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## Appendix E

# Errata

**Page 13, Exercise 1.5.6 (8) should read:**

Prove that  $f$  is a regulated function on  $I = [a, b]$  if and only if both of the limits

$$\lim_{x \rightarrow c+} f(x) \quad \text{and} \quad \lim_{x \rightarrow d-} f(x)$$

exist for every  $c \in [a, b)$  and every  $d \in (a, b]$ .

**Page 91, line 5 should read:**

$$\dots \leq 4n^2\mu(A) + \delta^2\mu(A^c) \leq 4n^2\delta + (b-a)\delta^2.$$

**Page 91, line 7 should read:**

$$\|f_n - g\| \leq \sqrt{4n^2\delta + (b-a)\delta^2} < \frac{\varepsilon}{2}.$$

**Page 102, line 7 should read:**

$s_n = \sum_{i=0}^n \langle w, u_i \rangle u_i$ . Then since the family is orthogonal, we know

**Page 108, line 5 should read:**

$$= \|w\|^2 - \sum_{n=0}^N |a_n|^2 + \sum_{n=0}^N (a_n - c_n) \overline{(a_n - c_n)}$$

**Page 109, line -4 should read:**

$$\sum_{i=0}^{\infty} |\langle w, u_i \rangle|^2 = \|w\|^2$$

**Page 116, lines 13–14 should read:**

with  $|g(x) - p(x)| < \varepsilon/4$  for all  $x$ . So

$$\|g - p\|^2 = \frac{1}{\pi} \int (g - p)^2 d\mu \leq \frac{1}{\pi} \int \frac{\varepsilon^2}{16} d\mu = \frac{\varepsilon^2}{8}.$$

Hence,  $\|f - p\| \leq \|f - g\| + \|g - p\| < \varepsilon/2 + \varepsilon/\sqrt{8} < \varepsilon$ .

**Page 118, the last line of exercise 6.1.10 (1) should read:**

$$\sin \theta_1 \sin \theta_2 = \frac{1}{2} (\cos(\theta_1 - \theta_2) - \cos(\theta_1 + \theta_2)).$$

**Page 135, the second paragraph of the proof of 7.2.2 should read:**

Conversely, if  $f$  is a  $T$ -invariant measurable function then its real part,  $u(x)$ , and imaginary part,  $v(x)$ , are  $T$ -invariant. If  $f$  is not  $\nu$ -almost everywhere constant, then at least one of  $u(x)$  and  $v(x)$  is not  $\nu$ -almost everywhere constant. Suppose, without loss of generality, that it is  $u(x)$ . Then there is a  $c \in \mathbb{R}$  such that the set  $A = u^{-1}([-\infty, c])$  satisfies  $\nu(A) > 0$  and  $\nu(A^c) > 0$ . The set  $A$  is  $T$ -invariant.

**Page 138, line 4 of Corollary 7.3.2 should read:**  $A \cap \{T^k(x)\}_{k=0}^{m-1}$ , i.e., the number of points in  $A$  from the orbit segment

**Page 169, line 2 should read:**

$$|\langle v, w \rangle|^2 = \|\langle v, w \rangle w\|^2 \leq \|\langle v, w \rangle w\|^2 + \|v - \langle v, w \rangle w\|^2 = 1$$

**Page 187, Lemma B.3.9 and its proof should read:**

**Lemma B.3.9.** *Suppose  $A_0$  and  $B_0$  are disjoint sets in  $\mathcal{M}_0$  and  $X_0$  is an arbitrary subset of  $\mathbb{R}$ . If  $A = A_0 \cap X_0$  and  $B = B_0 \cap X_0$  then*

$$\mu^*(A \cup B) = \mu^*(A) + \mu^*(B).$$

*The analogous result for a finite union of disjoint sets is also valid.*

**Proof.** Since  $A_0$  and  $B_0$  are disjoint, so are  $A_0$  and  $B$ , and therefore  $A_0 \cap B = \emptyset$  and  $A_0^c \cap B = B$ . Hence

$$A_0 \cap (A \cup B) = (A_0 \cap X_0) \cup (A_0 \cap B \cap X_0) = A_0 \cap X_0 = A.$$

Likewise,

$$A_0^c \cap (A \cup B) = (A_0^c \cap A_0 \cap X_0) \cup (A_0^c \cap B) = B.$$

Hence, the fact that  $A_0$  is in  $\mathcal{M}_0$  (using  $A \cup B$  for  $X$  in the definition of  $\mathcal{M}_0$ ) tells us

$$\begin{aligned} \mu^*(A \cup B) &= \mu^*(A_0 \cap (A \cup B)) + \mu^*(A_0^c \cap (A \cup B)) \\ &= \mu^*(A) + \mu^*(B). \end{aligned}$$

The result for a finite collection  $A_0 \cap X_0, A_1 \cap X_0, \dots, A_n \cap X_0$ , where  $A_i \in \mathcal{M}_0$  follows immediately by induction on  $n$ .  $\square$

**Page 188, the first displayed equation should read:**

$$B_{n+1} = A_{n+1} \setminus \bigcup_{i=1}^n A_i = A_{n+1} \cap \left( \bigcup_{i=1}^n A_i \right)^c.$$

**Page 197, item [C]:**

Replace “sumas” with “sums”

**Page 197, item [M]:**

Replace “Erdodic” with “Ergodic”

*May 13, 2014.*