TENSORS: GEOMETRY AND APPLICATIONS CORRECTIONS AND ADDITIONS, LAST UPDATED 8/12

J.M. LANDSBERG

1. Chapter 1

- p32, §2.3.1, line 4, $W^* \to V$ change to $W^* \to V^*$

2. Chapter 2

- p51, Eqn. (2.7.1) denominator $2^n n!$ change to $2^m m!$ and 2^n change to 2^m.
- p65, Fig. 2.11.14: of the six realizations, only three are distinct.

3. Chapter 3

- p82, §3.8.1.4: In Thm, It remains to show that the equations are not identically zero. This was partially shown in [4].
- p89, first displayed matrix, indices in $(2,3)$ slots of blocks 1,3,4,6 are incorrect. They should respectively be $(122), (122), (012), (012)$.

4. Chapter 4

5. Chapter 5

- p127, Thm 5.5.1.1(4) $\mathbf{ab} - \mathbf{a} - \mathbf{b} - 2$ change to $\mathbf{ab} - \mathbf{a} - \mathbf{b} + 2$ two times.

6. Chapter 6

- p140, first bullet: "the number of elements in its conjugacy class" change to "the dimension of the module".
- p140, (6.2.3) $+e_{(132)}$ change to $-e_{(132)}$.
- p140, Exercise 6.2.1(1) $v\rho_{\boxed{1}\boxed{2}\boxed{3}} = \rho_{\boxed{1}\boxed{2}\boxed{3}}$ change to $v\rho_{\boxed{1}\boxed{2}\boxed{3}} = \lambda\rho_{\boxed{1}\boxed{2}\boxed{3}}$ for some $\lambda \in \mathbb{C}^*$, and similarly for the rest of the exercises.
- p153, §6.7.1, displayed eqn., $\sum$ change to $\bigoplus$
- p154, Exercise 6.7.1.1: $c^{\nu}_{\pi,\mu}$ change to $c^{\pi}_{\nu,\mu}$, also Hint: Say $|\pi| = k$, expand $(A \otimes B)^{\otimes k}$, and reshuffle to have all A's first, then all B's, and consider the subgroup of $\mathfrak{S}_{2k}$ that preserves the shuffling.
- Exercise 6.7.3(2) $\sum$ change to $\bigoplus$
- p160, Exercise 6.8.1.3 $(x_1 \wedge \cdots \wedge x_p)^p$ change to $x_1^p \cdots x_k^p$
- p161, Prop. 6.8.2.1/Exercise 6.8.2.2. What is proved is that a nonzero *weight* vector is a highest weight vector iff $\mathfrak{n}.v = 0$. To do the exercise, need the fact that $B = exp(\mathfrak{b})$.
- p165, Exercise 6.9.2(2). To make the exercise easier, first do the case of the projection $GL(V)/B$ to a next-to minimal parabolic (show the fiber is a $\mathbb{P}^1$).
- p168, Prop. 6.10.4.1, remove the $S_{2d}A$ terms from the ideal.

- p170, Exercise 6.10.6.6 is difficult, as the natural realizations of the modules will not lie in $S^d(A\otimes B)$, one must show their projections into $S^d(A\otimes B)$ are nonzero.

7. Chapter 7

- p180, $\Pi\mathbf{a}_i - \sum \mathbf{a}_i - n + 1$ change to $\Pi\mathbf{a}_i - \sum \mathbf{a}_i + n - 1$ in Thm 7.3.1.4(1) and above statement of Thm.

8. Chapter 8

9. Chapter 9

- p235/6, Prop. 9.3.1 is not correct as stated because $\underline{\mathbf{R}}(xyz) = 4$, not 3 as stated in proof p 236.
- p236, §9.3.2, the symmetric border rank lower bounds via flattenings are easily seen to be $\binom{n}{\lfloor n/2 \rfloor}^2$ in both cases, as the image of flattenings are the appropriate sized minors (resp. sub-permanents) for the determinant (resp. permanent).

