

**GSM 95 QUANTUM MECHANICS FOR  
MATHEMATICIANS  
ERRATA**

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Here I correct the proof of Corollary 1.2 in §1 of Chapter 2 and list some other smaller corrections. I intend to update this file from time to time and so welcome further comments.

*Page 71, Line 5.* Replace ‘finite measures’ by ‘finite complex measures’.

*Page 71.* Clarification of the proof of Proposition 1.1. Instead of complex measures  $\mu_n$  one can use ‘ordinary’ measures  $\tilde{\mu}_n$ , defined as follows. Put  $M = B^2$ , where in accordance with (1.1), non-negative operator  $B$  is given by

$$B = \sum_{n=1}^N \sqrt{\alpha_n} P_{\psi_n}.$$

Then  $\mu_A(E) = \text{Tr } P_A(E)B^2 = \text{Tr } BP_A(E)B$  and

$$\mu_A(E) = \sum_{n=1}^{\infty} (BP_A(E)Be_n, e_n) = \sum_{n=1}^{\infty} (P_A(E)Be_n, Be_n) = \sum_{n=1}^{\infty} \|P_A(E)Be_n\|^2.$$

Thus we get

$$\mu_A(E) = \sum_{n=1}^{\infty} \tilde{\mu}_n(E),$$

where  $\tilde{\mu}_n$  are finite Borel measures on  $\mathbb{R}$  defined by  $\tilde{\mu}_n(E) = \|P_A(E)Be_n\|^2$ , and the proof of Proposition 1.1 goes without change.

*Page 71, Line 17.* Change the proof of Corollary 1.2 as follows. Define Borel measures  $\nu_n$  on  $\mathbb{R}$  by  $\nu_n(E) = (P_A(E)Me_n, Me_n) = \|P_A(E)Me_n\|^2$ . Since

$$\begin{aligned} \sum_{n=1}^{\infty} \nu_n(E) &= \text{Tr } MP_A(E)M = \text{Tr } P_A(E)M^2 = \sum_{n=1}^N \alpha_n^2 (P_A\psi_n, \psi_n) \\ &\leq \sum_{n=1}^N \alpha_n (P_A\psi_n, \psi_n) = \text{Tr } P_A(E)M = \mu_A(E), \end{aligned}$$

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*Date:* August 15, 2010.

we get

$$\int_{-\infty}^{\infty} \lambda^2 d\nu_n(\lambda) \leq \int_{-\infty}^{\infty} \lambda^2 d\mu_A(\lambda) = \langle A^2 | M \rangle < \infty.$$

Thus  $Me_n \in D(A)$ , i.e.,  $e_n \in D(AM)$ , and the result follows from the proof of Proposition 1.1.

*Page 71*, Line 4 from the bottom. Replace  $A_n = f_n(A)$  by  $A_n = Af_n(A)$ .

*Page 89*, Lines 14-16. Replace  $T$  by  $B$ .

*Page 95*, Last line. Replace  $\sqrt{\frac{m}{t}}$  by  $\frac{m}{t}$ .

*Page 140*, Line 5. Replace  $\mathcal{A}_t = \mathbb{C}[[t]] \otimes_{\mathbb{C}} \mathcal{A}$  by  $\mathcal{A}_t = \mathcal{A}[[t]]$ .

*Page 156*, Second line of equations in part (ii) of Theorem 2.1. Replace

$$\lim_{x \rightarrow -\infty} e^{ikx} f'_1(x, k) = -ik \quad \text{by} \quad \lim_{x \rightarrow -\infty} e^{ikx} f'_2(x, k) = -ik.$$

*Page 161*, In the inequality in the sixth line replace  $\frac{1}{4\mathfrak{z}} e^{-\mathfrak{z}A}$  by  $\frac{1}{4\mathfrak{z}_n} e^{-\mathfrak{z}_n A}$ .

*Page 169*, Line 14 from the bottom. Replace “...the normalization condition (2.14)...” by “...the analog of normalization condition (2.14)...”

*Page 169*, Line 13 from the bottom. In the right hand side of the formula, replace  $\delta_{jl}$  by  $2\pi\delta_{jl}$ .

*Page 179*, Line 4 from the top. Replace “...are the corresponding normalized eigenfunctions...” by “...are the analogs of normalized eigenfunctions...”

*Page 179*, Line 7 from the top. Replace “...are the corresponding normalized eigenfunctions...” by “...are the analogs of normalized eigenfunctions...”

*Page 179*, Line 10 from the top. In the right hand side of the formula, replace

$$\int_0^{\infty} \quad \text{by} \quad \frac{1}{2\pi} \int_0^{\infty}$$

*Page 179*, Line 11 from the top. In the right hand side of the formula, replace

$$\int_c^{\infty} \quad \text{by} \quad \frac{1}{2\pi} \int_c^{\infty}$$

*Page 180*, Line 6 from the top. In the right hand side of the formula, replace

$$\int_0^{\infty} \quad \text{by} \quad \frac{1}{2\pi} \int_0^{\infty}$$

Page 189, Second line from the bottom. Delete “with the boundary condition  $f_l(0) = 0$ ”.

