LIST OF CORRECTIONS

MOST OF CORRECTIONS ARE MADE IN
THE RUSSIAN EDITION OF THE BOOK, MCCME, MOSCOW, 2009.
SOME OF THEM APPLY TO BOTH EDITIONS, AS INDICATED

- page xiv, line -9:

  V. O. Tarasov (1985, 1986)

  $\Longrightarrow$

  V. O. Tarasov (1984, 1985)
- page xv, lines 10-12:

  An essential ingredient here is the *fusion procedure* for the symmetric group due to Cherednik (1986), which we also discuss in detail.

  $\Longrightarrow$

  An essential ingredient here is the *fusion procedure* for the symmetric group going back to the work of Jucys (1966) and developed by Cherednik (1986), which we also discuss in detail.
- page xvi, line -10:

  J. G. Nagel and M. Moshinsky (1964)

  $\Longrightarrow$

  J. G. Nagel and M. Moshinsky (1965)
- page xvii, line -13:

  see also Brundan and Kleshchev (2007)

  $\Longrightarrow$

  see also Brundan and Kleshchev (2008)
- page xviii, line -12:

  Alan Lascoux

  $\Longrightarrow$

  Alain Lascoux
- page 11, line -5:

  The antipode and counit for Δ' are the same as for the comultiplication Δ.

  $\Longrightarrow$

  The counit for Δ' is the same as for the comultiplication Δ, while the antipode is S^{-1}.

- page 77, lines 1-5:

  PROPOSITION 2.13.10. *For any* $1 \leqslant M \leqslant N$ *there exist expansions of the minors*

  $$s^{\,1\dots M}_{\,1\dots M}(u) \qquad \textit{and} \qquad \check{s}^{\,1\dots M}_{\,1\dots\widehat{k}\dots M,l}(u), \qquad k,l\in\{1,\dots,M\}$$

  in terms of the generator series $s_{ij}(u)$ *of the algebra* $\mathrm{X}(\mathfrak{g}_N)$ *which only involve the series with the indices satisfying* $1 \leqslant i,j \leqslant M$.

  $\Longrightarrow$

  PROPOSITION 2.13.10. *For any* $1 \leqslant M \leqslant N$ *in the algebra* $\mathrm{X}(\mathfrak{g}_N)$ *there exist expansions of the minors*

  $$s^{\,1\dots M}_{\,1\dots M}(u) \qquad \textit{and} \qquad \check{s}^{\,1\dots M}_{\,1\dots\widehat{k}\dots M,l}(u), \qquad k,l\in\{1,\dots,M\}$$

  in terms of the generator series $s_{ij}(u)$ *which are independent of* N.

- page 84, line 5:

  summed over the indices $a_i, b_i \in \{-n,\dots,n\}$

  $\Longrightarrow$

  summed over the indices $a_i, b_i \in \{1,\dots,N\}$

- page 92 line -12:

  quantum quality principle

  $\Longrightarrow$

  quantum duality principle

- page 122, line 14:

  COROLLARY 3.4.6. *Every finite-dimensional ...*

  $\Longrightarrow$

  COROLLARY 3.4.6. *Up to twisting with an automorphism of the form* (1.20)*, every finite-dimensional ...*

- page 129, lines 14-17:

  Example 3 is contained in Billig, Futorny and Molev [35] and Brundan and Kleshchev [53]. Example 4 is taken from the paper [35], which also contains corresponding results for all Drinfeld Yangians.

  $\Longrightarrow$

  Example 3 is taken from Billig, Futorny and Molev [35] while Example 4 is contained in Brundan and Kleshchev [53] and the paper [35]. The latter paper also contains corresponding results for all Drinfeld Yangians.

- page 132, line 18:

  quantum comatrix

  $\Longrightarrow$

  Sklyanin comatrix

- page 147, line -11:

  Taking the coefficient of $(E_{1,-1})^p\zeta_1$ in (4.33) gives

  $\Longrightarrow$

  Taking the coefficient of $(E_{1,-1})^p\zeta_1$ in the expansion of $s_{-1,1}(u)\eta$ gives
- page 156, line -7:

  taking the coefficient of $(E_{1,-1})^p\zeta_1$ in (4.53) gives

  $\Longrightarrow$

  taking the coefficient of $(E_{1,-1})^p\zeta_1$ in the expansion of $s_{-1,1}(u)\eta$ gives
- page 165, line -17:

  However, this is possible only if the parameter γ takes half-integer values.

  $\Longrightarrow$

  However, this is possible only if the parameter γ takes integer or half-integer values.
- page 187, line -2:

  The (5.9) implies

  $\Longrightarrow$

  The relation (5.9) implies
- page 192, line -16:

  were

  $\Longrightarrow$

  where
- page 196, line 5: For these values we take the numbers

  $\Longrightarrow$

  For these values we take the numbers negative to
- page 201 line -1, and page 388:

  replace the reference Jimbo [199] by

  [199] M. Jimbo, *Quantum R matrix related to the generalized Toda system: an algebraic approach.* Field theory, quantum gravity and strings (Meudon/Paris, 1984/1985), 335–361, Lecture Notes in Phys., 246, Springer, Berlin, 1986.
- page 234 line -10, twice:

  J_{UU}

  $\Longrightarrow$

  $J_{UU}(0)$
- page 238, lines 4-5:

  Theorem 6.4.7 goes back to Cherednik [72, 73, 74]; see also Jucys [203].

