Updates and additional information on “Seifert Fiberings”

p. 38, 2.6.4.

See the discussion in this updates (p.373) for a precise definition of lens spaces.

p. 42, Exercise 2.7.10

Last formula $q\hat{x} = h^{-1} \circ \hat{x}$ should change to $q\hat{x} = h^{-1} \bullet \hat{x}$

p. 52, after Corollary 3.1.17

Theorem 3.1.16 and Corollary 3.1.17 are also valid for closed aspherical cohomology manifolds over $\mathbb{Z}$. In fact, many places in the book, results for manifolds remain valid when manifold is replaced by cohomology manifold because many of the properties of manifolds are solely cohomological in nature, and consequently are also enjoyed by cohomology manifolds.

p. 53, Add to Remark 3.1.19.

In a recent preprint : arXiv:1108, 2321v1 [math.GT] Aug 11 2011, Sylvian Cappell, Shmuel Weinberger, and Min Yan show that in any dimension ≥ 6, there are closed aspherical manifolds (CAM) with fundamental groups having $\mathbb{Z}$ as centers, yet do not possess any non-trivial topological circle actions. Thus their theorem is a negative answer to the first question in 3.1.19 (2). This question was first proposed in [CR 69]. See also the Remark 11.7.5, page 249.

To construct such a manifold M, they form the mapping torus $T(h)$,where h is a self homeomorphism of N. N is a CAM with centerless fundamental group and h_* is a non-conjugation automorphism of $\pi_1(N)$ of order 2. This N is a refinement of a CAM N' constructed by Jonathon Block and Shmeul Weinberger (B-W) , On the generalized Nielsen realization problem, Comment. Math. Helv. 83 (2008) 21-33, for which the Nielsen realization problem fails. (See 11.3 especially p.217 and 11.3.13.). Now if $M = T(h)$ admits a circle action, it is easy to see that the action is homologically injective, 11.6. By 11.6.2, M would fiber equivariantly over the circle and its fiber N would admit a $\mathbb{Z}_2$ involution inducing h_* as an outer automorphism. In (B-W) it is shown that neither N', nor any ANR homology manifold N'' homotopically equivalent to N', admits an involution inducing h_* or a conjugate of h_*.

The major technical arguments are the modification of N' to N and any ANR homology manifold homotopy equivalent to it will have the desired properties to yield a contradiction to the existence of a circle action on M.

The universal covering of N is contractible but may fail to be Euclidean space. However, the universal covering of M is homeomorphic to Euclidean space. This follows from Manifolds covered by Euclidean space, Topology 14 (1975) by Ronnie Lee and Frank Raymond where it is shown that the universal covering of a CAM ,of dimension > 4, whose fundamental group has a non-trivial finitely generated normal abelian subgroup is homeomorphic to Euclidean space..

p. 53, Add to Remark 3.1.20.

Mike Davis, in his book The Geometry and Topology of Coxeter Groups Princeton University Press (2008), presents a detailed account of his and his collaborators' techniques in constructing closed aspherical manifolds that are very unlike those arising from discrete subgroups of Lie groups. This accessible and well written book covers many topics of current interest in aspherical manifold theory.

p. 54 Line 9 of the proof of Proposition 3.1.21.

The fact that $C_{G^*}(\mathit{\Pi})$ is Abelian comes from the general fact p.103, Corollary 5.5.3.

p. 56 Theorem 3.2.5 (2).

Add an assumption that $\mathrm{Fix}(G, M) \neq \emptyset$. Only then the evaluation map (at a fixed point) $\theta : G \to \mathrm{Aut}(\Phi)$ is defined.

p. 59. Proposition 3.2.14.

Is the converse true? That is: If N is admissible and M covers N, is M admissible?

p. 67 Line -10

The second X' should read $\widetilde{X}$.

p. 67 Line -9

X' should read $\widetilde{X}$.

P 67, Theorem 3.5.2.

Line 2 below diagram, $\pi_1(T^k, x')$ should read $\pi_1(T^k, 1)$.

P 67, Add after Theorem 3.5.2.

The proof of this theorem does not require the finite generation of the center of the fundamental group of X. The splitting was proved earlier in [CR 69 section7] for aspherical manifolds.

As an illustration where the center is not finitely generated, consider the mapping telescope T of a sequence of circles each being mapped onto the next by a 2-fold covering. The fundamental group of the telescope is the set of rationals of the form $Q' = \{m/2^n : \ m \text{ and } n \text{ integers}\}$. T is a $K(Q', 1)$-space. There is an obvious injective circle action on the telescope with orbit space the interval $[0, \infty]$. The action is free over $[0, 1)$, of isotropy group $\mathbb{Z}_2$ over $[1, 2)$, and isotropy $\mathbb{Z}_2^n$ over $[n - 1, n)$ for each n.

