

# Errata for **Understanding Numbers in Elementary School Mathematics**

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**Page 10:** lines 1 and 2.

“hundreds place” *should be* “leftmost place” (*twice*).

**Page 16:** Replace lines  $-5$  and  $-4$  (not counting the footnote) with:

900,000) takes the same number of steps as the passage from 0 to 100,000, which is exactly 100,000 steps.

**Page 18:** line 8.

“ $173 \rightarrow 174 \rightarrow 175 \rightarrow$ ” *should be* “ $172 \rightarrow 173 \rightarrow 174 \rightarrow$ ”

**Page 19:** line  $-14$ .

“3 2” *should be* “32”

**Page 19:** lines  $-6$  and  $-5$ .

“To count to the eighth row, one must first count to 700,” *should be*  
“To count to 728, one must first count to 700 in the eighth row,”

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**Page 22:** line 3.

“pages 4–6]” *should be* “pages 4–6])”

**Page 28:** line 15.

“3 added to itself five times” *should be* “the sum of five copies of 3”

**Page 28:** line 16.

“5 added to itself three times” *should be* “the sum of three copies of 5”

**Page 31:** line 17.

10 times the *place value* of the digit to its right  
*should be*

10 times the *place value* of this same digit if it were put in  
the next place to its right

**Page 32:** lines –6 to –5.

$$\begin{aligned} &0 \times 10^8 + 0 \times 10^7 + 0 \times 10^6 + 8 \times 10^5 \\ &+ 3 \times 10^4 + 1 \times 10^2 + 5 \times 10^1 + 9 \times 10^0 \end{aligned}$$

*should be*

$$\begin{aligned} &(0 \times 10^8) + (0 \times 10^7) + (0 \times 10^6) + (8 \times 10^5) + (3 \times 10^4) \\ &+ (0 \times 10^3) + (1 \times 10^2) + (5 \times 10^1) + (9 \times 10^0) \end{aligned}$$

**Page 43:** line 4.

*Delete* “(Theorem 2.1)”.

**Page 43:** Replace the two activities near the middle of the page by:

**Activity.** Compute  $37 + 189 + 163$  and  $275 + 892 + 225 + 4211 + 108$ .

**Activity.** Compute  $666,666,667 + 788,646,851,086 + 333,333,333$ .

**Page 44:** line 12.

“876190” *should be* “871690”

**Page 46:** Replace the first four lines of the **Activity** *by*:

One can use the distributive law to multiply a one-digit number by a two-digit number using mental math. For example, to compute  $6 \times 43$ , we break up 43 into  $(40 + 3)$  so that  $6 \times 43 = 6 \times (40 + 3) = (6 \times 40) + (6 \times 3)$ , and the last sum is just  $240 + 18 = 258$ .

**Page 46:** Replace the last two lines of the page *by*:

Therefore,  $6 \times 43 = 258$ . Now follow this example and use mental math to compute: (a)  $8 \times 24$ , (b)  $7 \times 53$ , (c)  $6 \times 39$ , (d)  $79 \times 5$ , (e)  $94 \times 9$ , (f)  $47 \times 8$ .

**Page 58:** line 3.

“...this is 12 added to itself 17 times” *should be* “...this is the sum of 17 copies of 12”

**Page 62:** line  $-2$ .

“resulting numbers” *should be* “resulting number”

**Page 64:** lines  $-11$  to  $-10$ .

“the sum of the digits in the second column” *should be*  
“10 times the sum of the digits in the second column”

**Page 64:** lines  $-4$  to  $-3$ .

“the sum of the digits in the third column” *should be*  
“100 times the sum of the digits in the third column”

**Page 70:** line 3.

“nine even numbers are 2, 4, 6” *should be*

“ten even numbers are 0, 2, 4, 6”

**Page 72:** the displayed equality on line –12, namely,

$$k + n = m$$

*should be*

$$n + k = m$$

**Page 79:** below the 9th line, i.e., the line

$$11 - 9 \quad 11 - 8 \quad 11 - 7 \quad 11 - 6 \quad 11 - 5 \quad 11 - 4 \quad 11 - 3 \quad 11 - 2$$

*insert:*

$$10 - 9 \quad 10 - 8 \quad 10 - 7 \quad 10 - 6 \quad 10 - 5 \quad 10 - 4 \quad 10 - 3 \quad 10 - 2 \quad 10 - 1$$

