Mathematical World: Game Theory (Math 110)
May 2009

Instructor
David Housman will available afternoons we have class. It is best to call ahead or schedule an appointment. His office is SC 112, email address is dhousman@goshen.edu, office telephone is 574-535-7405, and home telephone is 574-875-0339.

Description
Quantitative skills are important for understanding our world and making wise personal, business, and societal decisions. This course will improve your quantitative skills through a study of the mathematics of conflict and cooperation. You will critically read and complete exercises in preparation for intense, team-oriented learning experiences during class time.

Prerequisite
Mathematics competency established by college credit in a mathematics course, SAT Math score of 550 or greater, ACT Math score of 23 or greater, or passing the GC Math Competency Exam.

Learning Objectives and Outcomes
1. Modeling: Model situations of conflict and cooperation as games, that is, define important assumptions, convert assumptions into mathematical formalism, identify and use appropriate mathematical techniques and solution concepts, and interpret the results of the mathematical analysis.
2. Game Theory: Describe and compute solutions for strategic, sequential strategic, bargaining, coalition, and fair division games.
3. Quantitative Tools: Use arithmetic, graphs, algebra, probability, and expected value correctly in applied contexts.
4. Communication: Communicate quantitative information and analysis effectively in written and oral forms.
5. Collaboration: Become more effective at achieving goals, obtaining help, contributing information, and supporting others in a collaborative learning environment.
6. Critical Thinking: Improve ability to critique an argument and to evaluate a purported solution.
7. Appreciation: Develop a greater appreciation for the usefulness of mathematics for making personal decisions, understanding our world, and engaging in public discourse.
8. Enjoyment: Experience a vibrant and enjoyable learning environment for mathematics in which new ideas are explored, techniques are evaluated, and assumptions are challenged.
9. Integration: Identify situations that a quantitative approach would enhance performance in making a decision, understanding a phenomenon, or solving a problem; seek and obtain the required knowledge to apply to the situation; build, analyze, and interpret appropriate quantitative models; and then communicate this new understanding to others in order to actually enhance performance.

Resources
The required textbook is *Models of Conflict and Cooperation* by Rick Gillman and David Housman. A copy of the current draft of this book can be obtained from Housman on the first day of class for $17. Please bring cash or a check made out to Goshen College. There are a number of good game theory books in the Good Library. Moodle is the course web site.

Grade
Your grade will be based upon participation (20%), homework (20%), project (20%), and two exams (20% each). Minimum percentages to guarantee letter grades: 95% (A), 90% (A-), 87% (B+), 83% (B), 80% (B-), 75% (C+), 70% (C), 68% (C-), 65% (D+), and 60% (D).

Participation
The study of mathematics is not a spectator sport! Essential to your success is (1) regular study and reflection outside of class, and (2) an active and thoughtful participation during class activities, lecture, and discussion.   Attend class and devote three hours outside of class for each 1.5 hour class to prepare and synthesize. For participation points, attend and participate in class activities (5 points per class $\times$ 25 classes = 125 points), maintain a time log (20 points), perform well on announced games (25 points), serve as a class recorder (2 classes alone or 4 classes with a partner for 20 points), and submit end-of-semester evaluations (10 points).

Homework
Solving problems, writing explanations, reflecting upon ideas, and feedback are essential to learning mathematics. Completion of the homework will prepare you for class discussion and the exams. Designated homework will be collected and graded.

Project
The project will be an opportunity, either individually or collaboratively, to examine some topic in greater depth. The final product will consist of a paper, presentation, and evaluations. Potential topics will be mentioned as the term progresses. Details about the mechanics and grading criteria will be distributed in a separate document.

Exams
The exams will be opportunities to exhibit your ability to describe and apply some of the basic concepts and techniques of game theory.

Extra Credit
For extra credit points, attend a talk or participate in an activity that has quantitative content and write a description of that content (one-half to one page); describe errors in or suggest improvements to the text; and/or provide a written/oral review of the quantitative aspects of a current article, movie, play, musical performance, or other media item related to course topics. These points will be added to your participation or homework grades.

Academic Integrity
For homework, collaboration is encouraged, but copying is prohibited. It is okay for a group to work together to solve a problem, but written materials produced during the group interaction should be destroyed before each person writes his or her own solution. Acknowledge in writing any collaborations or other resources used. Exams are closed book, closed notes, and the only human assistance is the instructor.

Cell phones
Cell phones and all other electronic devices (except calculators) should be turned off during class.

Schedule
See Moodle for the continuously evolving schedule.

Schedule

1. Rules of the Game

1. Book, Questionnaire, and Introductions
2. Syllabus and Schedule
3. Nim
4. Record content discoveries and process reflections
5. Hex
6. Record content discoveries and process reflections
7. Keypad, Poison, and Chomp

Read: First page and a half of the Preface, 1.1-2 (this means chapter 1, sections 1 through 2).
Exercises: 1.1 #1-4; 1.2 #1-4, 6, 8. Turn in 1.1 #2-3 and 1.2 #3ab (20 points).

