A First Course in Sobolev Spaces, First edition
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For the original text I use the color Red, for corrections the color Blue, and for improvements and additions the color Cyan. Names in brackets refer to the persons who called the error to my attention (to the best of my recollection) or suggested improvements and additions.[1]

CHAPTER 1:

p. 5 In Exercise 1.5 one should assume that u is bounded from below, since otherwise the series could diverge to ∞. Alternatively, one should choose a point $x_0 \in I^\circ$ at which u is continuous and then take

$$u_{\mathrm{J}}(x) := \begin{cases} +\sum_{y\in I,\, x_0\le y<x} (u_+(y) - u_-(y)) + u(x) - u_-(x) & \text{if } x \ge x_0, \\ -\sum_{y\in I,\, x<y\le x_0} (u_+(y) - u_-(y)) + u(x) - u_+(x), & \text{if } x \le x_0. \end{cases} \tag{$\boxplus$}$$

p. 32 In Exercise 1.44 Consider the Banach space

$$X := \{u : [0,1] \to \mathbb{R} : \ u \text{ is continuous, } u(0) = 0, \text{ and } u(1) = 1\},$$

where we take the supremum norm

$$\|u\|_\infty := \max_{x\in[0,1]} |u(x)|.$$

should be replaced by Consider the complete metric space

$$X := \{u : [0,1] \to \mathbb{R} : \ u \text{ is continuous, } u(0) = 0, \text{ and } u(1) = 1\},$$

where we take the metric induced by supremum norm

$$\|u\|_\infty := \max_{x\in[0,1]} |u(x)|.$$

[Rinat Kashaev]

CHAPTER 3:

p. 109 In Corollary 3.74 one should either assume that u is bounded from below or replace (3.33) with ($\boxplus$) (above in this file).

CHAPTER 4:

p. 116 In line 4 By requiring parametric representations and parameter changes should be replaced by By requiring parametric representations, parameter changes and their inverses .

[1] The style of this file is inspired by http://www.hss.caltech.edu/ kcb/IDA-Errata.pdf

CHAPTER 6:

p. 188 In Proposition 6.1(ii) If u vanishes at infinity, then

$$\lim_{s\to\infty} \varrho_u(s) = 0$$

should be replaced either by If u vanishes at infinity, then

$$\lim_{s\to\infty} \varrho_u(s) = \mathcal{L}^1(\{x \in E : u(x) = \infty\})$$

or by If u vanishes at infinity and is real-valued, then

$$\lim_{s\to\infty} \varrho_u(s) = 0$$

[Allen]

p. 189 In the proof of In Proposition 6.1(ii) Then $E_n \supset E_{n+1}$ and

$$\bigcap_{n=1}^{\infty} E_n = \emptyset.$$

Since $\mathcal{L}^1(E_n) < \infty$, it follows by Proposition B.9(ii) that

$$\lim_{n\to\infty} \varrho_u(s_n) = \lim_{n\to\infty} \mathcal{L}^1(E_n) = \mathcal{L}^1(\emptyset) = 0.$$

should be replaced either by Then $E_n \supset E_{n+1}$ and

$$\bigcap_{n=1}^{\infty} E_n = \{x \in E : u(x) = \infty\}.$$

Since $\mathcal{L}^1(E_n) < \infty$, it follows by Proposition B.9(ii) that

$$\lim_{n\to\infty} \varrho_u(s_n) = \lim_{n\to\infty} \mathcal{L}^1(E_n) = \mathcal{L}^1(\{x \in E : u(x) = \infty\}).$$

or one should assume in the statement that u is real-valued.

p. 190 One line after formula (6.6): one should either replace Note that if u vanishes at infinity, then $\varrho_u(s) \to 0$ as $s \to \infty$ by the previous proposition, and so for $t > 0$ the set

$$\{s \in [0,\infty) : \varrho_u(s) \leq t\}$$

is nonempty. Thus, $u^*(t) < \infty$ for all $t > 0$, while $u^*(0)$ could be infinite. with Note that if u vanishes at infinity, then $\varrho_u(s) \to T$ as $s \to \infty$, where $T := \mathcal{L}^1(\{x \in E : u(x) = \infty\})$, by the previous proposition, and so for $t > T$ the set

$$\{s \in [0,\infty) : \varrho_u(s) \leq t\}$$

is nonempty. Thus, $u^*(t) < \infty$ for all $t > T$. or replace Note that if u vanishes at infinity with Note that if u vanishes at infinity and is real-valued

CHAPTER 7:

p. 217 An alternative proof of Step 2 in the proof of Lemma 7.3:

Step 2: In the general case, let (a_n, b_n) be an increasing sequence of open intervals such that $[a_n, b_n] \subset (a_{n+1}, b_{n+1})$ and

$$\bigcup_{n=1}^{\infty} (a_n, b_n) .$$

Then for every n,

$$\int_{(a_n, b_n)} u\varphi' \, dx = 0$$

for all $\varphi \in C_c^1((a_n, b_n))$. Hence, by Step 1, there exists $c_n \in \mathbb{R}$ such that $u(x) = c_n$ for $\mathcal{L}^1$ a.e. $x \in (a_n, b_n)$. But since, $[a_n, b_n] \subset (a_{n+1}, b_{n+1})$, it follows that $c_n = c_{n+1}$ for all n.