**Page 86:** the third line *of footnote 1*.

“memorization the  $12 \times 12$ ” *should be*

“memorization of the  $12 \times 12$ ”

**Page 92:** line –4. Replace “15” by “17”.

**Page 103:** line –12.

“ $q$  as that particular multiple  $qd$  that equals  $a$ ” *should be*

“ $q$  as that whole number so that the  $q$ -th multiple of  $d$ —which is  $qd$ —is equal to  $a$ ”

**Page 104:** line –7.

“first  $q$  multiples” *should be* “first  $q$  positive multiples”

**Page 105:** line 4.

“whole numbers  $a$  and  $d$  with  $d > 0$ ” *should be*

“whole numbers  $a$  and  $d$  with  $d > 0$  and  $a$  not a multiple of  $d$ ”

**Page 108:** line –14.

“*divisions-with-remainders*” *should be* “*divisions-with-remainder*”

**Page 110:** line –1 and line –16.

“*divisions-with-remainders*” *should be* “*divisions-with-remainder*”

**Page 112:** line –10. Replace

Recall that  $q$  is the largest multiple of 3 which is less than or equal to 586.

*by*

Recall that  $q$  is the largest whole number so that  $q$  times 3 is less than or equal to 586.

**Page 115:** line –2.

“Finally, give” *should be* “Finally, we give”

**Page 116:** line 6. Replace

Recall that  $q$  is the largest multiple of 35 which is less than or equal to 1308.

*by*

Recall that  $q$  is the largest whole number so that  $q$  times 35 is less than or equal to 1308.

**Page 117:** line 9.

“dividend 195” *should be* “quotient 195”

**Page 117:** line –8.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 118:** line –15.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 119:** middle of page, third line below section 7.6.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 119:** line –7. Replace

the largest multiple of the divisor *not to exceed*

by

largest whole number  $q$  so that  $q$  times the divisor *does not exceed*

**Page 120:** This page needs several corrections.

(i) Line 8. “the dividend of 129” *should be* “the quotient of 129”

(ii) Line 17. Replace

*largest multiple of 35 not to*

by

*largest whole number  $q$  so that  $q \cdot 35$  does not*

(iii) Line –8. Replace

*largest multiple of  $d$  not to*

by

*largest whole number  $q$  so that  $qd$  does not*

**Page 123:** lines 8 and 9. Replace

“1234,5497,2086” by “123,454,972,086” (*twice*), and  
“8026,5937” by “80,265,937”.

**Page 124:** Exercise 14.

“division of 652 by 8” *should be*  
“division-with-remainder of 652 by 8”

**Page 124:** second line of Exercise 16.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 145:** line  $-5$  to line  $-2$ . Replace

the **leading digit** (i.e., the left digit) *... therefore  $m + n$  is at least 2000*. Thus

*by*

we know  $n \geq 1000$  and  $m \geq 1000$ , so that

**Page 147:** line 1.

“\$7,500,000,000” *should be* “\$7,500,000,000,000”

**Page 147:** line 13.

“is 102,743” *should be* “is listed as 102,743 (in year 2010)”

**Page 147:** line  $-19$ . Replace

“*at the least,*” by “*at the most,*”

**Page 156:** line 12. Replace

whole number  $b$ , *using only multiples  $< b$* .

*by*

whole number  $b > 1$ , with the restriction that in any such multiple  $qb^m$ , where  $q$  and  $m$  are whole numbers, *we require  $q < b$* .

**Page 159:** This page needs several corrections.

(i) Between lines 6 and 7, insert:

$$1 = 0 \cdot 7 + 1$$

(ii) Line 7.

Because  $1 < 7$  in the last equation, the process stops. Also observe

*should be*

It is clear from the last equation that there is no need to continue. Observe

(iii) Line 8.

“divisor (i.e., 4, 2, 4, 3) is” *should be* “divisor—4, 2, 4, 3, 1—is”

(iv) Line –15 to line –14.

Notice that among the coefficients 1, 3, 4, 2, 4 of the base 7 representation of 3644, {3, 4, 2, 4} are

*should be*

Notice that the coefficients 1, 3, 4, 2, 4 of the base 7 representation of 3644 are

(v) Line –14 to line –13.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 160:** line –9 to line –8.

