Errata from *The Stationary Tower.*

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- On page 4, in Exercise 1.1.7, replace '(i.e., preserving the $\in$-relation)' with '(i.e., preserving formulas whose quantifiers are bounded)'.
- On page 6, add to the hypothesis of Lemma 1.1.13 that $\kappa \setminus \alpha$ is in U for each $\alpha < \kappa$.
- On page 8, add to the hypothesis of Lemma 1.1.18 that M satisfies 'For every ordinal α, V_α exists' and 'For every set x, there is an ordinal α such that x is in V_α.'
- On page 13, in the statement of Lemma 1.1.27, insert 'on κ' after the first occurrence of μ.
- On page 18, line -7, put 'when $\lambda \geq 2^\omega$.' after 'under the Axiom of Choice'.
- On page 23, in the third bulleted point at the top of the page, the tower should concentrate on T_x.
- On page 24, in the last sentence of the statement of Lemma 1.3.13, insert 'if σ contains a nonprincipal measure then' after 'in particular'.
- On page 24, add 'on λ' after 'λ-complete measure' in the statement of Lemma 1.3.14.
- On page 25, the key point in the proof of Lemma 1.3.14 doesn't follow from the Lemma 1.1.25, but it does follow easily from the fact that $\bar{\pi}$ in the proof of Lemma 1.1.25 is a bijection.
- On page 29, M should be N in the sentence beginning with 'Then E_j is a function' in the second to last paragraph.
- On page 37, V should be M on the third-to-last line.
- On page 43, line -14, $s^{<\omega}$ should be $2^{<\omega}$.
- On page 43, in the sentence in the second-to-last paragraph starting with 'Assuming that we have', 'σ_t for all t in $2^{G(i-1)}$' should be 'σ_t in $2^{G(i-1)}$ for all t in 2^i'.

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- On page 46, in the last line of the proof of lemma 1.6.26, b_S should be b_T.
- On page 47, switch the order of the first two sentences of the proof of Lemma 1.7.3.
- On page 49, in the second paragraph after Definition 2.1.2, X and M are used interchangeably. Converting all M's to X's would fix the problem.
- On page 50, in the last sentence of proof of Lemma 2.1.3 each instance of g should be g'.
- On page 55, in Corollary 2.2.11, 'is club' should be 'contains a club' (this happens in other places as well).
- On page 55, in Corollary 2.2.12: α should be removed from the statement of the lemma, δ should be κ, and the sets in (1) and (2) should be the union of the sets written there with $\mathcal{P}(\cup a) \setminus Z$ (also the same 'is club'/'contains a club' issue).
- On page 62, in the last line of Definition 2.5.1, replace 'is club in' with 'contains a club in'.
- On page 62, replace the last sentence of the paragraph after Definition 2.5.1 with the following (what is written there is correct, but the following is better). 'If D is semi-proper, then for each $a \in \mathbb{P}_{<\kappa}$ the set of Y capturing D with $Y \cap (\cup a) \in a$ is stationary. The corresponding map $Y \mapsto d$ is constant on a stationary set b, which is below both a and the constant value d in $\mathbb{P}_\infty$.'
- On page 62, in the hint for Exercise 2.5.3, the 'D' at the end of the second-to-last sentence should be a 'd'.
- On page 63, part (2) of the statement of Lemma 2.5.4 should also require that M satisfies the statement that V_α exists for all α.
- On page 63, in the hint for Exercise 2.5.5, instead of letting $\langle a_\alpha : \alpha < \delta \rangle$ be an enumeration of $\mathbb{P}_{<\delta}$, let a_α be the set of ordinals of cardinality $\aleph_\alpha$.
- On page 64, in the proof of Lemma 2.5.6, in the 3rd bulleted point, $X \cap A_\alpha$ should be $X \cap A_\alpha \cap \mathbb{P}_{<\gamma}$.
- On page 66, in item (a) in the proof of Theorem 2.5.9, "for all $\alpha < \delta$" should be "for $\alpha < \gamma$".
- On page 67, in lines 15-16, remove '$b \in \mathcal{P}_{<\delta}$, i.e.,'. (The point is that the proof as written does not literally ensure that $j(f)(\gamma) + \omega < \delta$, but it doesn't require it, either. One can ensure this by assuming that j is generated from an extender in V_δ, or even that j is definable from a set (which indeed is the usual assumption, but is not explicitly written here) or by appealing to Theorem 1.5.6 and choosing j such that $j(f)(\gamma) = f(\gamma)$.)

