Teaching School Mathematics: Algebra by Hung-Hsi Wu

Index

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Index

Symbols

| N (whole numbers) 119 | AA criterion (= angle-angle criterion) 62 , 69, |
|----------------------------------------------------------------------|--------------------------------------------------------|
| Q (rational numbers) 160 | 255, 269 |
| R (real numbers) 1 | above a line 165 |
| β^x (β to power x) 198 | abstract (abstraction) xii, 16, 27, 118, 186 |
| $\sqrt[n]{\alpha}$ (the positive <i>n</i> -th root of α) 202 | absolute value 65–66, 111, 125, 161 –162 |
| $x \cdot y$ (product of x and y) 20 | achieve a maximum 239 , 250 |
| x (absolute value of x) 161 | achieve a minimum 239 , 250 |
| [a, b] (closed interval) 266 | achieving the maximum value 159, 181–182, |
| (a,b) (open interval) 267 | 186–188 |
| < (less than) 160 | achieving the minimum value 159, 187–188 |
| \leq (less than or equal to) 160 | algebraic expression 221 |
| > (greater than) 160 | algebraic identity (= <i>identity</i>) xiii, 6, 7–10, |
| ≥ (greater than or equal to) 160 | 12, 15–16, 29, 199, 220, 234 |
| $\{ax + by \ge c\}$ (the graph of $ax + by \ge c$) | algorithm xix, 23 |
| 163 | cross-multiplication 45, 79, 92, 99, 149, |
| (is parallel to) 54 | 151–152, 215, 261, 270 |
| ⊥ (is perpendicular to) 54 | standard algorithms in arithmetic 41 |
| $\{A\}^{-1}$ (multiplicative inverse of $\{A\}$) 7 | alternate interior angles 54, 95, 265 |
| | angle(s) |
| $\binom{n}{k}$ (binomial coefficient) 11 | alternate interior 54, 95, 265 |
| \cong (triangle congruence) 266 | corresponding 62, 67, 113, 266 |
| $\sim \text{(similar)} 254$ | angle-angle criterion $(= AA \ criterion)$ |
| $\angle AOB$ (angle with vertex O) 265 | annual interest rate 213 |
| $A \subset B$ (A is contained in B) 242 | associated double-inequality (of an absolute |
| PQ (the segment joining P to Q) 268 | value) 190 |
| PQ (the length of segment PQ) 54 | associative law 6, 40, 196 |
| L_{PQ} (the line joining P to Q) 54 | average rate 49 |
| R_{OP} (the ray from O to P) 82 | average speed (in a time interval) 46 |
| \overrightarrow{AB} (the vector from A to B) 269 | axis of symmetry xv, 266 |
| \cup (union) 175 | axis of symmony xv, 200 |
| \cap (intersection) 175 | В |
| 6) | D.1.1 |
| \mathbf{R}^2 (coordinate plane) 56 | Babylonians 104, 224, 236 |
| (x,y) (coordinates of a point in the plane) 56 | base (of an exponential function) 195 , 196, |
| \mathcal{L}^+ , \mathcal{L}^- (half-planes of a line L) 164 | 198 |
| L L Haif-blanes of a fille L/ 104 | basic isometries 57, 224 |

Α

| Basic Protocol in the use of symbols 4, 28, 41 basic rules (for doing exercises 5, 10, 16 Behr, Merlyn J. 144, 273 below a line 165 big numbers 214 bilateral symmetry (= reflectional symmetry) 241, 243, 245, 248, 266 binomial coefficients 11, 266 binomial theorem 11 boundary 187 bounded region 178, 187–189 | continuous (function) 198 conventions 7, 19–20, 24, 47, 49, 67, 104, 202 for coordinates of a point 67 for order of operations 7, 20 for rate problems 47, 49 for the radical symbol 202 for writing polynomials 20 for writing Pythagorean triples 104 for writing rational expressions 24 when conventions should be ignored 30, 236 convex (figure) 177, 180, 188 coordinate axes 55 |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| CCSSM (= Common Core State Standards for Mathematics) xvi-xvii, 273 clearing the denominators 42 closed half-plane 175 co-domain of function 117 coefficient 19, 20, 91 leading 225 coherence, coherent xii, xiv, 194, 199 importance of 16, 60, 98, 102, 104–105, 137–140, 223, 196–198, 223–224 collect terms 18, 21 Common Core State Standards for Mathematics (= CCSSM) xvi-xvii, 273 commutative law 2, 6, 40, 196 | convention for coordinates of a point 67 system (in the plane) 54 scaled 131, 134 , 197 coordinates of a point 55 corollary xix corresponding angles (of a transversal) 62, 67, 113, 266 criterion for solvability of a linear system 101 cross-multiplication algorithm 45, 79, 92, 99, 149, 151–152, 215, 261, 270 cube root 202 cubic, cubic polynomial 19 , 130 |
| completing the square xvi, 223–224, 229, 247, 262 reason for the name 236–237 congruence 62, 239, 254–258 conic section 132, 274 constant rate 49 constants 5 constant speed 47 constant term 139 constraints 158 continuous problems (in proportional reasoning) 145–146, 148, 152–154 | data points 310–312, 318 decimal (finite) 221 defining equation of a line 80 definition xii–xv, xix, 46, 66, 70–71, 118 importance of xiv, xvi, 53, 58–59, 64, 85– 87, 109, 126–127, 144–145, 163–164, 166, 178, 192–193, 199, 223, 249, 254 degree of polynomial 19 –20, 23 degree of a product of polynomials 23 Descartes, René 2, 14, 40 determinant 100 –101 |

