Enumerative Geometry and String Theory

by Sheldon Katz

Errata for the *first* printing as of November 7, 2011:

Page 30 line 10: After "quintic threefold." add "Hermann Schubert determined this number explicitly in [Schubert2, P. 72], noting in a footnote that the number can also be determined by the methods appearing earlier in [Schubert1, P. 192]." Thanks to Steve Kleiman for supplying this history and the references given later in this errata listing.

Page 40 line 14: Replace "but it was found for plane curves of arbitrary degree only after the techniques of stable maps inspired by physics were developed!" with "but the degree 4 characteristic numbers were not rigorously verified until after the techniques of stable maps inspired by physics were developed [Vakil]. The general problem remains unsolved." Thanks to Paolo Aluffi (who had previously rigorously verified the degree 3 characteristic numbers) for this correction.

Page 41 lines -3 to -1: Replace "degree h" with "degree e" and "degree $d+h$" with "degree $d+e$". Thanks to Yuji Shimizu.

Page 80 line 6: Replace "$Z[x_1]-Z[x_0]$." with "$Z[x_1]-Z[x_0]$, where the $Z[x_i]$ are $(2n-2)$-cycles representing the $[Z(x_i)]$.". Thanks to Yuji Shimizu.

Page 80 line 11: Replace "$H^{2n-2}(\mathbf{P}^{n-1})$" with "$H_{2n-2}(\mathbf{P}^{n-1})$". Thanks to Jinwon Choi.

Page 80 lines 17–18: Remove "where the $Z[x_i]$ are $(2n-2)$-cycles representing the $[Z(x_i)]$ deduced from the chosen triangulation". Thanks to Yuji Shimizu.

Page 100 line 10: Replace "Then" with "Then, assuming that the complex codimensions of $\sigma_{a,b}(V)$ and $\sigma_{c,d}(V')$ sum to $2(n-1)$,". Thanks to Jinwon Choi.

Page 108 line 8: Replace "$c_6(Q)$" with "$c_6\left(\mathrm{Sym}^5(Q)\right)$". Thanks to Yuji Shimizu.

Page 115 line -1 and Page 121 line 15: Replace "$(0,\ldots,0,x_{n-k+1},\ldots,x_n)$" with "$(0,\ldots,0,x_{k+1},\ldots,x_n)$". Thanks to Yuji Shimizu.

Page 126 line -3: Add the sentence "An example of such a vector bundle E is the bundle E_5 mentioned at the bottom of Page 127, whose fiber $(E_5)_f$ is defined in the last paragraph on Page 128." Thanks to Radoslav Kirov.

Page 129 line −10: Replace "$E|_U$" with "E_U". Thanks to Yuji Shimizu.

Page 130 line −9: Replace "$\mathbf{P}^n$" with "$\mathbf{P}^4$". Thanks to Yuji Shimizu.

Page 143 line −3 – page 144 line 5: Exercise 3a is not correctly formulated. A simultaneous translation $\delta t = \epsilon$ of t is required to obtain a symmetry, which can be recognized as an infinitesimal version of (95). However, the translation in t will require a corresponding infinitesimal shift in the limits of integration, a situation not discussed in the text. The reader is challenged to extend the description of Noether's method to apply to this situation. Thanks to Haojie Chen and Xiaolan Nie.

Page 151 line 3: Replace "$h(\infty)$" with "$h'(\infty)$". Thanks to Alexander Voronov.

Page 153 line 11 – page 154, line 16: The change of variables formula given in the display on Page 153 line 11 is incorrect, and in fact Z' does *not* vanish as incorrectly claimed in display (104). To preserve of the readability of this errata listing, the lengthy correction appears in the separate file "More on the localization of a supersymmetric integral", `http://www.ams.org/bookpages/stml-32/localization_susy_int.pdf`. Thanks to Kentaro Hori for assistance with the correction.