- On page 68, in Definition 2.5.12, insert 'strongly inaccessible' in between 'A' and 'cardinal'.
- On page 69, in the proof of Lemma 2.5.15, in the third dashed point of the third bullet, 'a' should be 'α'.
- On page 71, in Remark 2.5.19, in the sentence beginning with 'As before, the point', V_k should be V_κ.
- On page 73, in the proof of Theorem 2.5.20, in the second collection of bullets, second bullet, D should be $j(D)$.
- On page 73, in the proof of Theorem, 2.5.20, in the paragraph starting with 'Now let X =' towards the bottom of the page, 'Since $j\restriction V_{\kappa+2}$' should be 'Since $j\restriction V_{\kappa+2}$', likewise $X \cap V_{\kappa+1}$ should be '$X \cap V_{\kappa+2}$'.
- On the third line of page 74, '$X^* \cap V_\kappa$' should be '$j(X^*) \cap j(V_\kappa)$' (what's written is true, but the corrected version is the point).
- On page 74, change the last sentence of the proof of Theorem 2.5.20 to 'Lastly, it follows easily that $j(X^*) = j[X^*] \subset Y^*$.' This change induces the following changes earlier on the same page, though in each case what's written there is correct.
  - Change 'Also, $j(X^*) \subset X \cap j(V_{\kappa+1})$, so it suffices' to 'It then suffices'.
  - Change the first bulleted point to '$j(X^*) \subset Y^*$'.
- On page 74, it appears that I had the obvious proof in mind for Corollary 2.5.21. Unfortunately, I don't see why the obvious proof should work. So I don't know if the corollary is true. This necessitates some changes on pages 79 and 89, which are listed below. I probably should have just noted that Theorem 2.5.20 shows that strongly compact cardinals satisfy the hypotheses of Lemma 2.5.15.
- On page 76, nearly half way down: 'union of the first δ stages' should be 'union of the first γ stages'.
- On page 79, remove the clause beginning with 'and this holds for' from the statement of Theorem 2.7.6.
- On page 81, in the statement of Corollary 2.7.12, replace '$\zeta < \delta < \kappa$' with '$\zeta < \delta < cof(\kappa)$' (also 'is a limit ordinal' instead of 'is limit ordinal').
- On page 82, in the statement of Lemma 2.7.14: a should be the set of countable X elementary in V_{δ_1+1}, not V_{δ_1}.
- On page 83, in the second sentence of the proof of Lemma 2.7.16: 'each $b \in G \cap \mathbb{Q}_{<\delta_1}$' should be 'each $b \in \mathbb{Q}_{<\delta_2}$'.
- On page 86, line 10, 'to to' should be 'to'.

- On page 88, in the statement of Theorem 3.1.9, change 'strongly inaccessible cardinal and that for cofinally many' to 'strongly inaccessible limit of Jónsson cardinals and that for stationarily many'.
- On page 89, Corollaries 3.1.10 and 3.1.11 are true, but they aren't corollaries of Theorem 3.1.9. Rather, they follow from Lemma 3.1.5, Lemma 2.7.14, Theorem 2.5.20 and Lemma 2.5.15 (and its proof).
- On page 95, line -3, 'in' should be 'for.'
- On page 97, in Definition 3.3.1, $m(\kappa^{<\omega})$ should be $m(Z^{<\omega})$.
- On page 103, in the first paragraph of the proof of Theorem 3.3.8, $p[T]^*$ should be $p[T^*]$.
- On page 110, in the first sentence of the second paragraph, $\omega^{<\omega}$ should be ω^{ω}.
- I probably should have included an appendix on sharps. A note on this topic can be found at http://www.logic.univie.ac.at/ caicedo/sharps.pdf

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