Page 189, Last line. Add “The operator  $H_0$  should be supplemented by the boundary condition  $f_0(0) = 0$ , whereas in the case  $l > 0$  no boundary conditions are required.”

Page 207, Equation (6.10). Replace  $\varphi(q)$  by  $\log \varphi(q)$ .

Page 207, Equation (6.12). In the left-hand side replace  $\left(\frac{\partial}{\partial t} + \frac{\partial S_0}{\partial q}\right)$  by  $\left(\frac{\partial}{\partial t} + \frac{1}{m} \frac{\partial S_0}{\partial q} \frac{\partial}{\partial q}\right)$ .

Page 210, Line 3. Replace  $\sigma'^2 \ll \hbar \sigma''$  by  $\hbar |\sigma''| \ll (\sigma')^2$ .

Page 225, Line 6 from the bottom. Replace  $S_3(S_3 + I)$  by  $S_3(S_3 + \hbar I)$ .

Page 228, Line 5 from the bottom. Replace *exchange energy* by *exchange term*.

Page 229, Equation (3.5). Replace  $s(s+1)\Phi$  by  $\hbar^2 s(s+1)\Phi$ .

Page 233, Equation (3.8). Replace  $s(s+1)\chi_{mj}$  by  $\hbar^2 s(s+1)\chi_{mj}$  and  $m\chi_{mj}$  by  $\hbar m\chi_{mj}$ .

Page 254. Clarification of the periodic boundary conditions in equation (2.16). It immediately follows from the Euclidean version of (2.12) that

$$\begin{aligned} & \frac{1}{\pi \hbar} \int_{\mathbb{C}} U_n(\bar{a}, a; -iT) e^{-\frac{1}{\hbar} \bar{a} a} d^2 a \\ &= \int \dots \int_{\mathbb{C}^n} e^{-\frac{1}{\hbar} \sum_{k=1}^n (-\bar{a}_k (a_{k-1} - a_k) + H(\bar{a}_k, a_{k-1}) \Delta t)} e^{-\frac{1}{\hbar} \bar{a}_n a_n} \prod_{k=1}^n \frac{d^2 a_k}{\pi \hbar}, \end{aligned}$$

where one puts  $a_n = a = a_0$  and  $\bar{a}_n = \bar{a} = \bar{a}_0$ . In the limit  $n \rightarrow \infty$  one gets equation (2.16).

Page 258, Line 22. Replace

$$B_{1n-1} = B_{n-11} = \frac{1}{a_n} = \frac{\sin n\theta}{\sin \theta}$$

by

$$B_{1n-1} = B_{n-11} = \frac{1}{a_{n-1}} = \frac{\sin n\theta}{\sin \theta}$$

Page 292, Lines 14-15. Replace  $(1 + \varepsilon^2 \alpha_i^2)$  by  $(1 + 2\varepsilon^2 \alpha_i^2)$ .

*Page 297*, Line 18. The statement of Theorem 2.1 should read: "... is the conditional Wiener measure with the diffusion coefficient  $D = \frac{\hbar}{2m}$ ."

*Page 300*, Line 10. Replace 'bounded below' by 'bounded'.

*Page 300*, Line 12. Replace  $L_{\hbar}(\mathbf{q}', t'; \mathbf{q}, t)$  by " $\tilde{L}_{\hbar}(\mathbf{q}', t'; \mathbf{q}, t)$  for the operator  $\tilde{H} = H_0 - V$ ".

*Page 300*, Equation (2.9). Replace  $L_{i\hbar+\varepsilon}(\mathbf{q}', t'; \mathbf{q}, t)$  by  $\tilde{L}_{i\hbar+\varepsilon}(\mathbf{q}', t'; \mathbf{q}, t)$ .

*Page 332*, Line 18. Replace  $\tilde{\Phi}_{\alpha}(\boldsymbol{\theta})$  by  $\tilde{\Phi}_{\alpha}(\bar{\boldsymbol{\theta}})$ .

*Page 337*, Line 4. Replace  $\alpha(T)$  by  $\alpha_N$ .

*Page 337*, Lines 7-8. Clarification of the periodic boundary conditions in the formula for  $\text{Tr}_s e^{-iTH}$ . Same as on the page 254, where now one uses Theorem 4.1 and Problem 4.2.

*Page 346*, Line 18. Replace  $d\alpha_{n-1} = 0$  by  $i_V\alpha_1 = 0$ ,  $d\alpha_{n-1} = 0$ .

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