Corrections to the book "Tensor categories"
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Here are some corrections to the book "Tensor categories". Some of them appear at Ulrich Thiel's website

`http://ulthiel.com/math/teaching-org/tensor-categories/`

along with many useful comments and explanations. We thank him, as well as Johannes Berger, Ryan Kinser and Vanessa Miemietz for pointing out many of the corrections below.

If you see other mistakes, please let us know!

General comments. 1. When working with an additive category $\mathcal{C}$ over a field $\Bbbk$, we often use the fact that for any finite dimensional $\Bbbk$-vector space V we have a natural functor $V\otimes : \mathcal{C} \to \mathcal{C}$. Namely, for $X \in \mathcal{C}$ the object $V \otimes X$ is uniquely defined through the Yoneda lemma by the formula $\mathrm{Hom}(Y, V \otimes X) = V \otimes \mathrm{Hom}(Y, X)$ (and the existence of this object is checked by choosing a basis in V).

2. We often abuse terminology and refer to epimorphisms in abelian categories as surjections and to monomorphisms as injections.

3. We use the terms "natural morphism" and "functorial morphism" interchangeably.

Other comments.

Definition 1.8.3, typo: "any simple object" should be "any object".

Definition 1.8.13: $\Bbbk$ should be replaced by another field $\mathbf{k}$, usually of characteristic zero, and not necessarily equal to the field $\Bbbk$ of definition of $\mathcal{C}$.

Subsection 1.10 (Coend), clarification. In (1.9), the direct sum is taken over isomorphism classes of objects. Also the (co)limits in lines 6,7 of the section are taken for the following diagram J: the vertices v_X of J are labeled by representatives X of isomorphism classes of objects, and arrows are $\phi_f : v_X \to v_{X\oplus Y}$ and $\psi_f : v_Y \to v_{X\oplus Y}$ for every morphism $f : X \to Y$. The map $E_F : J \to \mathcal{C}$ is given by the formula $E_F(v_X) = \mathrm{End}(F(X))$, $E_F(\phi_f)(a) = f \circ a$, $E_F(\psi_f)(a) = a \circ f$, where we view $\mathrm{Hom}(X, Y)$ naturally as a subgroup of $\mathrm{End}(X \oplus Y)$. Then $\varinjlim \mathrm{End}(F(X))$ reproduces the usual definition of $\mathrm{End}(F)$: the set of collections $\{a_X \in \mathrm{End}(F(X))\}$ such that $a_Y \circ f = f \circ a_Y$ for all X, Y and $f : X \to Y$.

Proof of Corollary 2.2.5, typo at the beginning: $Y = Z = \mathbf{1}$ should be $Y = X = \mathbf{1}$.

Remark 2.4.2: Section 2.5 should be Section 2.6.

Subsection 2.6, l. 3, typo: $\mathcal{C}_i$ are not the categories of graded vector spaces (they are not linear). l. 5: the word "simple" should be deleted. Also we consider only monoidal functors which act trivially on A.

Remark 2.8.6, clarification. In the last sentence of p.38, "the same category with the trivial associativity morphism" should read "the same category with the trivial associativity morphism and a modified tensor product of morphisms". Namely, let $\mu : \widetilde{G} \times \widetilde{G} \to A$ be a 2-cochain trivializing the cocycle $f^*\omega$. Then the tensor product of morphisms in the strict category isomorphic to $\mathcal{C}^{\wr}$ is defined as follows. Given elements $g, g', h, h' \in \widetilde{G}$ with $f(g) = f(g')$, $f(h) = f(h')$ and $a \in \mathrm{Hom}(g, g') = A$, $b \in \mathrm{Hom}(h, h') = A$, we set

$$a \otimes b := ab\mu(g,h)\mu(g',h')^{-1} \in \mathrm{Hom}(gh, g'h') = A.$$

Exercise 2.9.1, typo: the correct answer is the $n-1$-th Catalan number, $\frac{1}{n}\binom{2(n-1)}{n-1}$.

Remark 2.10.9, typo: in line 3, V^* should be ${}^*V$.

Example 2.10.14, $\omega(g, g^{-1}, g)$ should be $\omega(g, g^{-1}, g)^{-1}$.

Paragraph after Example 2.10.14, typo: 2.10.7(ii) should be 2.10.7(b).

Exercise 2.10.16, typo: "left (respectively, right)" should be replaced by "right (respectively, left)".

Corollary 4.3.9: "monomorphisms" and "epimorphisms" should be interchanged.

Paragraph before Example 4.5.5: the ring category should be assumed finite.

Remark 4.5.6: "homomorphism of unital $\mathbb{Z}_+$-rings" should be "unital homomorphism of $\mathbb{Z}_+$-rings".

Definition 4.7.11: Tr^L in the first line is not needed, but Tr should be Tr^L in the formula right below.

Proof of Theorem 4.7.15, line 3: Y_i and V_i should be switched (I.e., V_i are objects and Y_i are vector spaces).

Exercise 4.9.6 is an example.

Exercise 5.3.7(ii): "is a commutative" should be "is commutative".

Exercise 5.3.13. The exercise is to prove the given statement.

Remark 5.4.3: Corollary 5.3.15 should be Proposition 5.3.15.

Theorem 5.6.2, clarification: there is a unique Hopf algebra structure on $U_q(\mathfrak{sl}_2)$ *such that...*

Theorem 5.10.2, proof, last two lines, typo: A_0 should be $S\mathfrak{g}$ in three places.

