

# Large Deviations for Stochastic Processes

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## Errata

- On page 12, line 6, a missing ”)” in ” $\partial_j f(x)$ ”, and a redundant ”)” in ” $\partial_i \partial_j f(x)$ ”.
- The proof of Lemma 7.8 on page 114 contains an error. The assertion that the conditions of that lemma imply  $\inf_x g(x) > -\infty$  is not in general correct. For example, let  $f(x) = g(x) = -x^2$  and  $\epsilon_0 = 1$ .

The lemma, however, is correct as stated.

As in the incorrect proof, let  $x_n$  and  $0 < \epsilon_n < \epsilon_0$  satisfy  $\epsilon_n \rightarrow 0$  and

$$\sup_x f(x) < f(x_n) - \epsilon_n g(x_n) + \epsilon_n^2. \quad (0.1)$$

That is,  $\varphi(0) \leq \sup_x f(x) < \varphi(\epsilon_n)$  where  $\varphi(\epsilon) = f(x_n) - \epsilon g(x_n) + \epsilon^2$ . By continuity of  $\varphi$ , we can assume that  $\sup_x f(x) \leq \varphi(\epsilon_n) \leq \sup_x f(x) + n^{-1}$ . We have

$$\lim_{n \rightarrow \infty} (f(x_n) - \epsilon_n g(x_n)) = \sup_x f(x). \quad (0.2)$$

Then

$$\limsup_{n \rightarrow \infty} g(x_n) = - \liminf_{n \rightarrow \infty} \frac{\sup_x f(x) - f(x_n)}{\epsilon_n} \leq 0. \quad (0.3)$$

Next, we show that

$$\lim_{n \rightarrow \infty} f(x_n) = \sup_x f(x). \quad (0.4)$$

By (0.3), either  $\limsup_{n \rightarrow \infty} g(x_n) = 0$  and (0.4) holds, or  $\limsup_{n \rightarrow \infty} g(x_n) < 0$ . In the latter case, let  $0 < \epsilon < \epsilon_0$ . Then

$$\sup_x (f(x) - \epsilon g(x)) \geq f(x_n) - \epsilon_n g(x_n) - (\epsilon - \epsilon_n)g(x_n),$$

and hence, by (0.2),

$$0 < - \limsup_{n \rightarrow \infty} g(x_n) \leq - \liminf_{n \rightarrow \infty} g(x_n) \leq \frac{\sup_x (f(x) - \epsilon g(x)) - \sup_x f(x)}{\epsilon}.$$

It follows that  $\lim_{n \rightarrow \infty} \epsilon_n g(x_n) = 0$  and  $\lim_{n \rightarrow \infty} f(x_n) = \sup_x f(x)$  completing the proof of Part (a) of Lemma 7.8.

Part (b) can be proved similarly.

- Page 130, line -2,  $\eta_n(z_n) \rightarrow yx$  should be  $\eta_n(z_n) \rightarrow x$ .
- In Lemma 9.16 on page 176, (9.46) must hold for all  $x, p \in R^d$  with  $p \neq 0$ . Additionally, one must assume  $H_{\ddagger}^*(x, 0) < \infty$  for each  $x \in R^d$ .
- Page 190, the  $\rho^p$  in (9.84) should be  $\dot{\rho}^p$ .

- In Lemma 10.5 on page 210, (10.20) must hold for all  $x, p \in R^d$  with  $p \neq 0$ .
- Lemma 12.5 on page 289 should assume Conditions 11.21.3 and 11.21.4 (not 11.34.3 and 11.34.4 which don't exist).
- On page 340, lines 11 and 12, the  $h(X(t), z)$  should be  $h(X(s), z)$  and the  $U(t)$  should be  $U(s)$ .
- On page 391, line -6, the  $\Psi_n$  should be  $\Psi_\epsilon$ .