Define a ring homomorphism φ form $\mathbb{Q}[x]$ to the filed $\mathbb{C}$ of complex numbers as follows:

$$\begin{array}{rcl} \varphi : \mathbb{Q} & \to & \mathbb{C} \\ f(x) & \mapsto & f(i), \end{array}$$

where i is the imaginary unit $i^2 = -1$. Then we have $\operatorname{Ker}\varphi = (x^2+1)$. Similarly, for $\overline{\varphi}$, defined by

$$\begin{array}{rcl} \overline{\varphi} : \mathbb{Q} & \to & \mathbb{C} \\ f(x) & \mapsto & f(-i), \end{array}$$

we have $\operatorname{Ker}\overline{\varphi} = (x^2+1)$. Hence the ideal (x^2+1) does not describe a difference between φ and $\overline{\varphi}$.