| diagonal line (in coordinate plane) 114 | exponential function xv, xvi, 193–194, |
|-----------------------------------------------------|---------------------------------------------------|
| dilated image of a parabola 255 | 198 , 200 |
| dilation 62, 82–83, 254–256, 267 | with base β 198 |
| Diophantus 2 | exponential notation 194–195, 214 |
| directrix 252 | expression (= number expression) xiii, 6–7, |
| discrete problems (in proportional reasoning) | 220–221 |
| 145 –152 | algebraic 221 |
| disjoint union 166 | number 6 |
| distance of a point from a line 54 , 56, 252 | |
| distance formula 57, 96, 258 | F |
| distance function 138–140 | factor 22 |
| distributive law 6, 8, 17–18, 21–23, 48–50, | factoring 10 |
| 196, 265 | quadratic polynomials 22–23, 233–235 |
| dot between numbers (for multiplication) | why you should not overdo it 23 |
| 20 | factorization 10 , 22 , 23, 233–235 |
| double inequality 161 | FASM (= Fundamental Assumption of |
| down parabola 249 | School Mathematics) 3, 8, 39, 47, 50, |
| _ | 68, 160, 195, 196, 206, 212, 220, 265 |
| E | Fermat, Pierre 2, 14 |
| elimination method (of solving linear | feasibility region 180 –187 |
| systems) 90–92 | figure (= $geometric\ figure$) 267 |
| empty figure 165 | bounded 187 |
| empty set 120 | convex 177 , 180, 188 |
| equal figures (sets) 61, 267 | empty 165 |
| equality of ordered pairs of points 57 | equal 61, 267 |
| equation xiii–xvi, 3, 4, 27–30, 28 , 57 | nonempty 165 |
| one equation equivalent to another 42 | similar 62 , 69, 254–255 |
| graphs of 58–60, 76–77, | finite decimal 216, 221 |
| linear 38 –45, 59 –60, 74–81 | importance in science 221 |
| quadratic 224–237, 225 , 261 | finite geometric series 15–16 |
| system of linear equations 85–87 | finite intersection of closed half-planes 187 |
| solving equations 37–44, 227–232 | focal length (of a parabola) 253 |
| equation in (of) one variable 29 | focus (of a parabola) 252 |
| equation in (of) two variables 121 | FOIL 21 |
| graph 121 | four quadrants 110 |
| equation of a line 60 | $FTA (= Fundamental \ Theorem \ of \ Arithmetic)$ |
| equivalent equations 42, 88 | 18, 270 |
| equivalent systems of equations 88 | function 117 –122 |
| exponent 195 | domain 117 |