“divisions-with-remainders” *should be* “divisions-with-remainder”

**Page 161:** line 1 to line 2.

“divisions-with-remainders” *should be* “divisions-with-remainder”



**Page 162:** the subtraction near the bottom of the page,

$$\begin{array}{r} \phantom{-} \phantom{2} \phantom{5} \phantom{6} \\ \phantom{-} \phantom{2} \phantom{5} \phantom{6} \\ \phantom{-} \phantom{2} \phantom{5} \phantom{6} \\ \hline \phantom{-} \phantom{2} \phantom{5} \phantom{6} \end{array}$$

*should be* (the changes are indicated in red):

$$\begin{array}{r} \phantom{-} \phantom{2} \phantom{5} \phantom{6} \phantom{12} \\ \phantom{-} \phantom{2} \phantom{5} \phantom{6} \phantom{12} \\ \phantom{-} \phantom{2} \phantom{5} \phantom{6} \\ \hline \phantom{-} \phantom{2} \phantom{5} \phantom{6} \end{array}$$

**Page 166:** line 14.

$$\underbrace{(11 \cdots 1)}_m \phantom{)}_2 + (1)_2 = (1 \underbrace{00 \cdots 0})_m$$

*should be* (the change is indicated in red):

$$\underbrace{(11 \cdots 1)}_m \phantom{)}_2 + (1)_2 = (1 \underbrace{00 \cdots 0})_m \phantom{)}_2$$

**Page 166:** The computation on line 18,

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \hline \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \end{array}$$

*should be* (the change is indicated in red):

$$\begin{array}{r} \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \\ \hline \phantom{+} \phantom{1} \phantom{0} \phantom{0} \phantom{0} \phantom{0} \end{array}$$

**Page 167:** the last computation on the page, i.e.,

$$\begin{array}{r} \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \times \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 1 \phantom{0} \phantom{1} \phantom{1} \phantom{0} \phantom{1} \end{array}$$

*should be* (the change is indicated in red):

$$\begin{array}{r} \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \times \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \phantom{\times} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ + \phantom{1} \phantom{1} \phantom{1} \phantom{1} \phantom{1} \\ \hline 1 \phantom{0} \phantom{1} \phantom{1} \phantom{0} \phantom{1} \end{array}$$

**Page 168:** Exercise 6 (a) and (c).

“ $1 + 2^2 + 2^3 + 2^4 + 2^5 = 2^6 - 1$ ” *should be*

“ $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 = 2^6 - 1$ ” (*twice*)

**Page 183:** line 14.

“this chapter” *should be* “Part 2”

**Page 184:** line -4.

“*nonzero* whole number” *should be* “whole number”

**Page 184:** line -3.

“1, 2, 3, 4, ...” *should be* “0, 1, 2, 3, 4, ...”

**Page 185:** line 6.

“for any whole number  $m > 0$  and  $\ell > 0$ ” *should be*

“for any whole numbers  $m$  and  $\ell$ , with  $\ell > 0$ ”

**Page 186:** line 15. Replace it by the following:

To the extent that we identify a fraction  $\frac{m}{n}$  with the segment  $[0, \frac{m}{n}]$ , we will abuse the language and use the same sentence to express

$[0, \frac{m}{n}]$  is the concatenation of  $m$  segments, each of length  $\frac{1}{n}$ .

This is the terminology that will be used in the rest of Part 2.

**Page 188:** line 11.

“We will do that in the next section” *should be* “We will do that in Section 13.2”.

**Page 200:** line 7.

“48” *should be* “47” (*twice*), and “203” *should be* “202”

**Page 205:** line -7.

“ $\frac{38}{57} = \frac{3}{2}$ ” *should be* “ $\frac{38}{57} = \frac{2}{3}$ ”

**Page 211:** line -14.

“ $\frac{5}{3}$  is  $5 \times 4$ ” *should be* “ $\frac{4}{3}$  is  $4 \times 5$ ”

**Page 214:** line 11.

“6.122959” *should be* “6.12295”

**Page 224:** lines 5 and 6.

“ $\frac{k}{l}$ ” *should be* “ $\frac{k}{\ell}$ ”

