COMMUNICATION

Remembering Mark Mahowald 1931–2013

Douglas C. Ravenel

with contributions by Martin C. Tangora, Stewart B. Priddy, Donald M. Davis, and Mark J. Behrens

Introduction

Mark Mahowald was a dominant figure in algebraic topology for decades. After being trained as an analyst he became a self-trained homotopy theorist in the early 1960s. His 1967 AMS Memoir *The Metastable Homotopy of S^n*, with its dozens of charts packed with arcane information, established him as a master of computations related to the homotopy groups of spheres. It came to be known in the field simply as "the red book." It was the unanimous opinion of experts that he knew this subject far better than anyone else. His insight and intuition were legendary. Countless coworkers benefited from his ideas and advice.

Mark first attended college at Carnegie Tech in Pittsburgh and planned to major in chemical engineering. After two years, knowing that his father could not afford to support him any longer, he joined the naval ROTC so he could support himself with the four-year scholarship that they offered. At the same time he transferred to the University of Minnesota and changed his major to mathematics "because I could get a degree faster that way."

In 1963 Northwestern hired him as a full professor at the age of thirty-one. Within a decade Northwestern hired four more algebraic topologists and became one of the leading centers of homotopy theory in the world. Mark was the prime mover in the creation of the Midwest Topology Seminar, which meets three weekends a year to this day.

Photo courtesy of Martin Tangora.

Mahowald sailing on Lake Michigan with colleague Carl Verhey in 1968.

He had a lifelong passion for sailing. He built himself a small sailboat as a teenager. After moving to Chicago he became involved in racing on Lake Michigan. He named three succesive sailboats *Thetajay*, a reference to the notation θ_j for certain hypothetical elements in the stable homotopy groups of spheres.

Recollections of Students and Colleagues

Martin C. Tangora

When Mark Mahowald came to Northwestern in 1963, I was very discouraged. It was not at all clear how I could finish my PhD before the time limit ran out. Around that time Peter May came to the Chicago area to talk about his thesis, and Mahowald and Bob Williams told me that

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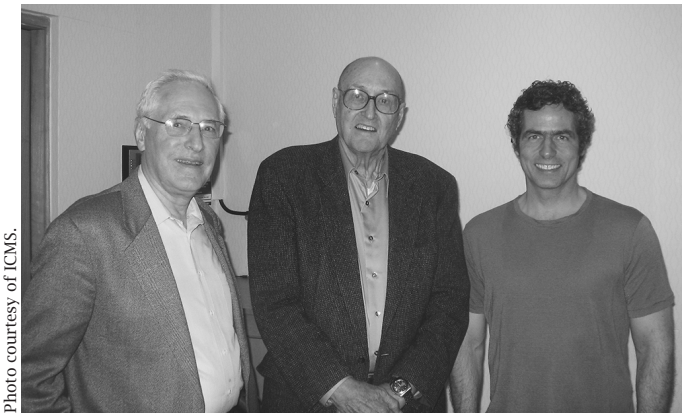

Photo courtesy of ICMS.

Mahowald (center) with his former students Marty Tangora (left) and Mike Hopkins (right) at a conference about the θ_js in Edinburgh in April 2011.

if I could find a way to use a computer to push the computations in May's thesis, I could have a thesis of my own.

The topic was perfect for me, and pretty soon I was extending the computations of Ext (by hand), and Mark was phoning me at night to ask me questions. One got the impression that Mark was always working, from before breakfast until late at night. And one quickly got the impression that he expected, or hoped for, the same diligence from everyone else. Before long, whenever we saw each other in the hallway, he would ask me, "Any new theorems?" He asked this with a smile, but it did put some pressure on. In August my diary says, "I never get any ideas—he gets so many!"

It was a very productive summer. Every day I would push Ext a little further and then drop in on Mark in his office, where he would show me stuff that I could not hope to understand. It was to be his famous Memoir. But he was also writing our first paper on differentials in the Adams spectral sequence, and in his bibliography he cited my thesis, although I had not even begun to write it, much less know that it would be accepted.

In June, I had a dream where we had gone to the University of Chicago for a seminar, and when we came out of the talk, Mark said to me, "Let me show you what I've got here." He pulled a crystal ball out of his pocket and started to pull from it long chains of brightly colored beads and flowers.

One day in November he gave me a problem at 10 a.m., and when he saw me again at 2 p.m., he asked me, "Any progress?" In the meantime I had taught two classes and gone to lunch. This kind of friendly pressure was wonderful but harrowing.

Mark knew that what he considered a proof was not always accepted as such by other mathematicians. I can still hear Frank Adams's loud protests coming from behind the closed door of Mark's office when Frank was visiting.

