New and Noteworthy Titles on Our BookShelf
September 2016

This month on the BookShelf, three books that aim to make the fascination of number theory accessible to a wide audience.

*Prime Numbers and the Riemann Hypothesis*, by Barry Mazur and William Stein (Cambridge University Press, April 2016). Several popular books about the Riemann Hypothesis have appeared in recent years, such as *The Riemann Hypothesis* by Karl Sabbagh (2002), *Prime Obsession* by John Derbyshire (2003), *The Music of Primes* by Marcus du Sautoy (2003), and *Stalking the Riemann Hypothesis*, by Dan Rockmore (2005; reviewed in the *Notices*, September 2006). These books told the story of the Riemann Hypothesis mainly through its history and through portraits of the mathematicians who worked on it. As William Stein explains on his web site, the book he has written with Barry Mazur is different in that it sticks more closely to the mathematics: “We aim...to explain, in as direct a manner as possible and with the least mathematical background required, what this problem is all about and why it is so important.” To really understand the entire book, considerable mathematical background is needed, more or less at the level of a senior undergraduate mathematics major. As Mark Hunacek commented in a review on the web site of the Mathematical Association of America: “even a student who can't follow every detail will likely emerge [from reading the book] with a good intuitive sense of what makes RH so important.” Written by two eminent number theorists, the book possesses a distinctive flair born of their deep insights and unending enthusiasm.

*Summing It Up: From One Plus One to Modern Number Theory*, by Avner Ash and Robert Gross (Princeton University Press, 2016). In addition to this book, Avner Ash and Robert Gross have written two other semi-popular books on number theory. In *Fearless Symmetry* (2006), they took readers with a background in calculus through much of the number theory needed to understand Wiles’s proof of Fermat’s Last Theorem, doing so “at a leisurely pace, with motivating examples, and with digressions on how mathematicians really think, and how mathematics is ‘made,’” wrote Dino Lorenzini in a review in the *Notices* (January 2007). Ash and Gross took that same precise yet unfussy approach in *Elliptic Tales* (2012), which treated elliptic curves, and have retained it in their latest book *Summing It Up*. While the three books can be read independently, they are linked: The first two alluded to but do not explain modular forms, while the ultimate goal of *Summing It Up* is to explain what modular forms are and how they are used in number theory. The first part of *Summing It Up* explores that basic bedrock of mathematics, addition, assuming no more than high school algebra and geometry. In the second part, which leads up to modular forms, familiarity with calculus is assumed. Ash and Gross put the emphasis on clear and intuitive development of the ideas, often omitting formal proofs in favor of keeping the exposition uncomplicated and informal. The basic spirit and structure of the book is captured in this sentence from the introduction: “It continues to amaze us what human beings have accomplished, starting with one plus one equals two, getting to two plus two equals four...and going far beyond into the realms of number theory that even now are active areas of research.”

*Prime Numbers, Friends Who Give Problems: A Trialogue with Papa Paulo*, by Paulo Ribenboim (World Scientific, 2016). This unusual and whimsical book presents some of the most beautiful classic results of number theory as a “trialogue”: a conversation among 3 characters, written in playscript style. The main character is Papa Paulo, who responds to a question from Eric, asking what prime numbers are. In 29 chapters covering more than 300 pages, Papa Paulo happily agrees to bring his wide and deep knowledge of number theory to bear in answering Eric’s question. In chapter 3, Eric brings his friend Paulo into the conversation, and the three proceed on a leisurely journey, starting with the definition of a prime number and proceeding to such topics as the sieve of Eratosthenes, primality testing, perfect numbers, Fermat’s Last Theorem, the Prime Number Theorem, and the Riemann Hypothesis. Along the way, Papa Paulo leavens his disquisitions with charming tales, some of them imaginary, such as the one in which Saint Peter condemns Fermat to Purgatory, contending...
that Fermat must have lied when asserting existence of a certain proof that did not fit into the margin of a book. Ribenboim’s book is sprinkled with inside jokes, such as the reference to a person who has a vanity license plate emblazoned with “FERMAT.” Papa Paulo’s comment: “Some people want to be famous by any means.” (Donald Knuth’s article “Mathematical Vanity Plates,” which appeared in 2011 in the Mathematical Intelligencer, mentions FERMAT license plates but had no photo. A reader having such a photo is invited to post it in the comments section of the Notices web site, www.ams.org/notices.)

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