“ $\frac{k+m}{l+n}$ ” *should be* “ $\frac{k+m}{\ell+n}$ ”

“ $l = n = 2$ ” *should be* “ $\ell = n = 2$ ”

**225:** line  $-15$  and line  $-11$ .

$\frac{r}{l}$  should be  $\frac{r}{\ell}$

**Page 232:** Exercise 11, second line.

$[4\frac{1}{3}, E]$  should be  $[3\frac{1}{3}, E]$

**Page 232:** Exercise 11, the number-line picture.

$4\frac{1}{3}$  should be  $3\frac{1}{3}$

**Page 246:** Activity at the bottom of the page. Add:

Your explanation for (b) should be directly in terms of the mixed number  $3\frac{3}{7}$ .

**Page 254:** line 7.

$\frac{k}{l}$  should be  $\frac{k}{\ell}$

$\frac{kn}{ln}$  should be  $\frac{kn}{\ell n}$

**Page 258:** the third line of Exercise 4.

$\frac{7}{8}$  should be  $\frac{17}{18}$

**Page 258:** the second line of Exercise 7.

“and 1” should be “or 1”

**Page 264:** bottom line.

$\frac{1}{\ell}$  should be  $\frac{1}{7}$

**Page 277:** line 9.

$\frac{39}{15}$  should be  $\frac{38}{15}$

**Page 284:** line -4.

“ $m > n$ ” *should be* “ $m \geq n$ ”

“ $A > B$ ” *should be* “ $A \geq B$ ”

**Page 287:** line -7 (not counting the number-line picture).

“ $\frac{7}{3} = \frac{5}{4} \times C$ ” *should be* “ $\frac{7}{3} = C \times \frac{5}{4}$ ”

**Page 291:** lines -6 and -5.

... nonzero whole number  $n$ ,

$$\frac{1}{\frac{1}{n}} = n.$$

*should be*

... nonzero whole number  $\ell$ ,

$$\frac{1}{\frac{1}{\ell}} = \ell.$$

**Page 294:** there are several corrections.

(i) Line 1.

“being 66 mph” *should be* “is 66 mph”

(ii) Line 2.

“in any time interval” *should be* “in all time intervals”

(iii) Line 6.

“it will *all* be 48 mph.” *should be* “it will *always* be 48 mph.”

(iv) Line 11.

“ $s$  miles.” *should be* “ $s$  feet.”

**Page 296:** line 3.  
“*m*” *should be* “*T*”

**Page 300:** line 15.  
“is worthwhile” *should be* “it is worthwhile”

**Page 302:** line 16.  
“the fraction  $\frac{2}{7}$ ” *should be* “the fraction”

**Page 302:** line -4.  
“the only hand” *should be* “the other hand”

**Page 306:** Exercise 9. Add the assumption that *Shawna drives at a constant speed.*

**Page 307:** line 1.  
“same speed” *should be* “same constant speed”

**Page 311:** line 7.  
“(a)  $\leftrightarrow$  (17.4)” *should be* “(a)  $\leftrightarrow$  equation above (17.3)”

**Page 316:** in the second displayed equation near the middle of the page, the left side

$$\frac{1}{1 - \frac{2}{3}} + \frac{1}{1 - \frac{2}{3}}$$

*should be*

$$\frac{1}{1 - \frac{2}{3}} + \frac{1}{1 + \frac{2}{3}}$$

**Page 317:** second line of Exercise 4.  
“(15.2) on page 240” *should be* “(14.5) on page 228”

**Page 320:** line –13.  
“to to” *should be* “to”

**Page 324:** line –4.  
“If the low price” *should be* “If the high price”

**Page 328:** Exercise 3.  
“20%” on the fourth line of the exercise *should be* “15%”

**Page 344:** lines –8 and –7.  
(see the introduction of Chapter 17) and in the definition of  
*should be*

(see the initial paragraph of Section 17.1) and in the discus-  
sion of

**Page 344:** line –6.  
“page 292.” *should be* “page 295.”

**Page 349:** line 7.  
“ $85\frac{1}{5}$ ” *should be* “ $85\frac{1}{15}$ ”

**Page 350:** the last sentence of *footnote 6*:  
“It something is true” *should be* “If something is true”

**Page 350–351:** replace the passage starting from the second para-  
graph, line 12 on page 350 (“We are going to show ...”) to line 7 on  
page 351 (“... is  $5\frac{5}{11}$  hours.”) by the following simpler argument.