Once Mark said in a talk, "Is this obvious to everybody?" (pause) "It's probably true!" Often he would start to give a proof, and then founder, and look around at his puzzled audience, and then say, "Well, how about this?" and try a completely different argument. There was also a time when someone asked him about a statement that he had just made in a talk, and Mark said, "Well, that's trivial. And if you give me a minute, I'll try to think why it's trivial."

Once a colleague who tended to use lots of different alphabets on the chalkboard was giving a talk and stopped to look in the lectern desk to see if there was any colored chalk. Mark said, "If you have colored chalk, you can use the same letter for everything!"

Mark was not strong on the Greek alphabet. He sometimes would write a squiggle on the board, and if asked, would say, "That's the universal Greek letter." (He meant either lowercase zeta or xi.) Once he asked what an uppercase xi would look like, and when someone showed him, he said, "That's perfect! That's the next best thing to not giving it a name at all."

Stewart B. Priddy

As a fresh PhD contemplating a job interview at Northwestern University, I had some questions: "What sort of man is Mark Mahowald?" "What about his mathematics?" Answers from my mentors: "Well, he is a big man; a former Marine; a force from the North." "He is a deep mathematician, but his arguments are often difficult to follow." I decided to consult his 1967 AMS Memoir. From this it was clear he was adept at computing homotopy groups of spheres, a notoriously complicated, difficult subject. It appeared that a profound but elemental force was at work here. This and all I had been told turned out to be true, in spades!

At this time, 1968, Mark was building a group in homotopy theory. Almost every algebraic topologist passing through Chicago came to lecture at the weekly seminar led by Mark, who absolutely insisted on a lecture every week without fail. Hiroshi Toda of Kyoto came for a year. Watching them draw curlicue cell diagrams was awesomely mystifying.

Mark was a leading force in the Midwest Topology Seminar, a quarterly Saturday meeting in the Chicago area with a yearly meeting at other universities in the Midwest. For a couple of decades Mark was the coordinator for this series, which contributed immensely to activity in the field. It was always de rigueur to invite an outside speaker with a hot new result in topology. During those decades (1970–2000) Northwestern had a series of Emphasis Years in various subjects rotating every four years or so. There were long-term senior visitors and two junior positions reserved for the year. Because of Mark's vigorous leadership, topology got more than its share of these resources. His strategy was to keep an excellent candidate in the wings in case there was an extra slot available. He was such a dominant player in the department that he often prevailed. Associated with these years was an international conference where many of the seminal results in the field were exposed for the first time. Gunnar Carlsson described his solution to

Photo courtesy of Donald M. Davis.

From L to R: Topologists Fred Cohen, Martin Bendersky, Mahowald, and his former student Mike Hopkins at a 2008 conference in Valle de Bravo, Mexico.

the Segal Conjecture, Haynes Miller explained his proof of the Sullivan Conjecture, the always-formidable Frank Adams gave his view of Mahowald's new infinite family of elements in the homotopy groups of spheres. All this was terribly exciting and as one of my former MIT instructors said, those conferences helped put Northwestern mathematics on the map. Central to all this activity was Mark's energetic leadership.

Many of our most talented students were attracted to the depth of Mark's lectures. One was W. H. Lin, whose solution to the first case of the Segal Conjecture started an extended industry engaging many others and culminating with Carlsson's work. Another was Michael Hopkins, who won a Rhodes Scholarship, then returned to get a second PhD with Mark; it was difficult to escape Mark's spell. Young people from the University of Chicago—Paul Goerss, Mark Behrens, and others—were often found in his office. During the heyday of homotopy theory at Northwestern, Mark was one of the most important people in shaping careers. He continued to be active and influential until the last weeks of his life.

Donald M. Davis

I was very fortunate to have a forty-year collaboration with Mark Mahowald, resulting in thirty-five published papers, with publication dates extending from 1975 to 2014. He taught me more about homotopy theory than I will ever know.

My Stanford thesis advisor, Jim Milgram, knew that Mark and I would be a good pair, and they arranged for my postdoctoral position at Northwestern in 1972–74. My thesis was related to the problem of finding the smallest Euclidean space in which real projective n-space could be immersed, and Mark was a top expert on this question.

In my first meeting with Mark he asked me what I was working on. I explained a certain approach to the immersion problem (that never amounted to much), and he suggested a totally different approach, which I immediately began to study. It involved an inductive argument, largely numerical but with a lot of underlying topology to justify the induction. We were never able to prove the part that he really wanted, one with which he was quite obsessed over the years. In the end, we needed a compatibility condition that he felt should be satisfied, but he could never convince me that it was provable.

