Updates and additional information on “Seifert Fiberings”

Feb 17, 2011

Lens spaces. Lens spaces are the quotients of free linear actions by finite abelian groups on spheres. The 3-dimensional lens spaces $L(p,q)$ are defined on p. 373. The lens space $L(p,q)$ admits an S^1-action with a circle of fixed points and one singular E-orbit with slice invariant (μ,ν) where $\mu = p$ and $\nu q \equiv 1 \mod p$. This is described in 14.5.1 on p. 312. The lens spaces $L(p,q)$ are oriented according to our conventional orientation. For example, $L(p,1)$ is the principal S^1-bundle over S^2 with euler class $-p$. These lens spaces are classified up to orientation preserving homeomorphisms by

$$L(p,q) \approx L(-p,-q) \approx L(p,q') \approx -L(p,-q) \approx -L(p,p-q)$$

where $qq' \equiv 1 \mod p$ or $q \equiv q' \mod p$, and $\approx$ denotes orientation preserving homeomorphism.

Using Orlik's formula (as corrected) on p. 368, but with invariants in Chapter 15, we actually obtain, as to be expected, the oriented lens space $L(-p,q) \approx -L(p,q)$.

On pp. 366–368, several lens spaces have the wrong sign if we insist on orientation preserving homeomorphisms. In the following, we list which signs must be changed to give the correct orientation.

p. 366, 15.3.1 second paragraph

line -4 change $L(r,1)$ to $L(-r,1)$
line -3 change “positive” to “negative”

p. 367, 15.3.2(b) last line should be

space $-L(2m,2m-1) = L(2m,1)$

p. 368, second line above (e)

change $L(\beta,1)$ to $L(-\beta,1)$

p. 368, two lines above Exercise 15.3.3.
course by 15.3.1, $L(p,q)$ has some Seifert fibering of the form $\langle(1,0),(m,\beta),(m,r-\beta)\rangle$, where $mr = p$. Note...

p. 369, 15.3.4. In 4th line below the diagram

change $Q\backslash P_{2r}$ to $Q\backslash P_2$

p. 373, line 16

change $''e_2(\gamma) = -1$ to $''e_2(\gamma) = 1$

Other typos.
p. 322, 14.8.1, line 5

change $(\mathbb{Z}_2 \times \mathbb{Z}_2)\backslash \mathrm{SO}(3)/\mathrm{SO}(3)$ to $(\mathbb{Z}_2 \times \mathbb{Z}_2)\backslash \mathrm{SO}(3)/\mathrm{SO}(2)$.