Because Regina mows at a constant rate, if she mows  $B$   
sq. ft. in  $t$  hours, then  $\frac{B}{t} = R$ , so that  $B = Rt$  sq. ft.  
Thus the amount of lawn mowed by Regina in  $t$  hours is  $Rt$

sq. ft. Similarly the amount of lawn mowed by Eric in  $t$  hours is  $E t$  sq. ft. The amount of lawn mowed by Regina and Eric together in  $t$  hour is therefore

$$(R + E)t \text{ sq. ft.}$$

Suppose now Regina and Eric together finish mowing the lawn in  $T$  hours. Then in  $T$  hours they mow  $A$  sq. ft. Hence

$$(R + E)T = A$$

But we know the values of  $R$  and  $E$ , so

$$\left(\frac{A}{10} + \frac{A}{12}\right)T = A$$

It follows that

$$\begin{aligned} T &= \frac{A}{\frac{A}{10} + \frac{A}{12}} \\ &= \frac{1}{\frac{1}{10} + \frac{1}{12}} \quad (\text{by (b) on page 310}) \\ &= \frac{1}{\frac{11}{60}} = 5\frac{5}{11} \end{aligned}$$

So the time it takes both of them to do it together is  $5\frac{5}{11}$  hours.

**Page 353:** line  $-5$ .

“16 HOURS” *should be* “12 HOURS”

**Page 354:** line 2.

“ $\frac{H}{16}$ ” *should be* “ $\frac{H}{12}$ ”

**Page 354:** replace the passage from line 4 (“exactly as in ...”) to the end of the page by the following:



exactly as in the preceding Problem 6, we conclude that, if they paint together, they'd paint  $(J + L + M)t$  sq. ft. in  $t$  hours for any number  $t \geq 0$ . Now using the values of  $J$ ,  $L$ , and  $M$  above, we see that, working together, they'd paint

$$\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) Ht \quad \text{sq. ft. in } t \text{ hours.}$$

If they finish painting the whole house ( $H$  sq. ft.) in  $T$  hours, then

$$\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) HT = H.$$

Since  $\left(\frac{1}{18} + \frac{1}{15} + \frac{1}{12}\right) = \frac{37}{180}$ , we get,

$$T = \frac{1}{\frac{37}{180}} = 4\frac{32}{37} \quad \text{hours.}$$

The answer is therefore 4 and  $\frac{32}{37}$  of an hour, or approximately 4 hours and 52 minutes.

**Page 355:** replace the second and third lines of Exercise 1 by:

proved that (22.1) holds. Now prove the converse of (22.1), i.e., if  $\frac{A}{\ell} = \frac{B}{m} = \frac{C}{n}$ , then  $A : B : C = \ell : m : n$ .

**Page 358:** line -13 (not counting the footnote).

*Change* the period at the end of the line to a comma.

**Page 361:** line 9 (not counting the picture).

“look a the” *should be* “look at the”

**Page 364:** Exercise 3, second line.

“make this increase” *should be* “make this reduction”

**Page 365:** line 1.

“alcohol is 70% of the liquid” *should be* “alcohol is 70% of the liquid in terms of volume”

**Page 370:** the first full paragraph near the middle of the page.

“Sometimes one could” *should be* “Sometimes one can”

**Page 384:** Line  $-8$  (counting the footnote but not counting the number lines).

“shorter than  $y$ ,” *should be* “shorter than  $\vec{y}$ ,”

**Page 387:** line 17.

“and (vii) above,” *should be*

“and the definition of the addition of rational numbers”

**Page 388:** there are several corrections.

(i) Lines 1.

“(vi)” *should be* “(viii)”

(ii) Lines 3 and 4.

“picture drawing” *should be* “picture-drawing”

(iii) Line 10.

Delete “This also follows from (vi).”

**Page 389:** second number-line picture.

The notch to the immediate right of the number 1 should be labeled as  $\frac{7}{6}$ .

