $L_{i\hbar+\varepsilon}(\boldsymbol{q}', t'; \boldsymbol{q}, t)$ by $\tilde{L}_{i\hbar+\varepsilon}(\boldsymbol{q}', t'; \boldsymbol{q}, t)$.

Page 332, Line 18. Replace $\tilde{\Phi}_{\boldsymbol{\alpha}}(\boldsymbol{\theta})$ by $\tilde{\Phi}_{\boldsymbol{\alpha}}(\bar{\boldsymbol{\theta}})$.

Page 337, Line 4. Replace $\boldsymbol{\alpha}(T)$ by $\boldsymbol{\alpha}_N$.

Page 337, Lines 7-8. Clarification of the periodic boundary conditions in the formula for $\mathrm{Tr}_s\, e^{-iTH}$. Same as on the page 254, where now one uses Theorem 4.1 and Problem 4.2.

Page 346, Line 18. Replace $d\alpha_{n-1} = 0$ by $i_V\alpha_1 = 0,\ d\alpha_{n-1} = 0$.

Department of Mathematics, Stony Brook University, Stony Brook, NY 11794-3651, USA
E-mail address: leontak@math.sunysb.edu