| graph 120 | Н |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| of one variable 117, 196–198, 223, 238–251 of two variables 121 of three variables 125 value of a function at a point 117 why functions? 122–126 Fundamental Assumption of School Mathematics (= $FASM$) 3, 8, 39, 47, 50, 68, 160, 195, 196, 206, 212, 220, 265 fundamental principles of mathematics xii Fundamental Theorem of Algebra xvi , 198, 227 | half-plane (of a line) xiv, xvi, 56, 164 , 165 , 166, 172, 175, 178, 187 closed 175 in terms of the equation of the line 172 in terms of the line itself 166 open 175 horizontal line 54 , 55 , 59 house-painting problems 48–49, 139, 141 Huygens, Christiaan 14 |
| Fundamental Theorem of Arithmetic $(=FTA)$ 18, 270 | identity (= algebraic identity) xiii, 6, 7–10, 12, 15–16, 29, 199, 220, 234 |
| geometric figure (= figure) 267 geometric series (finite) 15 Gladwell, Malcolm 64 273 global warming 126 golden ratio 261–262 graph of a function of one variable 120 vertical line rule 120 of a linear equation of two variables 58, 59 | Ina's distance function 138 interpolate, interpolation 128, 197, 196–198, 200 intersection (of sets) 129, 132, 163, 175, 176–178, 180, 187–189, 267 intuition, the need of 234–235 inverse (= multiplicative inverse) isolating the variable 43, 161, isometry (see basic isometry) isoperimetric inequality for rectangles 233 K |
| of a linear inequality of two variables 159, 161, 163 of an equation in two variables 60, 121 of a collection of inequalities 163 graph of a quadratic function is a parabola 254 graphs of quadratic functions are similar 254 Great Internet Mersenne Prime Search 14, 273 Greenes, Carole xiii, 53, 273 | "key words" 27 key word syndrome 27 Kepler's Third Law 191 L Lamon, Susan 145, 273 laws of exponents 192–194, 199 for positive integer exponents 195 for real (number) exponents 205 leading coefficient 225 leading digit (of a whole number) 216 left half-plane 164 |

| lanama win | 100 111 114 115 197 199 200 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| lemma xix Lesh, Richard 144, 273 | 109, 111, 114–115, 127, 182, 200, 229–231, 233, 235, 259, 283 |
| level set 129 , 172, 178, 181 | maximum 158 , 168 |
| line defined by an equation 60 , | maximum point 159 |
| line of symmetry 249–251, 258, 266 | maximum value 159 |
| linear equation | achieving the maximum value 159, |
| in (of) one variable 38–45 | 181–182, 186–188 |
| in (of) two variables 57, 59 –60, 74–81 | Mersenne, Marin 14 |
| system of 85 –87 | Mersenne prime 14, 273 |
| linear function | Meyer, Dan 127, 273 |
| of one variable 127 –128, 137–143, | minimum point 159 |
| 148-154 | minimum value 159 |
| of two variables 158 –159, 168, 172, | achieving the minimum value 159 , 187– |
| 181, 187 | 188 |
| linear function without constant term | modeling 47 |
| 139, 148–154 | monomial 19 |
| linear inequality 159 | multiple (of a number) 18 |
| graph of 159, 161, 163 | mumple (or a number) 18 |
| linear polynomial 19 , 21 | N |
| local slope (of a line at a point) 64 –66, 68–69 | |
| lower half-plane 165 | National Council of Teachers of Mathematics |
| lowest terms 26 | (= NCTM) xii, 3, 137, 144, 150, 151, |
| lowest terms 20 | 273 |
| M | National Mathematics Advisory Panel xi, 1, |
| | 273 |
| magnitude (of a measurement) 214 | NCTM (= National Council of Teachers of |
| order of 216 | Mathematics) xii, 3, 137, 144, 150, |
| manufacturing problem 155–159, 187, 197 | 151, 273 |
| solution 180–186 | Newton's law of cooling 124 |
| mass | Newton's law of universal gravitation 262 |
| of the earth 219 | nonempty figure 165 |
| of the inner planets 219 | normal form (of a quadratic equation) 231 |
| of the outer planets 219 | n at a time 1 convention 7 10 20 24 67 104 |
| C 1 010 | notational convention 7, 19–20, 24, 67, 104, |
| of the sun 219 | 202 |
| mathematical coherence xii, xiv, 194, 199 | 202 exception to 30, 236 |
| mathematical coherence xii, xiv, 194, 199 importance of 16, 60, 98, 104–105, | 202 exception to 30, 236 n-th root 201 |
| mathematical coherence xii, xiv, 194, 199 importance of 16, 60, 98, 104–105, 137–140, 223, 196–198, 223–224 | 202 exception to 30, 236 n-th root 201 convention for the notation 202 |
| mathematical coherence xii, xiv, 194, 199 importance of 16, 60, 98, 104–105, 137–140, 223, 196–198, 223–224 mathematical reasoning ix, xi, xix, 98 | 202 exception to 30, 236 n-th root 201 convention for the notation 202 positive n-th root 201 , 209, 211 |
| mathematical coherence xii, xiv, 194, 199 importance of 16, 60, 98, 104–105, 137–140, 223, 196–198, 223–224 mathematical reasoning ix, xi, xix, 98 importance of xiii, 2, 5, 6, 15, 26, 34, | 202 exception to 30, 236 n-th root 201 convention for the notation 202 |
| mathematical coherence xii, xiv, 194, 199 importance of 16, 60, 98, 104–105, 137–140, 223, 196–198, 223–224 mathematical reasoning ix, xi, xix, 98 | 202 exception to 30, 236 n-th root 201 convention for the notation 202 positive n-th root 201 , 209, 211 |