**Page 389:** line  $-5$ .

“ $\overrightarrow{(t-s)}$ ” *should be* “ $\overrightarrow{(t-s)^*}$ ”

**Page 392:** line –13.

“Using (27.8) twice in succession” *should be* “Using (27.1) twice in succession”

**Page 394:** Exercise 5, first line.

“then  $x = 0$ ” *should be* “then  $y = 0$ ”

**Page 399:** line 7.

“positive” *should be* “nonzero”

**Page 402:** Exercise 3 should be moved to the Exercises of Chapter 29 on page 412 (because this exercise requires that we know how to multiply rational numbers).

**Page 404:** line 5.

“This sections” *should be* “The sections”

**Page 404:** line –5 (counting footnote).

“(by (M2))” *should be* “(by (M2))”

**Page 406:** lines 14–15. These two lines,

$$\begin{aligned} &= 0 \cdot (-1) && \text{(by (M3))} \\ &= 0. \end{aligned}$$

*should be*

$$\begin{aligned} &= 0 \cdot (-1) \\ &= 0, && \text{(by (M3)).} \end{aligned}$$

**Page 414:** line 4.

“Basic Fact 4” *should be* “Basic Fact 5”

**Page 415:** line –14.

“ $x, y z$ ” *should be* “ $x, y$ ”

**Page 422:** line –11

“ $-(x - y) < 0$ ” *should be* “ $-(x - y) > 0$ ”

**Page 424:** line –2.

“By (E),” *should be* “By (D),”

**Page 426:** There are several corrections.

(i) Line 3.

“ $1 > x^3, 1 > x^3$ ” *should be* “ $1 > x^3, 1 > x^4$ ”

(ii) Line –9.

is equivalent to the two inequalities  $-b < x \leq 0$  and  
 $0 \leq x < b$ ,

*should be*

is equivalent to  $x$  not being equal to  $-b$  or to the left of  $-b$ ,  
and not being equal to  $b$  or to the right of  $b$ ,

(iii) Line –5.

“ $[a, b]$ ” *should be* “ $[-b, b]$ ”

(iv) line –3.

“ $x \in (-b, b)$ ” *should be* “ $x$  lies in  $(-b, b)$ ”

**Page 433:** second paragraph.

“Among the highlights of this chapter” *should be*

“Among the highlights of Part 4”

**Page 440:** line –6.

“ $84 = 26 \times 3$ ” *should be* “ $84 = 28 \times 3$ ”

**Page 441:** line –2.

“Lemma 32.1. If 572” *should be:*

Lemma 32.1. Since we know 4 does not divide 86, 4 does not divided 93,386 either, and vice versa. If 572

**Page 443:** line 4.

“ $k \geq 2$ ” *should be* “ $k \geq 1$ ”

**Page 444:** Exercise 1.

The order of (a) and (b) should be reversed. In other words, the first part of the exercise *should read:*

(a) *Prove* that every odd number is of the form  $2n + 1$ , where  $n$  is a whole number. (b) Prove that the product of two odd numbers is odd, the product of two even numbers is even, and the product of an odd number and an even number is even.

**Page 446:** There are several corrections.

(i) Line 19:

“*can divide 97.*” *should be* “*can be a proper divisor of 97.*”

(ii) Line –18:

“a divisor of 97.” *should be* “a proper divisor of 97.”

(iii) Line –16:

“*a divisor of n.*” *should be* “*a proper divisor of n.*”

(iv) Line –11:

“no divisors” *should be* “no proper divisors”

**Page 449:** line 14.

“ $2 \geq \ell \leq \sqrt{k}$ ” *should be* “ $2 \leq \ell \leq \sqrt{k}$ ”

**Page 449:** line –16.

*what remains cannot be divisible by*

should be

*what remains cannot be proper multiples of*

**Page 450:** There are three lists of certain whole numbers up to 144, and there are errors in the first two lists. In both lists of numbers, 135 and 141 *should be deleted, and 133 and 137 should be added.*

**Page 451:** line –2 of the footnote.

“beyond 4” *should be* “beyond 2”

**Page 452:** bottom line.

“ $2 \times 10^{10}$ ” *should be* “ $1.5 \times 10^{10}$ ”

**Page 453:** lines 8–10. These three lines *should read:*

both primes, but 5040, 5041, 5048, 5049, and 5050 are not (the fact about 5041 is not obvious, see Exercise 7 below). So we get in fact a string of not 6 but 11 consecutive composites from 5040 to 5050 as a bonus. A more startling fact is that it is not at