Page 170 line 7: Replace

$$\text{``}\sum_{\beta \in H_2(X)} \left(\int_\beta \mathcal{F}_1(z_1) \cdots \mathcal{F}_k(z_k) \mathcal{D}x \mathcal{D}\chi \mathcal{D}\psi \right) e^{-\int_\beta \omega}, \text{''}$$

with

$$\text{``}\sum_{\beta \in H_2(X)} \left(\int_\beta \mathcal{F}_1(z_1) \cdots \mathcal{F}_k(z_k) \mathcal{D}x \mathcal{D}g \mathcal{D}\chi \mathcal{D}\psi \right) e^{-[\omega]\cdot\beta}, \text{''}.$$

Thanks to Alexander Voronov.

Page 175 second display: replace

$$\text{``}\Omega = \mathrm{Res}\,(\Lambda) = \frac{F(z_1, z_2, z_3, z_4)\, dz_2 \wedge dz_3 \wedge dz_4}{\partial f(z)/\partial z_1}\Big|_X.\text{''}$$

with

$$\text{``}\Omega = \mathrm{Res}\,(\Lambda) = \frac{g(z_1, z_2, z_3, z_4)\, dz_2 \wedge dz_3 \wedge dz_4}{\partial f(z)/\partial z_1}\Big|_X.\text{''}$$

Thanks to Alexander Voronov.

Page 178 line −15 (this is display (134)): replace

$$\text{``}\left(q\frac{d^3}{d^3q}\right)F_0(q) = \frac{1}{\varpi_0^2}\int_{X_\psi}\Omega(\psi)\wedge\frac{d^3\Omega(\psi)}{dt^3},\text{''}$$

with

$$\text{``}\left(q\frac{d}{dq}\right)^3F_0(q) = \frac{1}{\varpi_0^2}\int_{X_\psi}\Omega(\psi)\wedge\frac{d^3\Omega(\psi)}{dt^3},\text{''}$$

Thanks to Haojie Chen and Xiaolan Nie.

Page 190 line 12: Replace "$\langle H^{a_1}, H^{a_2}, H^{a_3}\rangle$" with "$\langle H^{a_1}, H^{a_2}, H^{a_3}\rangle_d$". Thanks to Yuji Shimizu.

Page 190 line −12: Replace "$\langle H^2, H^2, H\rangle = 1$" with "$\langle H^2, H^2, H\rangle_1 = 1$". Thanks to Yuji Shimizu.

Page 191, line 12: In (148), replace

$$\Phi(\gamma) = \sum_{\beta, n\geq 3}\frac{1}{n!}\langle\gamma,\ldots,\gamma\rangle q^\beta.$$

with

$$\Phi(\gamma) = \sum_{\substack{\beta,n \\ n\geq 3 \text{ if } \beta=0}}\frac{1}{n!}\langle\gamma,\ldots,\gamma\rangle_\beta q^\beta.$$

and add the sentence "The restriction to $n \geq 3$ if $\beta = 0$ has been made because stable maps with $\beta = 0$ and $n < 3$ do not exist." Thanks to Marcin Hauzer.

Page 193 last line: replace "for any $D \in H^2(X)$." with "for any $D \in H^2(X)$ and either $\beta \neq 0$ or $n \geq 4$." Thanks to Alexander Voronov.

Page 196 line 5: replace "$K_d = 1$" with "$K_1 = 1$". Thanks to Alexander Voronov.

Page 200 line 9: Insert two new references, provided by Steve Kleiman:

[Schubert1] H. Schubert, *Das Correspondenzprincip für Gruppen von n Punkten und von n Strahlen*, Math. Annalen **12** (1877), 180–201.

and

[Schubert2] H. Schubert, *Die n-dimensionale Verallgemeinerung der Anzahlen für die vielpunktig berührenden Tangenten einer punktallgemeinen Fläche m-ten Grades*, Math. Annalen **26** (1886), 52–73.

Page 200 line 13: insert a new reference

[Vakil] R. Vakil, *The characteristic numbers of quartic plane curves*, Canad. J. Math. **51** (1999), 1089–1120.