| 0 | $\operatorname{in} x, y, z 20$ |
|--------------------------------------------------|------------------------------------------------------|
| one-step equations 44 | linear 19 , 21 |
| one-to-one correspondence 56 | root of 130 |
| open half-plane 175 | quadratic 19 , 223 ff. |
| optimization problem 155 , 157 | polynomial equation xvi, 130, 224, 227 |
| the inherent push-pull in optimization | polynomial in the number 10 20 |
| problems 156–158 | polynomial in x, y, z 20 |
| order of magnitude 216 | positive <i>n</i> -th root 201 , 211 |
| order of operations 19–20, 274 | positive square root 201, 202 , 220, 227, 231 |
| why it has no mathematical merit 20 | positive x -axis 55 |
| origin of coordinate system 55 | positive y-axis 55 |
| origin or coordinate system 30 | Post, Thomas R. 144, 273 |
| P | Postelnicu, Valentina xiii, 53, 273 |
| 1 1 004 070 074 | power 195 |
| parabola 224, 252 , 254 | precise definitions xii–xv, 46, 66, 70–71, 118 |
| and graph of quadratic function 254 | importance of xiv, xvi, 53, 58–59, 64, |
| directrix of a parabola 252 | 85-87, 109, 126-127, 144-145, 163- |
| down parabola 249 | 164, 166, 178, 192–193, 199, 223, |
| focal length of a parabola 253 | 249, 254 |
| focus 252 | precision xii, xiv, 118, 144–145, |
| sketching a parabola 253 | examples of xv-xvi, 4, 28, 33, 123–126, |
| up parabola 249 | 125, 145–146, 156–159, 165, 172, 178, |
| vertex of a parabola 253 | 240, 254 |
| parallel lines xiv–xv, 55, 62, 67, 100, 113, 253 | prime 13–14, 102 |
| characterization by slope 94 | Mersenne 14, 16 |
| Parallel Postulate 56 | proportional reasoning xiv, xvi, 137, 144–154 |
| parallelogram 54, 94 | proportional relationship 139, 148 |
| partial fraction decomposition 102–103 | proposition xix |
| Pascal, Blaise 14 | purposefulness xii |
| perpendicular lines xiv–xv, 54, 109–114 | examples of 13, 63–66, 122–126, 193, |
| characterization by slope 109 | 196–199, 206, 216–217, 236, 246, 251 |
| π 8, 201, 262 | push-pull in optimization problems 156–158 |
| plane separation (theorem) 166, 172 | Pythagorean Theorem 104 |
| Plimpton 322 cuneiform tablet 104 | Pythagorean triple 104–107 |
| plotting points on a graph 127 | primitive 104 –107 |
| polynomial 19 , 24, 102, 103, 198 | convention for writing 104 |
| cubic 19 , 130 | Q |
| degree of 19 –20, 23 | |
| equation xvi, 130, 224, 227 | quadrants 110 |
| factorization of 22 | |

