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# A Primer on the Calculus of Variations and Optimal Control Theory: Errata

Mesterton-Gibbons, STML 50  
(April 23, 2017)

**Page 33, Exercise 4.4:** Replace “ $x(1) = 0$ ” by “ $x(1) = 2$ ”

**Page 63, (8.36):** Replace “ $n$ ” by “ $\eta$ ”

**Page 105, two lines below (13.16):** Replace “the second of which requires  $k \geq 4$ ” by “which together require  $k > 4$ ”

**Page 105, first unnumbered display:** Replace “ $l \geq \frac{9}{2}$ ” by “ $l > \frac{9}{2}$ ”

**Page 105, three lines below second unnumbered display:** Replace “ $l \geq \frac{9}{2}$ ” by “ $l > \frac{9}{2}$ ”

**Page 106, first line:** Replace “(because  $y > 0$ )” by “, because we can safely assume  $y > 0$  on  $\Gamma$  when maximizing (13.8)”

**Page 131, first line of new paragraph:** Insert “at rest” immediately after “initially”

**Page 132, (16.21):** Replace “sgn” by “ $\mathfrak{S}$ ” (twice); replace “ $t_c$ ” by “ $t_s$ ” (twice) in the first two lines of the display; replace

“ $t_s$ ” by “ $t_c$ ” (twice) in the last two lines of the display; and replace the period by a comma

**Page 132, immediately below (16.21):** Insert “where  $\mathcal{H}$  denotes Heaviside’s step function [7, p. 307].”

**Page 132, line below (16.21):** Replace “It is straightforward to” by “We” and delete “indeed”

**Page 134, Exercise 16.1, last line:** Replace “ $\dot{y} = tu$ ” by “ $\dot{y} = t^2u$ ”

**Page 168, third line:** Replace “ $R^m$ ” by “ $\mathfrak{R}^m$ ” (twice)

**Page 233, Lecture 14:** Replace “1.” by “2.”

**Page 238, Exercise 19.1 (c), second line:** Replace “admissible” by “potentially admissible”

**Page 238, Exercise 19.1 (c), sixth line:** Replace “For (b), the associated cost is  $J = \frac{1}{2}\{8\sqrt{2} - 11\}^{1/2} + \frac{3}{5}(\{4\sqrt{2} - 2\}^{1/2} - \{\sqrt{8} - 1\}^{1/2}) - \frac{1}{10} \approx 0.5161$ ” by “(b) is inadmissible because it violates  $|u| \leq 1$  between  $(1, 1)$  and  $(1, -1)$ , where  $x_1 > 1 \implies u_s < -1$ ”

The following errata concern only the original printing (not the corrected 2014 reprint):

**Page 50, Exercise 6.4:** The lower integration limit is “ $a$ ,” not “0”

**Page 52, line below (7.10):** Replace “ $k = \eta'(x)$ ” by “ $k = \epsilon\eta'(x)$ ”

**Page 53, first line of (7.11):** Replace “ $\eta'(x)$ ” by “ $\epsilon\eta'(x)$ ”

**Page 56, first display:** Replace

$$\frac{1}{2} \left\{ -\frac{\pi^2 K_1}{\delta^2} \cdot 1^2 + K_2 \cdot 1^4 \right\} \{c + \delta - (c - \delta)\} = -\frac{\pi^2 K_1}{\delta} + \delta K_2,$$

by

$$\frac{1}{2} \int_{c-\delta}^{c+\delta} \left\{ -\frac{\pi^2 K_1}{\delta^2} \sin^2 \left( \frac{2\pi\{x-c\}}{\delta} \right) + K_2 \cdot 1^4 \right\} dx = -\frac{\pi^2 K_1}{2\delta} + \delta K_2,$$

**Page 59, (8.12):** Replace “ $w(b)$ ” by “ $w(c)$ ”

**Page 61, second line of (8.22):** Replace “ $k$ ” by “ $\sqrt{k}$ ”

**Page 81, two lines below (11.1):** Replace “case.” by “case; and so here we regard an extremal as admissible if, in lieu of (2.2), it satisfies appropriate endpoint conditions that we are about to determine.”