2. Heuristics and Strategies

1. Recorder volunteer(s)?
2. Nim Strategies
3. Hex Strategies

Read: 1.3
Exercises: 1.3 #1-7, 9, 10, 12. Turn in #1abc, 3, 5abc, 6, 9, 10 (50 points).

3. Game Trees

1. [2,1,1] Nim game tree
2. [3,2,1] Nim "shortened" game tree
3. Hex game trees and winning strategies
4. Keypad restricted game tree using Fatima's winning strategy

Read: 1.4
Exercises: 1.4 #1-4, 8-10, 12, 14, 16. Turn in #4, 8ac, 9, 10abcd, 12a, restricted game tree for 14d (50 points).

4. Winning Strategies

1. Nim competition
2. Nim-sum strategy
3. Zermelo, Nash, and Hex

Read: 1.5-6.
Exercises: 1.5 #1-8 and 1.6 #1. Turn in 1.5 #4, 5, 6 (30 points).

5. Coalition Games

1. EPA Negotiation
2. EPA Arbitration

Read: 7.1-2. Don't worry about the comparison to bargaining games on page 294.
Exercises: 7.2 #1a, 2a, 2h, 3a, 3i (verify that Tabitha receives a payoff of zero in any rational allocation), 4a. Turn in: Describe an allocation method, use it to determine the allocation for each of the five coalition games (EPA, Apartment Sharing, Veto Power, Loner Power, and Not Finding Rational), and check whether these allocations are efficient and/or rational (50 points, may work in groups).

6. Allocation Methods

1. Group presentations of their methods
2. Shapley method and properties

Read: 7.3.
Exercises: 7.3 #1-5. Turn in 7.3 #2abcd, 4abcd, 5ab (50 points).

7. Ordinal Preferences

1. Prizes scenario

2. Acquaintance scenario
3. Die scenario (2.2 exercise #11)
4. Ultimatum game

Read: 2.1-2.
Exercises: 2.1 #1-3 and 2.2 #1-4, 6-9. Turn in 2.1 #2 and 2.2 #1, 2, 6befh, 7, 9ab (55 points). Please record your answers to 2.2 #9a in the questionnaire below <u>before</u> you read and complete 2.2 #9b.

8. Cardinal Preferences

1. New Prizes scenario
2. Utility transformations
3. Money scenario
4. Allais' experiment (exercise 2.3 #9)

Read: 2.3 and the last three paragraphs of 2.4.
Exercises: 2.3 #4-8, 10. Turn in 2.3 #4 (include your cardinal utilities and two outcome-lottery pairs for which you are indifferent, include the cardinal utilities for someone not in this class and two outcome-lottery pairs for which that person is indifferent; this one exercise is worth 30 points), 7, 10acd (50 points). Please record your answers to 2.3 #10a in the questionnaire below <u>before</u> you read and complete 2.3 #10bcd.

9. Catch Up

Discussion about coalition games and cardinal preferences.

10. Strategic Game Models I

1. Fingers (describe with participation points, practice, play, model)
2. Matches (describe, model)
3. Process decisions: pairs or quads, random or self-selected, recorder or not
4. Students model Tosca, Mating, War, and Three-Player Fingers

Reading: 3.1-2.
Exercises: 3.2 #1, 10. Present assigned model (25 points).

11. Strategic Game Models II

1. Students model Tosca, Mating, War, and Three-Player Fingers
2. Students present models
3. Work on homework

Reading: 3.1-2.
Exercises: Turn in 3.2 #7, 5 or 11 (50 points).

12. Exam 1 Preparation

1. Compare homework strategic models
2. Questions and answers
3. Games for participation points
   o Nim
   o Hex
   o Poison
   o Chomp
   o Matches
4. Time log reminder
5. Exam 1 instructions

13. Strategic Game Solutions I

Each solution concept will be defined and illustrated on Ad Hoc and Matches, and then students will apply to the games they previously modeled and present their findings to the rest of the class.
1. prudential strategy and security level
2. undominated strategy
3. Nash equilibrium
4. efficient strategy pair

Reading: 3.3.

14. Strategic Game Solutions II

Continue the previous agenda. Finish with a discussion of the Project.
Exercises: 3.3 #1-4, 6-8, 10-12, 14-15. Turn in 3.3 #2, 3, 8, 10, 12, 14 (124 points, each part is worth 4 points).