  $\Longrightarrow$

  Theorem 6.4.7 goes back to Jucys [203]. In the context of R-matrices the fusion procedure for the symmetric groups and Hecke algebras was developed by Cherednik [72, 73, 74].

- page 238, line 12:

  The fusion procedure for ...

  $\Longrightarrow$

  The argument actually establishes its equivalence with a construction of Jucys [J-1971] which was rediscovered by Murphy [331] (see Proposition 6.4.1). The fusion procedure for ...

  ADDED REFERENCE:

  [J-1971] A. Jucys, *Factorization of Young projection operators for the symmetric group*, Lietuvos Fizikos Rinkinys **11**, 5–10 (1971).
- page 240, line -16:

  proposition

  $\Longrightarrow$

  theorem
- page 255, line -18:

  to the center of

  $\Longrightarrow$

  to the center
- page 258, line 9:

  row tableau

  $\Longrightarrow$

  row diagram
- page 258, lines -8 (twice), -4, -3; page 259, lines 2; page 260, lines 3, 13

  Φ

  $\Longrightarrow$

  ϕ
- page 295, lines 18, 19:

  Its composition with the evaluation homomorphism (1.5) is a homomorphism $\mathrm{Y}(\mathfrak{gl}_M) \to \mathrm{U}(\mathfrak{gl}_{M+N})$

  $\Longrightarrow$

  Its composition with the evaluation homomorphism π_{M+N} given by (1.5), is a homomorphism $\mathrm{Y}(\mathfrak{gl}_N) \to \mathrm{U}(\mathfrak{gl}_{M+N})$
- page 295, lines 21–23:

  By Lemma 1.11.2, the image of $t_{ij}^{(1)}$ under the homomorphism (8.31) is $E_{M+i,M+j}$. Therefore, Corollary 1.7.2 implies

  $\Longrightarrow$

  Corollary 1.7.2 implies

- page 296, line -8:

  The action of $\mathfrak{gl}_N$ on the basis vectors is given by Theorem 5.4.1,

  $\Longrightarrow$

  By Lemma 1.11.2, the image of $t_{ij}^{(1)}$ under the homomorphism (8.31) is $E_{M+i,M+j}$. Therefore, the action of $\mathfrak{gl}_N$ on the basis vectors is given by Theorem 5.4.1 (applied to $\mathfrak{gl}_{M+N}$),
- page 297, line 11:

  μ_{m+1}

  $\Longrightarrow$

  μ_{M+1}
- page 302, lines 9–14 (in Russian edition page 404, lines -17–(-10)):

  Consider the double elementary symmetric polynomials $e_k(l_1^2,\dots,l_n^2\,|\,a)$ with the sequence $a=(\rho_1^2,\rho_2^2,\dots)$, where the numbers ρ_i are defined by (7.10) for any i; see Section 7.4. The relation (7.32) implies that

  $$o_N : e_k(l_1^2,\dots,l_n^2\,|\,a)\mapsto e_k(l_1^2,\dots,l_{n-1}^2\,|\,a).$$

  Hence, for any $k\geqslant 1$ the sequence $(e_k(l_1^2,\dots,l_n^2\,|\,a)\mid n\geqslant k)$ determines an element $e_k\in\Lambda^*$ of degree $2k$.

  $\Longrightarrow$

  Consider the double elementary symmetric polynomials $e_k(l_1^2,\dots,l_n^2\,|\,b^{(n)})$ with the sequence $b^{(n)}=(\rho_{-n}^2,\rho_{-n+1}^2,\dots)$, where the numbers ρ_i are defined by (7.10) for any i; see Section 7.4. The relation (7.32) implies that

  $$o_N : e_k(l_1^2,\dots,l_n^2\,|\,b^{(n)})\mapsto e_k(l_1^2,\dots,l_{n-1}^2\,|\,b^{(n-1)}).$$

  Hence, for any $k\geqslant 1$ the sequence $(e_k(l_1^2,\dots,l_n^2\,|\,b^{(n)})\mid n\geqslant k)$ determines an element $e_k\in\Lambda^*$ of degree $2k$.
- page 303, lines 4–5 (in Russian edition page 405, lines -7–(-6)):

  Write

  $$\mathcal{C}^{(n)}(u)=1+\mathcal{F}_1^{(n)}u^{-2}+\mathcal{F}_2^{(n)}u^{-4}+\dots,\qquad \mathcal{F}_k^{(n)}\in \mathrm{A}_0(N).$$

  $\Longrightarrow$

  Write

  $$\mathcal{C}^{(n)}(u)=1+\frac{\mathcal{F}_1^{(n)}}{u^2-\rho_1^2}+\dots+\frac{\mathcal{F}_n^{(n)}}{(u^2-\rho_1^2)\dots(u^2-\rho_n^2)},\qquad \mathcal{F}_k^{(n)}\in \mathrm{A}_0(N).$$