Subsection 6.1, line 3 should start as: "Let $\mathcal{C}$ be a finite multitensor category".

Definition 7.2.2: the diagram (7.8) defining morphisms of $\mathcal{C}$-module functors should read

$$\begin{array}{ccc} F(X \otimes M) & \xrightarrow{s_{X,M}} & X \otimes F(M) \\ {\scriptstyle \nu_{X\otimes M}}\downarrow & & \downarrow{\scriptstyle \mathrm{id}_X \otimes \nu_M} \\ G(X \otimes M) & \xrightarrow{t_{X,M}} & X \otimes G(M). \end{array}$$

Subsection 7.3, p.135, footnote 1: $\mathrm{End}_{\mathcal{C}}$ should be End_l.

Example 7.4.6: $\mathrm{End}_{\mathcal{C}}(\mathcal{M})$ should be $\mathrm{End}(\mathcal{M})$. Also Proposition 7.1.3 should be Proposition 7.3.3.

Example 7.8.3(4), typos: H should be L (in several places) to indicate it's a subgroup as in (3) and two instances of k should be $\Bbbk$.

Remark 7.8.6(i), line 3, typo: the target of q should be ${}^*M$.

Exercise 7.8.27, typo: $\otimes_A$ should be $\otimes_B$.

Exercise 7.10.4: "condition (2) above" should be "condition (ii) of Theorem 7.10.1".

Proof of Proposition 7.11.6, clarification: The category $\mathrm{Fun}_{\mathcal{C}}(\mathcal{M}_1, \mathcal{M}_2)$ has finitely many simple objects since any simple object of this category is a quotient of $A_1 \otimes P \otimes A_2$ for some indecomposable projective object $P \in \mathcal{C}$.

Example 7.12.26, last 4 lines: H^* should be $H^{*\mathrm{cop}}$ and vice versa.

Subsection 7.18, p.175, line 5: $\mathbf{1} \boxtimes \mathbf{1}$ is a direct summand of $A \otimes A^*$.

Subsection 7.19, p.178, line 9, typo: "V in the category of..."

Subsection 7.21, p.181, lines 27-28: "positive" instead of "non-negative" twice.

Lemma 8.10.5 is incorrect. For example, if $X \in \mathrm{Rep}_{\Bbbk}(G)$ is an irreducible representation of a finite group G over a field $\Bbbk$ of characteristic p such that $\dim V$ is divisible by p (e.g. the Steinberg representation of $SL_2(\mathbb{F}_p)$) then the composition in the lemma is, in fact, zero. In the proof, the map $\mathrm{Hom}(\mathbf{1}, X^* \otimes X) \to \mathrm{End}(\mathbf{1})$ is zero, even though $X^* \otimes X \to \mathbf{1}$ is surjective (which is possible since the functor $\mathrm{Hom}(\mathbf{1}, ?)$ is not right exact).

Lemma 8.10.5 is only invoked in the proof of Proposition 8.10.6, but it is not needed. Namely, the second displayed equation in this proof is incorrect, and instead we simply have $v_X \circ u_X = \mathrm{Id}_X$, for any object X. This is easy to prove by a direct computation, and the proofs can be found, say, in [BakK] and [Tu4].

Proof of Theorem 8.14.7, line 2, typo: K_O should be K_0.

Definition 8.17.1(3), typo: "coming from (2)" (not (Z)).

Subsection 8.18, p. 233 line 7 from bottom: T_s/T_pT_q should be T_pT_q/T_s.

Exercise 8.18.9(vii). This exercise requires an extension of the semisimplification procedure to non-spherical pivotal categories, see P. Etingof and V. Ostrik, Semisimplification of tensor categories, arXiv:1801.04409, Subsection 2.3. Also the answer is incorrect for even n. The correct answer is the subring of $\mathbb{Z}[\mathbb{Z}/2n] \otimes \mathrm{Ver}_{n-2}$ generated by $g \otimes L$, where g is a generator of $\mathbb{Z}/2n$ and L the tautological object of the Verlinde category (corresponding to the 2-dimensional irreducible representation of the quantum $\mathfrak{sl}_2$).

Subsection 8.18, p. 237, line 1, typo: "categories" should be "categorifies".

Proof of Lemma 8.20.8, p. 242, line 13, typo: $d_-(Z)^{-1}$ should be $d_+(Z)^{-1}$.

Proof of Lemma 8.20.9, p. 242, line 22, typo: $\dim(\mathcal{C})$ should be $\dim(\mathcal{D})$.

Proposition 8.20.16, line 5, typo: "the sum is over the simple objects X" (not of X).

p. 251, diagram (8.101), typo: $c_{M,N} \otimes A$ should be $c_{M,N\otimes A}$ on the left side of the diagram.

Subsection 8.25, p. 257, line 15 from bottom, typo: "find a basis x_i of V" (instead of "find a biproduct of V").

Remark 9.7.3 applies to the situation when $\omega = 1$. The general case is discussed in detail by S. Natale in arXiv:1608.04435.

Subsection 9.11, p. 303, line 18: $\mathrm{Coend}(F)$ should be $\mathrm{Coend}(F_1)$.

p.306, line 10: "$P \geq P'$" should be replaced with "$P \leq P'$".