10. Chapter 10

- p260, §10.5. A better, and older reference for this section is [1].
- p266, Thm. 10.9.2.1, S^3W change to S^dW.
- p269, displayed eqn. (4) $a_3\otimes b_1\otimes c_1 + a_1\otimes b_2\otimes c_2 + a_1\otimes b_1\otimes c_2 + a_2\otimes b_3\otimes c_1 + a_2\otimes b_1\otimes c_3$ change to $a_3\otimes b_1\otimes c_1 + a_1\otimes b_2\otimes c_1 + a_1\otimes b_1\otimes c_1 + a_2\otimes b_3\otimes c_1 + a_2\otimes b_1\otimes c_3$

11. Chapter 11

- p275, the current (as of 8/12) world record for the upper bound of the exponent of matrix multiplication is in [5].
- p276, the current (as of 8/12) world record for the lower bound of the border rank (resp. rank) of matrix multiplication is $\underline{\mathbf{R}}(M_{n,n,n}) \geq 2n^2 - n$ in [4]. For rank, the current (as of 8/12) world record is

  Theorem 11.0.1. [2] *Let $p \leq \mathbf{n}$ be a natural number. Then*

  $$\mathbf{R}(M_{n,n,n}) \geq (3 - \frac{1}{p+1})\mathbf{n}^2 - (4p+1)\mathbf{n}.$$

  This bound is maximized when $p = \lfloor \frac{\sqrt{\mathbf{n}}}{2} - 1 \rfloor$ or $p = \lceil \frac{\sqrt{\mathbf{n}}}{2} - 1 \rceil$, so if $\frac{\sqrt{\mathbf{n}}}{2} \in \mathbb{Z}$,

  $$\mathbf{R}(M_{n,n,n}) \geq 3\mathbf{n}^2 - 4\mathbf{n}^{\frac{3}{2}} + 3\mathbf{n}.$$

- p276, Thm. 11.0.2.12, note also that $23 \geq \mathbf{R}(M_{3,3,3})$, due to Laderman.
- p277, Remark: there are three families of lines on $Seg(\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1)$, lines from distinct families intersect. This gives more insight into the algorithm.
- p277, first displayed equation is for operator with $a_1^1 = 0$, second is for operator with $a_2^2 = 0$, in third, the first c_2^1 should be c_1^2.
- p279, Remark: since $T_1 + T_2 : A^* \times B^* \to C$ is surjective, in fact one has $\underline{\mathbf{R}}(T_1 + T_2) = \mathbf{a}_1\mathbf{b}_1 + 1$.
- p280, second and third displayed equations are missing $\otimes$ before many of the γ's.

- p281, Remark: Prop. 11.3.1.1 is a special case of the algebraic Peter-Weyl Thm 13.6.3.
- p281, §11.3.2, last line of paragraph, ${S_1}^{-1}S_3$ change to ${S_3}^{-1}S_1$.
- p284, first line, reference is incorrect, it should be [3]

12. Chapter 12

13. Chapter 13

14. Chapter 14

15. Chapter 15

16. Chapter 16

- p389, second line of proof, $\subseteq$ change to $=$. First displayed eqn. of proof, add summation over i in first term,

References

1. Josephine H. Chanler, *The invariant theory of the ternary trilinear form*, Duke Math. J. **5** (1939), 552–566. MR 0000221 (1,35e)
2. J. M. Landsberg, *New lower bounds for the rank of matrix multiplication*, preprint arXiv:1206.1530.
3. ______, *Geometry and the complexity of matrix multiplication*, Bull. Amer. Math. Soc. (N.S.) **45** (2008), no. 2, 247–284. MR MR2383305 (2009b:68055)
4. J.M. Landsberg and Giorgio Ottaviani, *New lower bounds for the border rank of matrix multiplication*, preprint, arXiv:1112.6007.
5. Virginia Williams, *Breaking the coppersimith-winograd barrier*, preprint.

E-mail address: jml@math.tamu.edu