ERRATA FOR LUSTERNIK-SCHNIRELMANN CATEGORY

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Chapter 1 Errata

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Chapter 6 Errata

As stated Lemma 6.26 is wrong and must be replaced by the following statement:

Lemma 6.26. *Let $r \geq q \geq 1$. Let X be a $(q-1)$-connected CW-complex of dimension at most r. Consider the cofibration*

$$S^r \xrightarrow{\alpha} X \xrightarrow{\rho} Y = X \cup_\alpha e^{r+1}\,.$$

For $n \geq 1$, except when $r = n = q = 1$, the map ρ induces $(r+1)$-equivalences $F_n(X) \to F_n(Y)$ between the fibres and r-equivalences $G_n(X) \to G_n(Y)$ between the total spaces of the Ganea fibrations.

This Lemma is used in three places:

- pages 179 and 180 where the previous correct statement is sufficient,
- page 184, where the argument at the end of the proof of Theorem 6.31 must be modified as follows.

We keep the notation of the quoted proof and first we add a new observation.

New Lemma. *The map $\mathcal{C}(G_n(\rho)) \to \mathcal{C}(\rho)$, induced by p_n^X and p_n^Y between the cofibres, is an isomorphism in homology up to degree $q+r-1$.*

With that result, at the end of the proof of Theorem 6.31, the sentence “Because the maps $\Sigma^i(\rho)$ and $\Sigma^i(G_n(\rho))$ are $(r+i+1)$-equivalences (for the second map, see Lemma 6.26), we obtain that ψ is also an $(r+i+1)$-equivalence,” must be replaced by what follows.

Consider the homotopy cofibres of the two left horizontal maps

$$\begin{array}{ccccc} \Sigma^i G_n(X) & \xrightarrow{\Sigma^i G_n(\rho)} & \Sigma^i G_n(Y) & \longrightarrow & \Sigma^i \mathcal{C}(G_n(\rho)) \\ \Big\downarrow{\scriptstyle \Sigma^i p_n^X} & & \Big\downarrow{\scriptstyle \Sigma^i p_n^Y} & & \Big\downarrow \\ \Sigma^i X & \xrightarrow{\Sigma^i \rho} & \Sigma^i Y & \longrightarrow & S^{r+i+1} \end{array}$$

From the New Lemma, we know that the map $\Sigma^i\mathcal{C}(G_n(\rho)) \to S^{r+i+1}$, induced by $\Sigma^i p_n^X$ and $\Sigma^i p_n^Y$, is an isomorphism in homology up to degree $q+r-1+i$. Therefore it is the same with its homotopy section induced by $\Sigma^i\sigma$ and $\overline{\sigma}'$. As a consequence, if $q \geq 2$, the homotopy cofibre of ψ is $(r+i+1)$-connected and the map ψ is an $(r+i+1)$-equivalence. □

We have now to prove the new Lemma.

Proof of the New Lemma. The proof goes in two steps.

(1) We consider the next diagram where the bottom line is formed with the homotopy cofibres of the top vertical arrows.

$$\begin{array}{ccccc} F_n(X) & = & F_n(X) & \xrightarrow{F_n(\rho)} & F_n(Y) \\ \Big\downarrow{\scriptstyle i_n^X} & & \Big\downarrow{\scriptstyle \rho_n \circ i_n^X} & & \Big\downarrow{\scriptstyle i_n^Y} \\ G_n(X) & \xrightarrow[\rho_n]{} & G_n(X) \cup e^{r+1} & \xrightarrow[\xi_n]{} & G_n(Y) \\ \Big\downarrow & & \Big\downarrow & & \Big\downarrow \\ G_{n+1}(X) & \xrightarrow[\rho_{n+1}]{} & G_{n+1}(X) \cup e^{r+1} & \xrightarrow[\xi_{n+1}]{} & G_{n+1}(Y) \end{array}$$

By induction on n, one shows that ξ_n is a $(q+r-1)$-equivalence.

(2) From the diagram

$$\begin{array}{ccccc} G_n(X) & \xrightarrow{\rho_n} & G_n(X) \cup e^{r+1} & \xrightarrow{\xi_n} & G_n(Y) \\ \Big\downarrow{\scriptstyle p_n^X} & & \Big\downarrow{\scriptstyle \tilde{p}_n} & & \Big\downarrow{\scriptstyle p_n^Y} \\ X & \xrightarrow[\rho]{} & Y & = & Y \end{array}$$

we deduce the next one by taking cofibres:

$$\begin{array}{ccc}
G_n(X) \cup e^{r+1} & \xrightarrow{\xi_n} & G_n(Y) \\
\downarrow & & \downarrow \\
\mathcal{C}(\rho_n) = S^{r+1} & \longrightarrow & \mathcal{C}(G_n(\rho)) \\
\| & & \downarrow \\
S^{r+1} & = = = & \mathcal{C}(\rho)
\end{array}$$

The map $\mathcal{C}(\rho_n) \to \mathcal{C}(G_n(\rho))$ has a left inverse and induces an injection in homology. On the other side, the upper square is a pushout, therefore the first step of the proof implies that $S^{r+1} = \mathcal{C}(\rho_n) \to \mathcal{C}(G_n(\rho))$ is a $(q+r-1)$-equivalence. So its left inverse $\mathcal{C}(G_n(\rho)) \to \mathcal{C}(\rho_n)$ is also a $(q+r-1)$-equivalence. □

Chapter 7 Errata

- Definition 7.35 should read as follows: Let $(W; V_0, V_1)$ be a compact cobordism and assume $f\colon W \to \mathbb{R}$ is a smooth function which is minimal, regular and constant equal to $a < 0$ on V_0 and maximal, regular and constant equal to $b > 0$ on V_1. We will say that the critical points of f can be *fused* if there is a point $T \in W$, diffeomorphisms $\phi_i\colon W \to W$ which preserve ∂W and $\psi_i\colon [a,b] \to [a,b]$ with $\psi_i(a) = a, \psi_i(b) = b$, $i \in \mathbb{N}$, such that the limit $\lim_{i\to\infty} \psi_i \circ f \circ \phi_i$ exists in the C^0 topology and the limit $\lim_{i\to\infty}[\psi_i \circ f \circ \phi_i|_{(W-\{T\})}]$ exists in the weak C^∞-topology and has no critical points.

  The only difference is the addition of "and has no critical points" in the last sentence.

- In Remark 7.36, the first sentence should read, "In the setting of Definition 7.35, if a function g is the C^0-limit of the sequence $f_i = \psi_i \circ f \circ \phi_i$, then g is smooth on $W - \{T\}$ and is continuous on W".

Chapter 8 Errata

Chapter 9 Errata

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