

Paul Larson, Jindrich Zapletal: Geometric set theory

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In Chapter 5, we introduce the calculus of virtual conditions, balanced pairs, and balanced virtual conditions for Suslin forcings. It turns out that it is possible to develop the whole calculus without change for the larger class of analytic forcings:

Definition 0.1. A pair $\langle P, \leq \rangle$ is *analytic forcing* if there is an ambient Polish space X such that $P \subset X$ is an analytic set and \leq is a preorder on P and an analytic subset of $X \times X$.

In other words, the class of analytic forcings obtains from Suslin forcings by dropping the analyticity demand on the incompatibility relation. In all theorems of the book, the phrase “Suslin forcing” in their assumptions can be replaced with “analytic forcing” and the conclusions remain in force. While it is true that all particular examples of analytic forcings in the book are Suslin, in further work we have encountered forcings which are clearly analytic, and where the analyticity of the incompatibility relation either results in burdensome verification or is even an open question.