

PROGRESS ON RESEARCH PROBLEMS

from Ramsey Theory on the Integers

Date: February 9, 2006

Note: Reference information is given at the end of this listing.

Problem 2.3

Several new mixed van der Waerden numbers have been found [LRC]:

$$w(11, 3; 2) = 114 \quad w(12, 3; 2) = 135 \quad w(13, 3; 2) = 160$$

$$w(4, 4, 3; 3) = 89 \quad w(5, 3, 3; 3) = 80 \quad w(6, 4, 2; 3) = 83 \quad w(7, 3, 2; 3) = 55$$

$$w(4, 3, 3, 2; 4) = 60 \quad w(6, 3, 2, 2; 4) = 48 \quad w(7, 3, 2, 2; 4) = 65$$

$$w(3, 3, 2, 2, 2; 5) = 20 \quad w(3, 3, 3, 2, 2; 5) = 41$$

Problem 2.5

The valid maximal-length colorings for several mixed van der Waerden numbers are given in [LRC].

Problem 5.4

That (1,1) is the only regular pair has been proven, independently, in [FLR] and [FR].

Problem 5.5

Table 5.2 has been expanded and several of the bounds improved by the results of [FLR] and [FR]. Specifically, in [FLR] it is demonstrated that $\text{dor}(2, 2) \in \{3, 4\}$ (in particular that $T(2, 2; 3) = 88$), and that, for $a \geq 1$ and $1 \leq j \leq 5$, $\text{dor}(a, 2a + j) \leq 4$.

In [FR] it is shown that $\text{dor}(a, b) \leq 5$ for all $(a, b) \neq (1, 1)$. In addition, the authors show that $\text{dor}(a, 2a - 2) = 2$ for $a \geq 4$, that $\text{dor}(a, 2a + 1) = 2$ for all a , and that $\text{dor}(3, 4) \leq 3$.

Problem 5.10

This conjecture has been proved by B. Kim and Y. Rho [KR].

Problem 5.11

All remaining cases have been completely solved in [KR].

Theorem 5.20 Improved

Gryniewicz [G] has, by elementary means, improved Theorem 5.19, by proving that for all even $b \geq 10$, $R(AUG_b, 3) \leq 2r + 10$. In fact, when combined with the lower bounds of Theorem 5.20, his result provides an exact formula for $R(AUG_b, 3)$ for all b .

Problem 10.9

This question has been answered in the affirmative (see [J]).

References

[FLR] N. Frantzikinakis, B. Landman, and A. Robertson, On the degree of regularity of generalized van der Waerden triples, *Advances Applied Math.*, to appear.

- [FR] J. Fox and R. Radoičić, On the degree of regularity of generalized van der Waerden triples, *Integers: Electronic J. Combinatorial Number Theory* **5** (1) (2005), Article A32, 6 pages.
- [G] D. Gryniewicz, On some Rado numbers for generalized arithmetic progressions, *Discrete Mathematics* **280**(1-3) (2004), 39-50.
- [J] V. Jungić, On Brown's conjecture on Accessible Sets, *J. Combinatorial Theory, Series A* **110**(1) (2005), 175-178.
- [KR] B. Kim, and Y. Rho, Van der Waerden's Theorem on Homothetic copies of $\{1, 1+s, 1+s+t\}$, preprint, [arXiv:math.CO/0410382](https://arxiv.org/abs/math/0410382).
- [LRC] B. Landman, A. Robertson, and C. Culver, Some new exact van der Waerden numbers, *Integers: Electronic J. Combinatorial Number Theory* **5**(2) (2005), Article A10, 12 pages.