This was very typical of our work. He had enormous insight but needed people like me to pin it down.

One time I didn't do a good enough job. In a 1982 paper [DM82], we proved that there could not be a spectrum with certain cohomology groups. In the late 1990s Haynes Miller and Mike Hopkins introduced some new spectra and, together with Mark, discovered that one of them had precisely the cohomology that we had shown to be not realizable. With great effort, Mark eventually realized that the mistake in our work had come from an error in the published information about the homotopy groups of spheres. Ironically, Mark and I were then able to use this new spectrum, *tmf*, to obtain some new nonimmersion results for real projective spaces [BDM02].

Working with Mark was great fun. We shared a common interest in sports, which we enjoyed discussing. Unfortunately, the first time he took me sailing I got seasick and never went again. We spent the fall of 1982 at the University of Warwick with our wives. I bought a car, and we frequently took the Mahowalds on weekend road trips. This and a 1990 stay together in Oxford were great experiences for me, thanks to Mark. In each case he was invited and then managed to extend the invitation to include me.

I visited Northwestern many times to work with Mark, usually staying with him and his wife, Zoe. I very much appreciated their hospitality. I also spent 1978–79 as visiting associate professor at Northwestern, with my family. That was possibly the coldest, snowiest winter ever in Chicago. But a new 20-inch snowfall on top of

Photo courtesy of Donald M. Davis.

Mahowald with Donald M. Davis in Mexico.

twenty that had already fallen did not deter Mark from driving me and the rest of the NU topology group to the Midwest Topology Seminar at the University of Chicago. We were about the only car on the road. This was Mark; nothing could stop him.

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Mark J. Behrens

I first met Mark Mahowald as a prospective graduate student visiting Northwestern University. As an undergraduate I had been studying homotopy theory with Norihiko Minami, a graduate from the Northwestern school of topology, and Mahowald met with me for a full hour to talk about his current research. I was immediately bewildered with some weird mixture of the Steenrod Algebra, certain polynomial equations over $\mathbb{F}_4$, and the random appearance of the group $SL_2(\mathbb{F}_3)$. Frankly, I had *no idea* what was going on but courteously pretended that I did.

Only later did I discover that he was trying to tell me about the supersingular elliptic curve at the prime 2 and his latest gadget, Topological Modular Forms (*tmf* for short, but back then it was called eo_2). I remember as a first-year graduate student at Peter May's sixtieth birthday conference joking with my fellow graduate students about the prospects of "eo_2 resolutions" after hearing a similarly incomprehensible talk Mahowald gave at the conference. Little did I know that in a few short years I would be hanging onto every word that this man said and shaping my career around the stuff he was talking about.

Indeed, I was bitten by the computation bug the following year, and my advisor, Peter May, suggested I go up to Northwestern to talk with Mahowald about possible thesis projects. It was not long until I was commuting up to Northwestern every week to talk to Mark to learn about his unique perspectives on the stable homotopy groups of spheres. Amazingly, he would devote his entire afternoons to talking with me. Talking with Mark was like learning a language by the immersion technique. At first, I had no idea what he was trying to tell me. Each week, I would ask him the same questions, and he would politely pretend I hadn't asked him the same question the week before and give a different explanation, until one took. His vision, passion, and mathematics will dominate my own work for the rest of my life.

I would later learn that he regularly took graduate students and postdocs under his gentle wing in this manner, initiating them in an oral tradition of homotopy theory that cannot be learned from any paper or book. Mark was generous with his ideas and would freely hand off amazing theorems to young mathematicians whose task was to provide a proper proof.

Mark's mathematics is best described as nontraditional. Arrows in his diagrams often lacked heads, for he was inevitably going to get the direction wrong. Once when he was talking about duality, I noticed he subconsciously started to write "*od*" instead of "*bo*." Rob Thompson recently told me that Mark said he wasn't concerned that the *bo*-based Adams spectral sequence didn't have an E_2-term expressible as an Ext group, because he "never understood homological algebra anyways." Indeed, for Mark everything could be thought of as a CW complex and drawn with his idiosyncratic "cell diagrams."

Mark's talks were always difficult for folks to understand. A geometer once told me about a talk Mark gave which began "In this talk, all spaces are mod 2…." He told me he didn't understand this or anything that followed. His writing was equally difficult to grasp at times. Perhaps he was gifted with an extraordinary vision and cursed with an inability to put it into words. I, however, believe that there are no words adequate to describe his incredible world-view of computational homotopy theory.