The splitting action $(S^1, T_{\mathbb{Z}}Z) = (S^1, S^1 \times W)$ where W is a tree. To understand this tree W, consider the pre-telescope T_n, i.e., the telescope for the first n maps of the circle. Starting with the identity over 0, we get the pre-telescope over $[0, 1]$. This is a circle action on the Möbius band which lifts to a splitting action where W_1 is an arc with a $\mathbb{Z}_2$ action – a reflection across the middle. The next stage W_2 is homeomorphic to the capital letter H, where $\mathbb{Z}_4 = \mathbb{Z}_{2^2}$ acts freely everywhere except for the crossbar of the H where the isotropy subgroup is $\mathbb{Z}_2$ except for the middle where it is $\mathbb{Z}_4$. One then continues inductively to construct the action of Q' on the tree, $\lim W_n$, with quotient $[0, \infty)$. At each stage one constructs the covering action of $\mathbb{Z}_2^n$ on $S^1 \times W_n$ commuting with the lifted free S^1 action. One can also construct a similar telescope to be a $K(Q, 1)$-space with an injective circle action with properties similar to T.

p. 68 Corollary 3.5.3.

Insert (cf. [CR 69 , 7.2]).

p. 90 Second display (Line 15)
first line of this display: Replace $\mathbb{Z}_2 \times \mathbb{Z}_2$ with H.

p. 147, Example 8.3.13 Line 4
change $G \rtimes (\mathbb{Z}_2 \times \mathbb{Z}_2)$ to $G \rtimes (\mathbb{Z}_4 \rtimes \mathbb{Z}_2)$.

p. 170 Line -6
$\mathrm{Aut}(\Gamma)$ should read $\mathrm{Out}(\Gamma)$.

p. 198 Line 1
"bbrk" should read $\mathbb{R}^k$.

p. 201 Line 1 of 10.5
$E_{p,q}$ should read $E^{p,q}$.

p. 220. 11.3.13.
The CAM N of Block and Weinberger as discussed in the addenda to 3.1.19 is an admissible CAM manifold which has an admissible extension which can not be realized by an action.

p. 247 11.6.15 Line 3
$\mathfrak{H}$ should read H.

p. 247 11.6.15 Line 6
"The set" should read "The cardinality of this set"
p. 249 11.7.5.
See the addenda to 3.1.19 above.

p. 322, 14.8.1, Line 5
change $(\mathbb{Z}_2 \times \mathbb{Z}_2)\backslash \mathrm{SO}(3)/\mathrm{SO}(3)$ to $(\mathbb{Z}_2 \times \mathbb{Z}_2)\backslash \mathrm{SO}(3)/\mathrm{SO}(2)$.
p. 327 Line 7
$p_1^{-1}(\Gamma_1)/G_\infty$ should read $p_1^{-1}(\Gamma_1)\backslash G_\infty$

p. 336 Line 2 and 3
α_j should read a_j.

p. 337 Line -1

$$-e(\Sigma(2,3,9)) = \frac{54}{18^2} = \frac{1}{6} = \frac{3}{2} - \frac{4}{3}.$$

p. 338 (ii) Add
We note that M is not homogeneous.

p. 350 Line 1 below diagram
Clearly, $\overline{\nu}^{-1}$ restricted to $D^2 - \partial D^2 =$

p. 350 Line 3 below diagram
M should read M'.

p. 366, 15.3.1 second paragraph

Line -4 change $L(r,1)$ to $L(-r,1)$
Line -3 change "positive" to "negative"

p. 367, 15.3.2(b) last line should be
space $-L(2m, 2m-1) = L(2m, 1)$

p. 368, second line above (e)
change $L(\beta, 1)$ to $L(-\beta, 1)$

p. 368, two lines above Exercise 15.3.3.
course by 15.3.1, $L(p,q)$ has some Seifert fibering of the form $\langle (1,0), (m,\beta), (m, r-\beta)\rangle$, where $mr = p$. Note...

p. 369, 15.3.4. In 4th line below the diagram
change $Q\backslash P_{2r}$ to $Q\backslash P_2$

p. 373, Line 16
change $''e_2(\gamma) = -1$ to $''e_2(\gamma) = 1$

p. 373. *Lens spaces.*

Lens spaces are the quotients of free linear actions by finite abelian groups on spheres. The 3-dimensional lens spaces $L(p,q)$ are defined on p. 373. The lens space $L(p,q)$ admits an S^1-action with a circle of fixed points and one singular E-orbit with slice invariant (μ, ν) where $\mu = p$ and $\nu q \equiv 1 \bmod p$. This is described in 14.5.1 on p. 312. The lens spaces $L(p,q)$ are oriented according to our conventional orientation. For example, $L(p,1)$ is the principal S^1-bundle over S^2 with euler class $-p$. These lens spaces are classified up to orientation preserving homeomorphisms by

$$L(p,q) \approx L(-p,-q) \approx L(p,q') \approx -L(p,-q) \approx -L(p,p-q)$$

where $qq' \equiv 1 \bmod p$ or $q \equiv q' \bmod p$, and $\approx$ denotes orientation preserving homeomorphism.

Using Orlik's formula (as corrected) on p. 368, but with invariants in Chapter 15, we actually obtain, as to be expected, the oriented lens space $L(-p,q) \approx -L(p,q)$. On pp. 366–368, several lens spaces have the wrong sign if we insist on orientation preserving homeomorphisms. In the following, we list which signs must be changed to give the correct orientation.

p. 374 15.4.1 (2)
$gT_2 = \cdots$ $e(gT_1) = 0$ should read $e(gT_2) = 0$
$gT_3 = \cdots$ $e(gT_1) = 0$ should read $e(gT_3) = 0$

Reference. An important paper was inadvertently left out in the reference.

G. Hamrick and D. Royster, *Flat Riemannian manifolds are boundaries*, Invent. Math. 66(1982), 405–413.