Errata, Corrections, and Additions to INVITATION TO ERGODIC THEORY

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Line numbers start counting from the first line right after the underlined title on each page.

Page 13, line 11 I_0 *should be* X_0

Page 29, line -4 **canonical atomic spaces** *should be* **canonical atomic Lebesgue spaces**

Page 30, line 6 the points x_i *should be* the points k

Page 45, line 5 $< \varepsilon$ *should be* $< \frac{\varepsilon}{2^k}$

Page 45, line 9 if and only if *should be* then

Page 64, line 11 $\lambda(L \cap A)$ *should be* $\lambda(L)\lambda(A)$

Page 81, line -3 $f(x) = x^2$ *should be* $f(x) = 4x(1-x)$

Page 83, line -12 a σ-finite *should be* a complete σ-finite

Page 88, line -4 A set *should be* A measurable set

Page 92, line -14 A set *should be* A measurable set

Page 94, line -17 *Exercises 3.6.2 and 3.6.3 are the same as Exercises 3.4.4 and 3.4.5*

Page 99, line 9 a nonatomic measure space *should be* a measure space

Page 107, line -9 are *should be* is

Page 117, line 7 $|\phi(x) - \phi(y)|$ *should be* $\lambda(F)\ |\phi(x) - \phi(y)|$

Page 117, line 7 $[0, x_0)$ *should be* $[0, y)$

Page 119, line 10 $A \in \mathcal{S}'(X_0')$ if and only if $\phi^{-1}(A) \in \mathcal{S}(X_0)$, *should be* $\phi^{-1}(A) \in \mathcal{S}(X_0)$,

Page 119, line 12 *After line* 12 *add:*

$$(3)\ \phi(T(x)) = T'(\phi(x)) \text{ for all } x \in X_0.$$

Page 119, line -4 Let $T : (X, \mu) \to (Y, \nu)$ and $\phi : X \to Y$ *should be* Let $\phi : (X, \mu) \to (Y, \nu)$

Page 122, line 4 $\mu(B \cap A_i)$ *should be* $(B \cap A_i)$

Page 137, line -4 two functions *should be* two measurable functions

Page 149, line -6 transformation T is *should be* transformation T on a σ-finite measure space is

Page 153, line 2 (*two instances of:*) (a, b) *should be* $[a, b)$

Page 153, line 3 $y \in (0, 1)$, *should be*

$$y \in (0,1),\ \nu(T^{-1}[0,y)) = \nu[0,y).$$

Page 153, line 4

$$\nu(T^{-1}[0,y)) = \nu[0,y) = \frac{1}{2}\int_{[0,y)} \frac{1}{1+x}\, d\lambda(x)$$

should be

$$\text{Now, } \nu[0,y) = \frac{1}{\ln 2}\int_{[0,y)} \frac{1}{1+x}\, d\lambda(x)$$

Page 153, line 5 $\frac{1}{2}$ *should be* $\frac{1}{\ln 2}$

Page 153, line 6 $\frac{1}{2}$ *should be* $\frac{1}{\ln 2}$

Page 159, line -8 all functions *should be* all measurable functions

Page 165, line 9 Lebesgue space *should be* measure space

Page 178, line -5 a measurable function *should be* a function (*while only used for measurable functions measurability is not needed in the proof*)

Page 178, line -3 *At the end of the line add:* for all $x \in X$

Page 178, line -1 *At the end of the line add:* for all $x \in X$

Page 179, line -8 a measurable function *should be* an integrable function

Page 193, line -16

$$\frac{1}{n} \textit{ should be } 2^n$$

Page 193, line -14

$$\|f_n - 0\|_p = \|f_n\| \textit{ should be } \|f_n - 0\|_p^p = \|f_n\|_p^p$$

Page 194, line 7

$$\|g_i\|_p = \lambda((K_i)) \textit{ should be } \|g_i\|_p^p = \lambda(K_i)$$

Page 195, line 7 $g \in L^2$ *should be* $g \in S$

Page 195, line 9 transformation *should be* projection

Page 195, line -15 $g \in L^2$ *should be* $g \in S$

Page 195, line -2 T *should be* T^i

Page 197, line 7 (h, h_k) *should be* $(h - h_k, f)$

Page 198, line -10 T *should be* T^i

Page 199, line 4 T *should be* T^i

Page 209, line -8 a probability space *should be* a measure space

Page 214, line 5 a countable sufficient *should be* a sufficient

Page 230, line -8 simples *should be* simpler

Page 253, line -16 [**13**] is weakly mixing *should be* [**19**] is weakly mixing

Additions

(1) E. Zoli suggests another proof for Lemma 3.7.3. We have changed notation somewhat to emphasize that the lemma holds in a measure space.

Lemma 3.7.3 1. *Let $(X, \mathcal{S}, \mu)$ be a measure space with a sufficient semi-ring $\mathcal{C}$. If $A \in \mathcal{S}$ is of finite positive measure, then for any $\delta > 0$ there exists $I \in \mathcal{C}$ such that I is $(1-\delta)$-full of A.*

Proof. We show the contrapositive. Let A be a measurable set and suppose that there is a constant $0 < c < 1$ such that

$$\mu(A \cap I) \leq c\, \mu(I)$$

for all I belonging to the sufficient semi-ring $\mathcal{C}$. Then, if $\{I_n\}$ is a sequence of elements of $\mathcal{C}$ such that $A \subset \bigcup_{n=1}^{\infty} I_n$, by hypothesis and by countable subadditivity,

$$\mu(A) \leq \sum_{n=1}^{\infty} \mu(A \cap I_n) \leq c \sum_{n=1}^{\infty} \mu(I_n).$$

As the sequence $\{I_n\}$ is arbitrary and $\mathcal{C}$ is a sufficient semi-ring, taking the infimum we obtain $\mu(A) \leq c\mu(A)$. Then, as $c < 1$, $\mu(A) = \infty$ or $\mu(A) = 0$. □

(2) Z. Lipecki points out that a shorter proof of Proposition B.1.8 can be found in W. Rudin, Amer. Math. Monthly 90, 41-42, 1983.