**Page 83, (11.13):** Replace “ $y_x(x, \epsilon)$ ” by “ $y_x(x, \epsilon)$ ”

**Page 86, two lines below (11.27):** Replace “ $dx_A, dy_A, dx_B$  and  $dy_B$ ” by “ $dx_A, dy_A$  and  $dx_B, dy_B$ ”

**Page 87, (11.35):** Replace “ $|k|$ ” by “ $k$ ”

**Page 87, line below (11.35):** Replace “ $k$ ” by “ $k (> 0)$ ”

**Page 90, Exercise 11.4:** Replace “ $xy'^2 + \sqrt{xy}$ ” by “ $xy'^2 + \sqrt{xy}'$ ”

**Page 99, last sentence of lecture:** Replace “Henceforward” by “In Lectures 13 and 14”

**Page 102:** Delete “{” from the first integral

**Page 104, Figure 13.1:** Replace “ $\Gamma_0$ ” by “ $\Gamma_*$ ”

**Page 112, second line:** Replace “(1,0)” by “(0,1)”

**Page 120, (15.3):** Replace  $\int_a^b y ds$  by  $\int_0^L 1 ds$

**Page 129, three lines below (16.8):** Replace “ $\dot{X}/X$ ” by “ $\frac{dX}{dt}$ ”

**Page 131, last two lines:** Replace “ $x >$ ” by “ $x_1 >$ ” and “ $x <$ ” by “ $x_1 <$ ” (twice)

**Page 132, first line:** Replace “ $x >$ ” by “ $x_1 >$ ”

**Page 137, (17.12):** Replace “ $\delta x_i(t) + o(\delta t)$ ” by “ $\delta x_i(t_1) + o(\delta t)$ ”

**Page 141, line below (17.27):** Replace “trancers” by “transfers”

**Page 148, first line:** Replace “ $x_3(0) = 0$ ” by “ $x_3(t_0) = 0$ ”

**Page 161:** Replace (19.15) by

$$\begin{aligned}\ddot{\sigma} &= e^{-\delta x_2} \{ (1 - 4x_1 - \delta + \theta) \dot{x}_1 - \delta \{ x_1(1 - 2x_1 + \theta) - \delta(x_1 - \theta) \} \} \\ &= e^{-\delta x_2} \{ 1 - 4x_1 - \delta + \theta \} \dot{x}_1 = -e^{-\delta x_2} \left( 2x_1 + \frac{\delta\theta}{x_1} \right) \dot{x}_1 \\ &= x_1 e^{-\delta x_2} \left( 2x_1 + \frac{\delta\theta}{x_1} \right) (qu - 1 + x_1)\end{aligned}$$

**Page 162, Exercise 19.1:** Replace “(with  $t_1$  unspecified)” by “for suitable  $t_1$ ”

**Page 169, fourth line:** Replace “ $TA^*$ ” by “ $T^*A^*$ ”

**Page 174, (21.20), lower integration limit:** Replace “0” by “ $t_0$ ”

**Page 175, (21.22):** Replace “ $x(t_1)$ ” by “ $x_0(t_1)$ ”

**Page 177, footnote, line 8:** Replace “ $\lambda_0 + K \sin(\theta) + u^* \tan(\theta)$ ” by “ $\lambda_0 + K \{ \sin(\theta) + u^* \tan(\theta) \}$ ”

**Page 177, footnote, line 9:** Replace “ $\lambda_0 + K \sin(\theta) \pm \tan(\theta)$ ” by “ $\lambda_0 + K \{ \sin(\theta) \pm \tan(\theta) \}$ ”

**Page 178, (21.38):** Replace “ $\lambda_2(t_1)$ ” by “ $\lambda_1(t_1)$ ” in denominator

**Page 225, Exercise 3.4, last line:** Replace “ $\phi(t) = \frac{4}{3}(t^4 - 1)$ ” by “ $\phi(t) = \frac{4}{3}(4t^4 - 1)$ ”

**Page 226, Exercise 4.5, end of 4th line:** Replace “ $\pm$ ” by “ $= \pm$ ”

**Page 227, Exercise 6.4:** Replace “ $\leq$ ” by “ $>$ ”

**Page 229, Exercise 10.1:** Replace all by “Here

$$E(x, \phi(x), \phi'(x), \omega) = \cos(2\omega) - \cos(2) + 2(\omega - 1) \sin(2)$$

fails to be nonnegative; for example, it is negative if  $\omega \leq 0$ .”

**Page 229, Exercise 10.6:** Delete this line

**Page 239, Exercise 21.4, second line:** Interchange **(a)** and **(b)**

**Page 241, top line:** Change " $\gamma a / (1 + \frac{1}{2} \gamma \pi)$ " to " $\gamma a \sin(t) / (1 + \frac{1}{2} \gamma \pi)$ "