PROGRESS ON RESEARCH PROBLEMS
from Ramsey Theory on the Integers

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Note: Reference information is given at the end of this listing.

Problem 2.3

Several new mixed van der Waerden numbers have been found [LRC]:

$w(11,3;2) = 114$ $w(12,3;2) = 135$ $w(13,3;2) = 160$

$w(4,4,3;3) = 89$ $w(5,3,3;3) = 80$ $w(6,4,2;3) = 83$ $w(7,3,2;3) = 55$

$w(4,3,3,2;4) = 60$ $w(6,3,2,2;4) = 48$ $w(7,3,2,2;4) = 65$

$w(3,3,2,2,2;5) = 20$ $w(3,3,3,2,2;5) = 41$

Problem 2.5

The valid maximal-length colorings for several mixed van der Waerden numbers are given in [LRC].

Problem 5.4

That (1,1) is the only regular pair has been proven, independently, in [FLR] and [FR].

Problem 5.5

Table 5.2 has been expanded and several of the bounds improved by the results of [FLR] and [FR]. Specifically, in [FLR] it is demonstrated that $\mathrm{dor}(2,2) \in \{3,4\}$ (in particular that $T(2,2;3) = 88$), and that, for $a \geq 1$ and $1 \leq j \leq 5$, $\mathrm{dor}(a, 2a+j) \leq 4$.

In [FR] it is shown that $\mathrm{dor}(a,b) \leq 5$ for all $(a,b) \neq (1,1)$. In addition, the authors show that $\mathrm{dor}(a, 2a-2) = 2$ for $a \geq 4$, that $\mathrm{dor}(a, 2a+1) = 2$ for all a, and that $\mathrm{dor}(3,4) \leq 3$.

Problem 5.10

This conjecture has been proved by B. Kim and Y. Rho [KR].

Problem 5.11

All remaining cases have been completely solved in [KR].

Theorem 5.20 Improved

Grynkiewicz [G] has, by elementary means, improved Theorem 5.19, by proving that for all even $b \geq 10$, $R(AUG_b, 3) \leq 2r + 10$. In fact, when combined with the lower bounds of Theorem 5.20, his result provides an exact formula for $R(AUG_b, 3)$ for all b.

Problem 10.9

This question has been answered in the affirmative (see [J]).

References

[FLR] N. Frantzikinakis, B. Landman, and A. Robertson, On the degree of regularity of generalized van der Waerden triples, *Advances Applied Math.*, to appear.

[FR] J. Fox and R. Radoičić, On the degree of regularity of generalized van der Waerden triples, *Integers: Electronic J. Combinatorial Number Theory* **5** (1) (2005), Article A32, 6 pages.

[G] D. Grynkiewicz, On some Rado numbers for generalized arithmetic progressions, *Discrete Mathematics* **280**(1-3) (2004), 39-50.

[J] V. Jungić, On Brown's conjecture on Accessible Sets, *J. Combinatorial Theory, Series A* **110**(1) (2005), 175-178.

[KR] B. Kim, and Y. Rho, Van der Waerden's Theorem on Homothetic copies of $\{1, 1+s, 1+s+t\}$, preprint, `arXiv:math.CO/0410382`.

[LRC] B. Landman, A. Robertson, and C. Culver, Some new exact van der Waerden numbers, *Integers: Electronic J. Combinatorial Number Theory* **5**(2) (2005), Article A10, 12 pages.