CHAPTER 10:

p. 282 In Exercise 10.11(4), $1 \leq p < \infty$ should be replaced by $1 < p < \infty$ [Alberto Venni]

p. 287 In formula (10.11) and in the formula on the last line of the page $(y', y_N) \in \Omega \cap A$ should be replaced by $(y', y_N) \in A$.

p. 290 In formula (10.12) $(y', y_N) \in \Omega \cap A_{x_0}$ should be replaced by $(y', y_N) \in A_{x_0}$.

p. 305 In Exercise 10.51, p should be ∞ . Also the fact that $W^{1,\infty}(\Omega)$ is a dual space is not obvious. See the file "Lipschitz functions as a dual space". [M.G. Mora]

CHAPTER 11:

p. 305 In line 8, by (1.15) (with u replaced by u_n) should be replaced by reasoning somewhat as in (1.15). [Maria Giovanna Mora]

p. 316 In Step 4

$$v_n(x) := \begin{cases} |u(x)| - \frac{1}{n} & \text{if } \frac{1}{n} \leq |u(x)| \leq n, \\ 0 & \text{if } |u(x)| < \frac{1}{n}, \\ n - \frac{1}{n} & \text{if } |u(x)| > \frac{1}{n}. \end{cases}$$

should be replaced by

$$v_n(x) := \begin{cases} |u(x)| - \frac{1}{n} & \text{if } \frac{1}{n} \leq |u(x)| \leq n, \\ 0 & \text{if } |u(x)| < \frac{1}{n}, \\ n - \frac{1}{n} & \text{if } |u(x)| > n. \end{cases}$$

[Robert Simione]

p.322 On line 6 of the proof of Theorem 11.10 we may find a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ should be replaced by we may find a subsequence of $\{u_n\}$, not relabeled, and then in the remaining of the proof u_{n_k} should be replaced by u_n .

p. 326 In the formula on line 4 of the proof of Theorem 11.21 $(y', y_N) \in \Omega \cap A_{x_0}$ should be replaced by $(y', y_N) \in A_{x_0}$.

p. 326 In the formula on line 19 $(y', y_N) \in \Omega \cap A$ should be replaced by $(y', y_N) \in A$.

CHAPTER 12:

p. 353 In Exercises 12.6 and 12.7, d_{reg} be its regularized distance should be replaced by d_{reg} be the regularized distance corresponding to $\mathbb{R}^N \setminus \overline{\Omega}$.

p. 353 In Exercises 12.7, $\lim_{t\to\infty} \frac{\psi(t)}{t^m} = 0$ should be replaced by $\lim_{t\to\infty} t^m \psi(t) = 0$ [Adrian Hagerty]

p. 354 In Line 2, $td_{\text{reg}}(x)$ should be replaced by $Ctd_{\text{reg}}(x)$, where C is the constant given in Exercises 12.6. See also the file "Extension domains for higher order Sobolev spaces" for a complete proof.

p. 354 In line 5 of Definition 12.9 $(y', y_N) \in \Omega \cap A$ should be replaced by $(y', y_N) \in A$.

p. 356 In formula (12.7), $\frac{1}{\varepsilon}$ should be replaced by $\frac{M}{\varepsilon}$.

p. 359 In lines 2-6 and in Remark 12.16, $\frac{1}{\varepsilon}$ should be replaced by $\frac{M}{\varepsilon}$.

p. 370 In Remark 12.35, $\frac{1}{4}$ should be replaced by $\frac{1}{2}$. [Marco Barchiesi]

CHAPTER 13:

p. 402 Thanks to Paolo Piovano for pointing out that the proof of Theorem 13.30 is too fast. It should be divided in two steps. Assuming first that $u \in L^{1^*}(\mathbb{R}^N)$, by mollification it follows that

$$\left(\int_{\mathbb{R}^N} |u(x)|^{1^*} \, dx\right)^{\frac{1}{1^*}} \leq C\,|Du|\left(\mathbb{R}^N\right).$$

To remove the additional hypothesis that $u \in L^{1^*}(\mathbb{R}^N)$, one should truncate u as in Step 4 of the proof of Theorem 11.2. The problem here is the fact the mollification of a function vanishing at infinity needs not vanish at infinity. See also the file "An extension of the Sobolev–Gagliardo–Nirenberg theorem".

CHAPTER 14:

p. 422 In Exercise 14.13 Prove that if g is decreasing, then there exists

$$\lim_{s\to 0^+} \|g\|_{s,\infty} = \lim_{h\to\infty} g(h).$$

What happens if we remove the hypothesis that g is decreasing?should be replaced by Prove that if g is increasing, then there exists

$$\lim_{s\to 0^+} \|g\|_{s,\infty} = \lim_{h\to\infty} g(h).$$

What happens if we remove the hypothesis that g is increasing?[Xiang Xu]

p. 422 In Exercise 14.13 Prove that if $\frac{g(h)}{h}$ is increasing, then there exists

$$\lim_{s\to 1^-} \|g\|_{s,\infty} = \lim_{h\to 0^+} \frac{g(h)}{h}.$$

What happens if we remove the hypothesis that $\frac{g(h)}{h}$ is increasing?should be replaced by Prove that if $\frac{g(h)}{h}$ is decreasing, then there exists

$$\lim_{s\to 1^-} \|g\|_{s,\infty} = \lim_{h\to 0^+} \frac{g(h)}{h}.$$

What happens if we remove the hypothesis that $\frac{g(h)}{h}$ is decreasing?[Xiang Xu]

More to come for sure... :(