- page 303, lines 11–17 (in Russian edition page 406, lines 1–9):

  PROPOSITION 8.7.3. *The elements* $\mathcal{F}_1, \mathcal{F}_2, \dots$ *are algebraically independent and generate the algebra* A_0. *Their images under the isomorphism* χ *are found by*

$$1 + \sum_{k=1}^{\infty} \chi(\mathcal{F}_k)\, u^{-k} = \prod_{i=1}^{\infty} \frac{u^2 - l_i^2}{u^2 - \rho_i^2}.$$

  PROOF. The Harish-Chandra image $\chi_N(\mathcal{F}_k^{(n)})$ coincides with the double elementary symmetric polynomial $(-1)^k\, e_k(l_1^2, \dots, l_n^2 \,|\, a)$ associated with the sequence $a = (\rho_1^2, \rho_2^2, \dots)$; see (7.32). Hence, $\chi(\mathcal{F}_k) = (-1)^k\, e_k$. So, the claims follow from Proposition 8.7.2.

  $\Longrightarrow$

  PROPOSITION 8.7.3. *The elements* $\mathcal{F}_1, \mathcal{F}_2, \dots$ *are algebraically independent and generate the algebra* A_0. *Their images under the isomorphism* χ *are found by*

$$1 + \sum_{k=1}^{\infty} \frac{\chi(\mathcal{F}_k)}{(u^2 - \rho_1^2) \dots (u^2 - \rho_k^2)} = \prod_{i=1}^{\infty} \frac{u^2 - l_i^2}{u^2 - \rho_i^2}.$$

  PROOF. The Harish-Chandra image $\chi_N(\mathcal{F}_k^{(n)})$ coincides with the double elementary symmetric polynomial $(-1)^k\, e_k(l_1^2, \dots, l_n^2 \,|\, b^{(n)})$ associated with the sequence $b^{(n)} = (\rho_{-n}^2, \rho_{-n+1}^2, \dots)$; see (7.32). Hence, $\chi(\mathcal{F}_k) = (-1)^k\, e_k$. So, the claims follow from Proposition 8.7.2.

- page 312, lines 14–15:

  the algebra of shifted symmetric functions in the variables $\lambda_{m+1}, \lambda_{m+2}, \dots$.

  $\Longrightarrow$

  the algebra Λ^*, associated with the variables $\lambda_{m+1}, \lambda_{m+2}, \dots$.

- page 319, line -8:

  matrices for the algebra $\mathrm{U}_q(\widehat{\mathfrak{gl}}_{M+N})$.

  $\Longrightarrow$

  matrices for the algebra $\mathrm{U}_q(\mathfrak{gl}_{M+N})$.

- page 321, line -19 (add):

  The generalized rings of symmetric functions Λ^a were introduced by Okounkov [O-1998].

  ADDED REFERENCE:

  [O-1998] A. Okounkov, *On Newton interpolation of symmetric functions: a characterization of interpolation Macdonald polynomials.* Adv. in Appl. Math. **20** (1998), no. 4, 395–428.

- page 325, line -18:
  complementary module $\mathfrak{p}$.
  $\Longrightarrow$
  complementary module $\mathfrak{p}$ and suppose that the corresponding weights $\nu_1, \dots, \nu_n$ of the basis vectors are written in a non-decreasing order; that is, $\nu_i \prec \nu_j$ implies $i < j$.
- page 325, line -12:
  form a basis of $\mathrm{Z}(\mathfrak{g}, \mathfrak{k})$.
  $\Longrightarrow$
  form a basis of the left (or right) $\mathrm{R}(\mathfrak{h})$-module $\mathrm{Z}(\mathfrak{g}, \mathfrak{k})$.
- page 326, line -7:
  $f_i \in \mathrm{U}(\mathfrak{h})$
  $\Longrightarrow$
  $f_i \in \mathrm{U}(\mathfrak{h}_N)$
- page 336, line 7:
  the Mickelsson–Zhelobenko algebra $\mathrm{Z}(\mathfrak{g}_N, g_{N-2})$
  $\Longrightarrow$
  the Mickelsson–Zhelobenko algebra $\mathrm{Z}(\mathfrak{g}_N, \mathfrak{g}_{N-2})$
- page 348, line -2:
  polynomial in u
  $\Longrightarrow$
  polynomial $A(u)$ in u
- page 349, line -16:
  highest vector ξ of $V(\mu)$
  $\Longrightarrow$
  highest vector ξ of $V(\lambda)$
- page 362, line 14:
  corresponding basis vectors ξ_ν
  $\Longrightarrow$
  corresponding basis vectors ζ_ν
- page 370, line -8:
  corresponding basis vectors ξ_ν
  $\Longrightarrow$
  corresponding basis vectors ζ_ν
- page 379, line -14 (add):
  the construction of an analogue of the Gelfand–Tsetlin basis for representations of $\mathfrak{sp}_{2n}$ announced in the paper by D. P. Zhelobenko (Russ. Math. Surv. 42 (1987), 247–248) is erroneous.