**Page 453:** line 11.

“9” *should be* “11”

**Page 454:** Exercise 4.

“20736” *should be* “21316 (=  $146^2$ )”

**Page 460:** line 6.

“from 2 to  $\frac{91}{2}$ ” *should be* “from 2 up to 13”

**Page 460:** line 8.

primes from 2 to  $\frac{n}{2}$

*should be*

primes from 2 to the counterpart of 13;

**Page 463:** lines  $-7$  and  $-6$  *should be* (the changes are indicated in red):

1, 2, 3, 4, 6, 8, 12, 24,  
1, 2, 3, 6, 9, 18, 27, 54.

**Page 463:** line -4.

“2, 6,” *should be* “1, 2, 3, 6,”

**Page 464:** lines 1 and 2 *should be* (the changes are indicated in red):

1, 3, 7, 9, 11, 21, 33, 63, 77, 99, 231, 693,  
1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210.

**Page 464:** line 4 *should be:*

1, 3, 7, 21,

**Page 465:** second sentence of the first full paragraph.  
“The is so” *should be* “This is so”

**Page 465:** line –17.  
“by iterating the process” *should be* “iterating the process”

**Page 469:** line –15 *should be:*

$$r_1 = q_3 \boxed{r_2} + 0$$

**Page 474:** There are several corrections.

(i) Line 5:

The first sentence of the **Lemma** *should be* “Let  $b$  and  $n$  be whole numbers  $\geq 2$ .”

(ii) Line 16:

“There is related concept.” *should be* “There is a related concept.”

(iii) Line –11.

“108, 216, 270, 324, ...” *should be* “108, 216, 324, ...”

**Page 475:** the listing of multiples of 210 in lines 14–16.  
*Insert* the number 2310 between 2100 and 2520 on line 14.

**Page 479:** line 15.

“ $\gcd(c, d) = 1$ ” *should be* “ $\gcd(a, b) = 1$ ”

**Page 479:** lines 20 and 21.

“Recall that we also have  $aB = Ab$ ,” *should be* “Since  $aB = Ab$ ,”

**Page 481:** line 6.

“prime decomposition of  $n$ ” *should be* “prime decomposition of  $m$ ”



**Page 481:** lines  $-14$  and  $-13$  (counting footnote).

until only one of  $a$  and  $b$  is even and the other odd.

*should be*

until at least one of  $a$  and  $b$  is odd.

**Page 482:** There are many errors.

(i) Line 7.

“of  $b^2$ ” *should be* “of  $b$ ”

(ii) The right side of equation (36.1) *should be* “ $3p_1^2 p_2^2 \cdots p_k^2$ ”

(iii) 4th line below equation (36.1), at the very end of the line,  
“ $p_1 p_2 \cdots p_k$ ” *should be* “ $p_1^2 p_2^2 \cdots p_k^2$ ”

(iv) 5th line below equation (36.1), *delete* “according to Lemma 36.3”

(v) 7th line below equation (36.1), “ $3p_1 p_2 \cdots p_k$ ” *should be* “ $3p_1^2 p_2^2 \cdots p_k^2$ ”

(vi) 8th line below equation (36.1), “ $3p_1 p_2 \cdots p_k$ ” *should be* “ $3p_1^2 p_2^2 \cdots p_k^2$ ”

(vii) 10th line below equation (36.1), “ $3p_1 p_2 \cdots p_k$ ” *should be* “ $3p_1^2 p_2^2 \cdots p_k^2$ ”

(viii) Two lines above equation (36.2).

“ $b^2$ ” *should be* “ $b$ ”

(ix) Line  $-7$ .

the prime decomposition  $q_1 q_2 \cdots q_k$

*should be*

the prime decomposition  $q_1^2 q_2^2 \cdots q_k^2$

(x) Line -7 to line -6, *delete* “according to Lemma 36.3”

(xi) Line -4.

