Invariant Theory, Corrections

Thanks to everybody who sent me comments, corrections, and typos!

Page 20, Line -10

Thus $a = b = 0$, but then T is not invertible.

Proposition 2.24

It is assumed, but not explicitely stated that the ring R is graded and the elements $f_1, \ldots, f_m$ are homogeneous.

Page 48, Line -11

Replace the display by

$$(\substack{+\\-}a, \substack{+\\-}b) \quad \text{and} \quad (\substack{+\\-}b, \substack{+\\-}a).$$

Page 57, Line -1

Replace the last line of the display by

$$-\begin{cases} 0 & \text{if } m-n \text{ is odd,} \\ \binom{m-n}{\frac{m-n}{2}}(x_1^2 x_2^2)^{\frac{m-n}{2}} & \text{if } m-n \text{ is even,} \end{cases}$$

Page 63, Line -1

The last n-tuple in the display has to look like

$$(v_{\sigma^{-1}(1)}, \ldots, v_{\sigma^{-1}(n)}).$$

Page 68, Line 8

The last n-tuple in the display has to look like

$$(v_{\sigma^{-1}(1)}, \dots, v_{\sigma^{-1}(n)}).$$

Page 68, Lines 11-12,14,17,20

In contrast, here the $v_{\sigma^{-1}(i)}$'s have to be replaced by $v_{\sigma(i)}$.

Page 81, Line 6

$$o(\mathbf{x}^I) = x_1x_2^3x_3 + x_1^3x_2x_4.$$

Page 81, Lines 10-12

$$\begin{aligned} o(x_1x_2^2x_3)s_1 &= (x_1x_2^2x_3 + x_1^2x_2x_4)(x_1 + \cdots + x_4) \\ &= x_1^2x_2^2x_3 + x_1^3x_2x_4 + x_1x_2^3x_3 + x_1^2x_2^2x_4 \\ &\quad + x_1x_2^2x_3^2 + x_1^2x_2x_3x_4 + x_1x_2^2x_3x_4 + x_1^2x_2x_4^2. \end{aligned}$$

Page 81, Line 17

$$x_1^3x_2x_3 \quad \text{and} \quad x_1x_2^3x_4$$

Page 101, Example 5.3

The image of the representation ρ lies of course in $\mathsf{GL}(2, \mathbb{C})$.

Page 110, Proposition 5.17

The coefficient in the display is not $\det(g)$ but $\det(\rho(g)^{-1})$.

Page 111, Lines 3,5, and 15

All the $\rho(g)$'s need to be replaced by $\rho(g)^{-1}$.

Page 115, Exercise 20 (iii)

The degree of the Hessian is $n(\deg(f) - 2)$.

Page 118, Line -12

Replace $(-1)^{i+1}x_2$ by $(-1)^i x_2$.

Page 119, Line 6

Replace ω^{12} by ω^{-3}.

Page 120, Line 6

Replace $\eta_g\pi^G$ by $\eta_G\pi^G$.

Page 123, Line -9

Replace $i = 1, 2$ by $j = 1, 2$.

Page 128, Line -9
$t_0 = 1$ and $t_d = 0$.

Page 136, Exercise 12
Since the representation of Q_8 this exercise refers to lives over the complex numbers, also the ring of invariants is an algebra over $\mathbb{C}$.

Lemma 7.6
We need to diagonalize the matrix R in order to conclude that $\lambda_1 = \cdots = \lambda_{n-1} = 0$.

Example 7.12[Groups of Type $G(m, p, 2)$]
The following should be a better discussion of the example than the one given in the book:
Let $m = pq$, $m, p, q \in \mathbb{N}$, and ω a primitive mth root of unity. The subgroup $G(m, p, 2) \leq \mathsf{GL}(2,\ \mathbb{C})$ is generated by the transposition

$$\mathsf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and the matrices

$$\mathsf{R}(a_1, a_2) = \begin{bmatrix} \omega^{a_1} & 0 \\ 0 & \omega^{a_2} \end{bmatrix}$$

for all $a_1 + a_2 \equiv 0 \bmod p$. We observe that this group consists of all matrices of the form $\mathsf{R}(a_1, a_2)$ and

$$\mathsf{S}(a_1, a_2) = \begin{bmatrix} 0 & \omega^{a_1} \\ \omega^{a_2} & 0 \end{bmatrix},$$

where $a_1 + a_2 \equiv 0 \mod p$. We have m choices for a_1. Once an a_1 is choosen, we have q choices for a_2. Hence the order of $G(m, p, 2)$ is $2qm$. We claim this group is a pseudoreflection group. The transposition T is a reflection of order 2 fixing the line $\mathbf{e}_1 + \mathbf{e}_2$, while the matrices $\mathsf{R}(a_1, a_2)$ are in general not, because they fix only the origin. However, the group can be generated by T, $\mathsf{R}(p, 0)$, $\mathsf{R}(0, p)$, and $\mathsf{S}(a, -a)$. We note that $\mathsf{R}(p, 0)$ fixes $\mathbf{e}_2$, the element $\mathsf{R}(0, p)$ fixes $\mathbf{e}_1$, and finally, $\mathsf{S}(a, -a)$ fixes $\mathbf{e}_1 + \omega^{-a}\mathbf{e}_2$. Furthermore, the invariants are

$$\mathbb{C}[x, y]^{G(m,p,2)} = \mathbb{C}[x^m + y^m, (xy)^q];$$

cf. Exercise 5 in this chapter. Note that for the special case $q = 1$ we obtain the dihedral groups.

Page 160, Lines 10 and 13

The upper bound on the integrals slipped down. They are supposed to read as

$$\int_{-1}^{1} h(t)\mathrm{d}t \quad \text{resp.} \quad \int_{-1}^{1} L_{n,i}(t)\mathrm{d}t.$$

Page 171, Line 8

We need to assume that the M_i's are submodules in some big module M.

Page 186, Line 8

Replace the display by:

$$(a + b\mathbf{i})^2 - 2a(a + b\mathbf{i}) + (a^2 + b^2) = 0$$

for $a,\ b \in \mathbb{R}$.

Page 192, Line -13

Replace $\mathbb{F}[V][t]$ by $\mathbb{F}[V][X]$.

Page 194, Line 5

Assume without loss of generality that $s_0 \neq 0$.

Page 196, Line 11

Replace $\mathfrak{p}_1 \supset \mathfrak{p}_2$ by $\mathfrak{p}_1 \subset \mathfrak{p}_2$.

Page 197, Line -8

Replace $\mathfrak{q}' \subseteq \mathbb{F}[V]^G$ by $\mathfrak{q}' \subseteq \mathbb{F}[V]$.

Page 197, Line -4

$g_0\mathfrak{q} \subseteq \mathfrak{q}'$

Page 212, Line -5

Replace the display by

$$\mathsf{FF}(A) = \mathbb{F}(V)^G.$$

Page 244, Exercise 11

Assume that the matrices $\mathsf{A}, \mathsf{B}, \mathsf{C}$, and D commute. Then $\det(\mathsf{M}) = \det(\mathsf{AD} - \mathsf{BC})$.

Page 251, Line -12

The elements in $V(J)$ are $n+1$-tuple, i.e., $(\alpha_1, \ldots, \alpha_{n+1})$.

Page 255, Line -9

Replace the coefficients $\mathbb{C}$ by $\mathbb{F}$.

Mara D. Neusel, March 4 2008