The payoff matrix in 3.3 #4 has several typos. Here is the correct payoff matrix:

	A	B	C	D	E
A	(5, 4)	(3, 2)	(4, 3)	(9, 2)	(3, 1)
B	(6, 6)	(1, 5)	(3, 4)	(8, 3)	(4, 4)
C	(4, 3)	(2, 6)	(4, 8)	(7, 7)	(5, 5)

15. Sequential Games I

1. Cuban Missile Crisis
2. Each student team models one of Sampson and Delilah, Square the Diagonal, Battle of Thermopylae, or The Taming of the Shrew (see pages 135-136) as a sequential game, determines the backward induction strategies and outcome, models as a strategic game, determines the Nash equilibria, and provides an interpretation.
3. Each student team presents their work.
4. Genesis Chapter 22

Reading: 3.4-5 (only through the "Trees to Matrices" subsection)

16. Sequential Games II

Finish the agenda from last class.
Exercises: 3.4 #1, 4, 5, 9, 11; 3.5 #1, 5, 6. Turn in 3.4 #4ab, 9; 3.5 #6 (100 points).

17. Mixed Strategies I

1. MARPS (describe, practice, model, play against the class)
2. Mixed strategies, Nash equilibrium, and prudential strategies in MARPS
3. Mixed strategies and Nash equilibria in Impromptu

Read: 4.1-2.
Exercises: 4.1 #1, 3; 4.2 #1-3, 5, 7, 8. Turn in 4.2 #2, 3abc, 7 (125 points).

18. Mixed Strategies II

Finish agenda from previous class.

19. Finding Nash Equilibria

1. Mating (p. 182)
2. Chicken (p. 184)
3. GAMBIT
4. Bargaining or fair division games next?

Read: 4.3 (skip the last subsection on prudential strategies)
Exercises: 4.3 #1-4, 6-7. Hand in 4.3 #3a (5 points), 3b (15 points), 3d (5 points), 3e (5 points), 6a (5 points), 6b (10 points), 6d (5 points) for a total of 50 points.

20. Fair Division I

1. truth versus Truth
2. An Inheritance Problem (describe, arbitrate, present)
3. Project work time

Read: 8.1
Exercises: 8.1 #1-4. Nothing to turn in.

21. Fair Division II

1. Methods (first-price auction, second-price auction, equal shares, Knaster's)
2. Properties (proportionate, envy-free, efficient, value-equitable, share-equitable)
3. Choosing a fair method

Read: 8.2-4 (skip the last subsection "Efficient, Envy Free, and Close to Share Equitable").

Exercises: 8.2 #1-3; 8.3 #1-3. Turn in 8.2 #2 (50 points), 8.3 #2 (50 points). Extra credit 8.2 #5 (20 points), 8.2 #7 (25 points), 8.3 #7 (25 points), 8.4 #2 (10 points), 8.4 #4 (10 points), 8.4 #5 (10 points).

22. Fair Division Games III

Finish the agenda from last class.

23. Climate Change Game

Read: 5.1

24. Repeated Prisoners' Dilemma & Exam 2 Review

Read: 5.2-3

25. Exam 2

26. Project Presentations

Math 110 Mathematical World (May 2009)
Project

Overview. The project is your opportunity, either individually or collaboratively, to examine some topic in greater depth. The final product will consist of a paper, presentation, and evaluations.

Mechanics.
1. The paper is due and the presentations will be given on the last day of class.
2. Groups of one to three students are permissible.
3. You are encouraged to discuss your ideas and progress with David often. David will answer questions, suggest literature, critique rough drafts, and attempt to provide any assistance necessary for your success.
4. There is no preordained number of pages for the paper; use the number of pages necessary to fully explain your work well.
5. Each group has $5 + 10 \times$ (number of group members) minutes for the presentation and audience questions and participation.
6. Each member of the group submits a written evaluation (project contributions, strengths, and improvement areas) for each member of the group, including him or herself.

Grading Rubric. The same rubric will be used no matter the size of the team, although the content quantity expectations will increase somewhat with the size of the team.

1. Mathematics and Modeling Content
   - 70 points: At a level above the most sophisticated work presented in the class or required reading with no errors in the mathematics or facts.
   - 60 points: At a level of the most sophisticated work presented in the class or required reading with no errors in the mathematics or facts.
   - 45 points: At a level of a class example or homework exercise with no errors in the mathematics or facts.
   - Errors in the mathematics or facts will result in point deductions.

2. Presentation
   - 10 points: Engaging, important aspects emphasized, interactive, organized, and clear.
   - Deviations from the above criteria will result in point deductions.

3. Paper Mechanics
   - 10 points: Complete, typed words and neatly drawn diagrams, organized, clear, no spelling or grammatical errors, and proper acknowledgements.
   - Deviations from the above criteria will result in point deductions.

4. Evaluations
   - 10 points: Accurate and clearly articulated project contributions, strengths, and improvement areas for each member of the group.
   - Deviations from the above criteria will result in point deductions.