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A first course in Sobolev spaces. (English)

Graduate Studies in Mathematics 105. Providence, RI: American Mathematical Society (AMS). xvi, 607 p. $ 85.00 (2009). ISBN 978-0-8218-4768-8/hbk

The core of this book is the definition and the main properties of the first order Sobolev spaces $W^{1,p}(\Omega)$, $1 \leq p \leq \infty$, where Ω is an open set in $\mathbb{R}^N$, and Besov spaces $B^{s,p,\theta}(\mathbb{R}^N)$, $0 < s < 1$. It is divided into two parts, entitled "Functions of one variable" and "Functions of several variables", respectively.
Part 1 (consisting of Chapters 1 to 7) provides an introduction to functions of bounded pointwise variation, absolutely continuous functions, functions of bounded variation, and the Sobolev space $W^{1,p}(\Omega)$ with $\Omega \subset \mathbb{R}$ (Chapters 1 to 3 and 7). Complements on curves, Lebesgue-Stieltjes measures, and decreasing rearrangements are introduced in Chapters 4, 5, and 6.
In Part 2 (consisting of Chapters 8 to 16), absolutely continuous functions and functions of bounded variation, in the case of several variables, are studied in chapters 8 and 13. Chapter 9 is dedicated to a brief introduction to the important theory of distributions; complements on the subject can be found in the classic [*L. Schwartz*, "Théorie des distributions" (Publications de l'Institut de Mathématique de l'Université de Strasbourg; Paris: Hermann & Cie.) (1966; Zbl 0149.09501)]. Chapters 10, 11, 12, 15 are devoted to the definition of the Sobolev spaces $W^{1,p}(\Omega)$, with $\Omega \subset \mathbb{R}^N$, and their classical properties, density, embeddings, extension domains, traces. The symmetrization in Sobolev spaces and in spaces of functions of bounded variation are studied in chapter 16.
Useful complements on functional analysis and measure theory are presented in three appendices. The book contains 183 references and more than 200 exercises.
This book constitutes a solid background for advanced students who are interested in analysis, partial differential equations and applied sciences.

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Keywords : first-order Sobolev spaces; Besov spaces; absolutely continuous functions; functions of bounded variation; measure theory
Classification :
∗46-01 Textbooks (functional analysis)
46E35 Sobolev spaces and generalizations
26A24 Differentiation of functions of one real variable
26A27 Nondifferentiability of functions of one real variable
26A30 Real functions of one real variable with other special properties
26A42 Ordinary integrals of functions of one real variable
26A45 Functions of bounded variation (one real variable)
26B30 Absolutely continuous functions, etc. (several real variables)