

Errata for  
*The Mathematics of Voting and Elections: A Hands-On Approach*

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**Chapter 4, page 68** – Condition 2 should be listed as Positive Association of *Social and* Individual Values.

**Chapter 5, page 90** – There is an error in Question 5.26 and the conclusions drawn from it.

Question 5.26 (and the discussion surrounding it) asserts that there cannot exist a voting system for three or more candidates that satisfies Arrow's conditions 1, 3, and 5, and the modified Pareto condition. This assertion is in error, and the argument outlined in Question 5.26 suffers from a subtle flaw.<sup>1</sup>

First, note that a system in which all candidates tie, regardless of votes, does in fact satisfy Arrow's conditions 1, 3, and 5, and the modified Pareto condition. (Call this system  $V$ .) If we apply the construction from Question 5.26 to this example, by breaking ties in favor of the unanimously preferred candidate, we will see that the resulting system (call it  $V'$ ) in fact violates universality. To illustrate, consider the following preference schedule:

Rank	Number of Voters	
	1	1
1	$A$	$C$
2	$B$	$A$
3	$C$	$B$

With system  $V$ , the resulting societal preference order would be  $A \approx B \approx C$ , since by definition all candidates must tie regardless of the votes. However, system  $V'$  requires  $A \approx C$  (since  $A$  is not unanimously preferred to  $C$ ),  $C \approx B$  (since  $C$  is not unanimously preferred to  $B$ ), and  $A \succ B$  (since  $A$  is unanimously preferred to  $B$ ). This, however, is a violation of transitivity, since in a transitive order,  $A \approx C$  and  $C \approx B$  would together imply that  $A \approx B$ .

Given the invalidity of the argument in Question 5.26 and the demonstrated existence of a voting system that does satisfy Arrow's conditions 1, 3, and 5, and the modified Pareto condition, the question remains: What other systems also satisfy these conditions? This question seems to be open, and a complete characterization of all such systems would be an interesting result.

**Chapter 8, page 167** – The answer to Question 8.9(a) should read:

There are a total of  $2^{51} = 2,251,799,813,685,248$  different coalitions in the system.

The original answer left out the empty coalition, which, according to Definition 6.5 on page 106, should be counted.

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<sup>1</sup>Many thanks to Alexander Woo for pointing out this error.