For Mark, mathematics was not so much about the big theorem as understanding the bigger picture. The homotopy groups of spheres were full of *mysteries*, and he was the oracle. His standards of proof were different than is typical in our field—at times it felt he was more like a physicist with brilliant intuition than an uptight mathematician bound by the restraints of rigor. Mark told me that the Kervaire invariant elements were "simultaneously v_2-periodic and v_n-periodic for all n"—a paradoxical situation that somehow strengthened Mark's hope to prove they all existed, but as Hill-Hopkins-Ravenel later discovered, forced their eventual nonexistence. For Mark, a theorem was proven by understanding *why* it was true, by placing it in a philosophical setting where its truth was so self-evident he could come up with a half dozen different arguments for its validity. He always told me that when doing a computation, arrive at the answer through two independent means so as to make sure you don't make a mistake. I think Adams summed up Mahowald's enigmatic brilliance the best in his review of his paper "The primary v_2-family":

> This is mathematics as it is lived; some of it seems like work in progress. May it continue to progress.

Douglas C. Ravenel

Mark Mahowald was an inspirational mathematician. I met him in the mid-1970s, early in my career. He was hugely encouraging and made me feel like I had just met a rich uncle I never knew I had. Like everyone else, I was impressed by the depth of his intuition and insight. I got the impression he could do the Adams spectral sequence in his sleep.

If you do not know what the Adams spectral sequence is, you will not learn it here. Suffice it to say it is an exquisite mystery that hard-core homotopy theorists like Mahowald have a long-term love-hate relationship with. He could draw charts of it with abandon, explaining

Photo courtesy of Mark Mahowald's estate.

Mahowald on the occasion of being named Henry S. Noyes Professor in Mathematics at Northwestern University, 1988.

patterns known only to him like they were mechanical devices. For him the homotopy groups of spheres were familiar tangible objects, like the furniture in his house.

In 1999 there was a homotopy theory meeting at Oberwolfach which, to my lasting regret, I missed. There was an evening lecture by a young German named Christian Nassau. He had a new computer algorithm for producing charts of the Adams spectral sequence. The talk was technical and possibly boring until the very end.

Then he produced a spectacular slide showing much more of the spectral sequence than anyone had ever seen before. Mahowald jumped out of his chair and ran to the front of the room, talking about newly revealed patterns in the picture and what they meant. It was as if he had been waiting his whole life to see that picture.

Talking to Mark was like riding a comet, a transporting but often bewildering experience. I heard many people say they had a five-minute conversation with him that was very profound and spent the rest of the day trying to figure out what he had said.

One of my favorite recollections is of a discussion we had at a conference at Northwestern in 1977. He was very excited about some things he had recently discovered. I could almost follow what he was saying, but it was not easy. Confusingly, he used the term "Hopf invariant" twice in the same sentence *with two different meanings!* When the conference ended, it was Haynes Miller's job to drive me to the airport. Mark gave him directions that involved virtual streets, but I did make my plane.

Listening to him lecture was even more puzzling. He had wonderful things to say, but nearly every line of it was garbled in some way, the terminology not quite right or the arrows going in the wrong direction. Polished elegance was not his style.

Some Mahowaldisms:

- Advice on traveling to Europe: "Pack two suitcases very carefully and leave one of them at home."
- When he was department chair, he was asked by the staff if they could throw out some old documents. His answer: "You can throw out anything you want, provided you make a copy of it first."
- When discussing a certain finite abelian 2-group in algebraic topology, he said it was a vector space. When told that it had elements of order 4, he said, "That's what I meant, a $\mathbf{Z}/4$-vector space."
- Referring to the notation in Toda's book on the homotopy groups of spheres, written just before Mark got into the game: "One should call these elements by their first names."
- "The element η is not in the Image of J for personal reasons."

His Relationship with Adams

Mark's contemporary Frank Adams was, up until his death in 1989, the preeminent algebraic topologist in the world. His style was quite different from Mahowald's. Unlike Mark, he would never discuss his intuition, only finished mathematics. They knew and respected each other and had a cordial relationship. However, there was a certain wariness on both sides that was fun to watch. Adams, who had his own unique sense of humor, wrote a poem about Mahowald.

The school at Northwestern
    is as fertile as manure,
full of deep insights,
    some rather obscure.

Mark loves those damn thetas
    like a sister or brother,
and if you don't like one proof
    he'll give you another.

While his mathematics was overwhelming, Mahowald's personality was quite gentle. There was not a trace of arrogance or intimidation. He was easy-going, unassuming, and very friendly. He never held a grudge. It was impossible not to like him. His sixtieth birthday celebration in 1991 had the best mathematical roast I have ever been to. There were dozens of stories about his quirky behavior, each told with great affection.

He was a major reason why it is fun to be an algebraic topologist. The subject is not the same without him.