But  $3b^2 = a^2$ , so by FTA,  $pq_1q_2 \cdots q_k$

*should be*

But  $pb^2 = a^2$ , so by FTA and equation (36.2),  $pq_1^2 q_2^2 \cdots q_k^2$

(xii) Line -1.

in  $pq_1q_2 \cdots q_k$

*should be*

in  $pq_1^2 q_2^2 \cdots q_k^2$ , which is the right side of (36.2),

**Page 483:** line 17.

Putting these prime decompositions

*should be*

Substituting these expressions of  $a^2$  and  $(5b)^2$  as products of primes

**Page 506:** line -12.

“ $76 < 79$ ” *should be* “ $77 < 79$ ”

**Page 506:** line -3.

“*Suppose two finite decimals*” *should be*

“*Suppose two unequal finite decimals*”

**Page 507:** line 10.

“*A and B*” *should be* “two decimals  $A$  and  $B$  with at most  $k$  decimal

digits”

**Page 507:** line 13.

“ $B$  has” *should be* “ $B \times 10^k$  has”

**Page 508:** line 6.

“0.000724” *should be* “0.0007624”

**Page 512:** line 12.

The exponent of 10 in each case

*should be*

The product of the digit with the power of 10 in each case

**Page 523:** in exercise 5, the order of (d) and (e) should be interchanged. In other words, it *should be*:

(d)  $0.\overline{142857}$ , (e)  $1.\overline{285714}$ ,

**Page 525:** third line of Theorem 42.1.

$m \times 10^k$  by  $n$ , where  $k$  is any whole number  $\geq n$

*should be*

$m \times 10^c$  by  $n$ , where  $c$  is any whole number  $\geq n$

(The change in notation from  $k$  to  $c$  is to avoid confusion in the proof of Theorem 42.1 on pp. 528 ff., where the letter  $k$  is used for a different purpose.)

**Page 526:** As a result of the change in the exponent from  $k$  to  $c$  on page 525, the exponents “ $k$ ” on lines 1, 3, 4, and 8 (twice) should be

changed to the exponent “ $c$ ”.

**Page 527:** line –3.

“ $\frac{1818}{1000}$ ” *should be* “ $\frac{1818}{10000}$ ”

**Page 528:** line –2.

“For definiteness,” *should be* “Since the two cases are similar, for definiteness,”

**Page 529:** Add to the end of **Remark** (1) in middle of page:

Thus the whole number  $k$  can be made as large as we like and the result does not change. This partly explains the choice of the whole number  $c$  in the statement of Theorem 42.1 to be larger than perhaps strictly necessary (it doesn’t matter, as we have just seen). The other reason why  $c$  is chosen to be so large is that in the *repeating case* of the theorem (see pp. 530 ff.),  $c$  may have to be this large in order to get a repeating block of the decimal.

**Page 530:** line 6 of the **Remark**.

The first two “ $<$ ” *should be* “ $\leq$ ”. In other words, this line *should read*:

Thus  $7q_n = (3 \times 10^n) - r_n \leq (3 \times 10^n)$ , so that  
 $q_n \leq \frac{3}{7} \times 10^n < 1 \times 10^n = 10^n$ ,

**Page 535:** line 3.

“ $3 \times 10^{108}$  (or  $3 \times 10^k$  for any” *should be* “ $2 \times 10^{108}$  (or  $2 \times 10^k$  for any”

**Page 535:** lines 15–16.

“by 7 above (42.2)” *should be* “on page 533”

**Page 535:** line -3.

“ $10^n - r_n < 10^n$ ” *should be* “ $10^n - r_n \leq 10^n$ ”

**Page 535:** Line -1 *should be* (the changes are indicated in red):

$$q_n \leq \frac{1}{28} \times 10^n < 1 \times 10^n = 10^n,$$

**Page 536:** First line to the right of the first long division.

$$1 \times 10^3 = (35 \times 28) + 2$$

*should be*

$$1 \times 10^3 = (35 \times 28) + 20$$

**Page 547:** left column. Above “division of finite decimals” (between line -15 and line -14), insert

division by zero, 132–133

**Page 547:** right column. Line -5.

“reduced, **205**” *should be* “reduced, **206**”

**Page 548:** insert right below line -12 (as the last item of the “ell” group, right below “117–119”)

lowest terms (fraction), **206**

**Page 549:** left column. Above “number” (between line -3 and line -2), insert

NSF-EHR, 367

**Page 550:** left column. Insert between line -4 and line -3 (above “reducing fractions”):

reduced fraction, **206**

**Page 551:** right column. “Williams, E. M., 22” *should be* “Willingham, D. T., 41”