Continuous Symmetry at Bowdoin College
Winter/Spring 2009

Continuous Symmetry is used as the textbook for Bowdoin's mathematics-major level *Introduction to Geometry* course, Mathematics 247. Here is a summary of the important structural elements of the course as offered during the Winter/Spring semester of 2009.

1. **Prerequisites**. The major prerequisite is *Introduction to Mathematical Reasoning* (Math 200). This allows us to assume familiarity with the techniques of mathematical proof and the basic material of naïve set theory (including injective functions, surjective functions, and equivalence relations). *Continuous Symmetry* can be used without this prerequisite material, but the pace needs to be slower and not as much of the text can be covered in one semester.

   Another prerequisite is *Multivariable Calculus* (Math 181). Predating our requirement of Math 200, this was included primarily to insure a desired sophistication in the students. It is not critical.

2. **Coverage**. The desire is to cover the full textbook (with just a few omissions) though we always fall a little short. The complete description sheet (Description_247_S09) contains a syllabus planned for the spring '09 that included all nine chapters. In fact, we did not cover Chapter IX and could only partially explore Chapter VIII. The actual syllabus covered is listed in a separate file (Actual_Syllabus_247_S09).

3. **Elementary plane geometry**. Chapter I presents a nearly complete development of axiomatic Euclidean plane geometry. (The only missing component is a discussion of area, which is postponed until Chapter IX.) At Bowdoin we move quickly through this long chapter (about 6 hours of class time), focusing on the *results*, and presenting only selected proofs. The purpose is to develop an unambiguous foundation of basic plane geometry on which to develop a rigorous treatment of transformational geometry.

   There are several additional goals of this chapter. The first is to expose students to an axiomatic development of a mathematical topic. With the demise of this approach in secondary school geometry, most of our students arrive at Bowdoin without ever having worked with mathematical axioms. A second goal is to present (if only briefly) rigorous developments of certain concepts such as *directed* angle measure and *ray translation*. The subtleties of these latter topics can be omitted.

4. **Core material**. As stated in the *Instructor Preface*, the heart of *Continuous Symmetry* is contained in Chapters II through V. These cover the basics of isometries, similarities, and their central role in the Kleinian transformational approach to Euclidean plane geometry. Most courses using *Continuous Symmetry* would cover these four chapters.

5. **Choices**. Following Chapters II through V, the remaining four chapters of *Continuous Symmetry* are divided into three essentially independent topics:
   a. **Applications to plane geometry** (Chapter VI). This chapter uses the full power of the transformational viewpoint to treat a selection of beautiful traditional topics such as the *nine-point circle*, *Euler's line*, the *orthic triangle*, and *Fagnano's Problem*. Most instructors will likely include at least parts of this chapter in their courses.
   b. **Symmetric figures** (Chapters VII and VIII). Chapter VII develops the fundamental notions of symmetry groups of figures in the plane, culminating in the classification of all finite symmetry groups of plane figures. The subsequent Chapter VIII, on frieze and wallpaper groups, is far deeper and more challenging. In the Spring 2009 version of the course we covered the first two sections (general theory and frieze groups) in a serious fashion, but only outlined the results of the last two sections (wallpaper groups). In other semesters I have simply summarized the full chapter. Both approaches worked: students like this material.
   c. **Area** (Chapter IX). The first three sections provide a complete rigorous axiomatic treatment of area in the plane which can stand on its own. Proving the *existence* of an area function is done in §IX.4 using Jordan measure. This is the most difficult portion of Chapter IX since it requires some intricate analysis arguments — the important results could easily be summarized, thereby avoiding the technical details. The behavior of area under similarity transformations is considered in §IX.5 — this lovely and powerful material only requires a few concepts from the previous section. The sixth section of Chapter IX is a brief introduction to fractals.

6. **Worksheets**. Most students better learn this material (and enjoy it more) when they work through it themselves with reasonable guidance. Though it takes more time than straight lecturing, I try as often as possible to have my students develop the course material by working in groups (three to fours students per group), each group at a chalkboard, using worksheets I've designed for this purpose. I don't have worksheets for every section (some sections work better in groups than others, and I've not had time to develop all the desired worksheets), but currently I have seventeen fully class tested sheets. These are on the website in pdf form; I'm happy to provide the TeX code for the sheets to any course instructor who requests it. I also have "solution "versions of most of these sheets, also available to instructors upon request.

7. **Assignments**. The assignments given during Spring 2009 are included on the website. Scanned copies of my handwritten solutions are available to instructors upon request.

7. **Handouts**. I've included copies of several text illustrations that I sometimes use during class sessions, either as paper handouts or overhead transparencies.

8. **Exams**. Copies of exams I've used are available to instructors upon request, along with scanned copies of my handwritten solutions.

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