| quadratic equation 224–237, 225 , 261 graph of 226 | S |
|-----------------------------------------------------------|-------------------------------------------------------------------------------------|
| vertex form of 231 , 235–236 | satisfy |
| quadratic formula 23, 223, 232 –235, 236, 251 | an ordered pair satisfying an equation |
| quadratic function xv-xvi, 57, 223 –224, 238– | or inequality 57, 58, 59 , 60 |
| 251 | an ordered pair satisfying a system of |
| graph of 239–246 | equations or inequalities 158, 159 |
| representations of 251 | scaled coordinate system 129–130, 133– 134 , |
| vertex form of 238 , 242, 246 | 197 |
| quadratic polynomial 19, 223 ff. | scientific notation 216 |
| quadratic polynomial function (= | set(s) |
| $quadratic\ function)$ | equality of 61, 267 |
| quantification 4 | intersection of 129, 132, 163, 175 , 176–178, 180, 187–189, 267 |
| R | union of 163, 166, 175 , 269 |
| IX. | set up a coordinate system 54–57 |
| range of function 117 | set up a coordinate system 34–37 set up a proportion 144, 145, 149 , 150, |
| rational exponents 192–194, 198, 200–204, | 152, 154 |
| 206–212 | Siegler, Robert 144, 274 |
| rational expressions 24, 26, 45, 102 | similar figures 62 , 69, 254–255 |
| convention about 7, 24 | similar triangles xv–xvi, 62, 70 |
| reduced to lowest terms 26 | similarity 254–258, 268 |
| real numbers (R) 1, 2 , 3, 7 | simultaneous (linear) equation (= system |
| real-valued function 117–122 | of linear equations) 85 |
| graph of 120–122, 126–129 | sketching a parabola 253 |
| reason multiplicatively 154 | slope (of a line) 66 –70, 72, 74–76, 80, 94 |
| reasoning (= proofs) xii, xix, 53, | identification from the graph xiii |
| importance of xiii, 37, 39, 60, 78, 87, | small numbers 214 |
| 154, 178, 193–194, 205, 223–224 | solution |
| reflection 57, 114–115, 126, 134, 239, 241, | |
| 243, 268 | of a linear equation of two variables 57, 59 |
| relation 120 | of a linear system 85–86 |
| relatively prime (whole numbers) | of a polynomial equation 130 |
| 108, 268 | of an equation of two variables 60 |
| representations of a quadratic function 251 | solve an equation 28 |
| right half-plane 164 | according to TSM 38–39 |
| Robson, Eleanor 104, 273 | correct procedure 39–44, 227–232 |
| root of a function 223 | solve a linear system 85 |
| root of a polynomial 130 | solving quadratic equations 223, 227–232, |
| Ross, Kenneth A. 189, 201, 273 | 236 |

| speculative reasoning 193 square root 201 positive 201, 202 , 220, 227, 231 standard algorithm in arithmetic 41 Stanley, Dick 145, 147, 274 Stump, Sheryl 53, 274 substitution method (of solving linear systems) 87–90 summation formula for finite geometric series 15 | translation of verbal data into symbolic language 27, 30–33 transposing terms 43–44 trichotomy law 160 trinomial 19 caution about 19 TSM (= Textbook School Mathematics) xi–xvii, 1, 10, 20, 28, 29, 37, 38, 41, 53, 54, 58, 79, 87, 117, 119, 121, 127, 128, 137, 139, 144, 147, 149, 150, 152– |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| symbols, correct use of 2–5 symmetry axis of xv, 266 bilateral 241 , 243, 245, 248, 266 | 154, 164, 178, 193, 194, 199, 206, 223, 224, 234, 249, 251, 254 |
| line of 249–251, 258, 266 symmetry of the similarity relation 256 system of linear equations (= simultaneous (linear) equations) 85 solution of 85 –86 solution by elimination 90–92 solution by substitution 87–90 | unbounded region 177 union (of sets) 163, 166, 175 , 269 unit rate 147 –148 unknown 4 , 48, 85–87, 103 up parabola 249 upper half-plane 165 |
| т | V |
| target of function 117 term 11, 19–20 constant 139 Teukolsky, R. 132, 274 Textbook School Mathematics (= TSM) xi–xvii, 1, 10, 20, 28, 29, 37, 38, 41, 53, 54, 58, 79, 87, 117, 119, 121, 127, 128, 137, 139, 144, 147, 149, 150, 152–154, 164, 178, 193, 194, 199, 206, 223, 224, 234, 249, 251, 254 | variable xii–xv, 1–3, 5, 28–29, 38–39, 43 vertex (of a parabola) 253 vertex form of a quadratic equation 231 , 235–236 vertex form of a quadratic function 238 , 242, 246 vertical line 55 , 59 left half-plane 164 right half-plane 164 vertical line rule 120 |
| theorem xix | W |
| time interval 46–47, 138 transformation(s) (of the plane) 126, 269 translation (along a vector) 64, 126, 242–243, 259 269 coordinate expression of 95–97 | water flow 49–49, 139–140 weak inequality 160 weather forecasting 126 why functions? 122–126 |

X, Y, Z

x-axis 55 x-coordinate of a point x-intercept y-axis 55 y-coordinate of a point y-intercept zero of a function