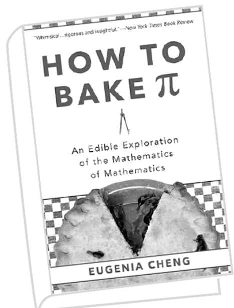

How to Bake π

A Review by Jeremy L. Martin

How to Bake π: An Edible Exploration of the Mathematics of Mathematics
Eugenia Cheng
Basic Books, 2016 (paperback edition), 304 pages
US$18.33
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Eugenia Cheng's book *How to Bake π* has an unusual form. Each chapter begins with a short recipe, which the author uses as a springboard to explain something about mathematics. The first part of the book is devoted to broad principles: chapters entitled "Abstraction" and "Generalization" begin with recipes for mayonnaise or hollandaise sauce and olive oil plum cake. In the second part, the author, a working category theorist, sets out to explain her field of research (regarded even by many mathematicians as "abstract nonsense") to a broad audience using analogies to custard and frozen eggs. The approach may sound whimsical, but in fact *How to Bake π* is a success at explaining what mathematics is and how it is done, using simple, appealing language. It should be a rewarding read for mathematicians and nonmathematicians alike.

> *"If you imagine drawing a circle in the air with a lightsaber, the surface you make is a vector bundle over a circle"*

Cheng's language is simple and chatty without being condescending. The book is packed with analogies: in addition to food, we see Lego, train tickets, road signs, the New York Marathon, and welding, among others. Occasionally the chattiness feels slightly forced, but most of the analogies are good and some of them are great, and teachers will find plenty to borrow for their own classrooms (indeed, the book could even serve as a side resource for an undergraduate proof-techniques course). Cheng frequently strips away technical details in order to show the big picture, which is particularly appealing in the book's frequent digressions into specific mathematical content (sometimes marked by gray boxes in the text): topology, Arrow's theorem, fair-division problems, Erdös numbers, the Poincaré Conjecture, the Riemann Hypothesis, and more. These analogies are not just toys: professionals can use the help too. I remember struggling with the formal definition of a vector bundle as a graduate student, and I wish someone had told me that "if you imagine drawing a circle in the air with a lightsaber, the surface you make is a vector bundle over a circle" (p. 175). That makes sense!

The first part of the book is about procedures that we mathematicians have internalized: abstraction, generalization, axiomatization. These are such habitual modes of thought for mathematicians that we take them for granted, yet unfamiliarity with these processes is precisely what makes mathematics look so difficult and inaccessible to many nonmathematicians. So Cheng finds simple, compelling ways to describe abstraction: "blueprints," "tidying away the things you don't need," "simplification," and this great little vignette (p. 20):

> I remember a wonderfully feisty mother at an elementary school I was helping at. She remarked on how frustrating it was when other mothers competitively declared that *their* child could count up to 20 or 30. "My son can count up to three," she said defiantly. "But he knows what three *is*."

Another explanation I particularly like comes from the second part of the book, in the chapter "Structure"—a word that is ubiquitous in mathematical thought and writing but hard to define. Cheng begins with a recipe for baked Alaska (solidly frozen ice cream baked in a meringue shell) and writes: "[Baked Alaska]…is food where the structure is integral to the food…different from a cake in the shape of a dinosaur, where the shape is more or less independent of the taste."

In the last chapter of the first part of the book, "What Mathematics Is," Cheng steps back from mathematics per se and raises the question of how accurately math can model the real world: is life difficult "because we haven't yet made ourselves logically powerful enough to understand it all," or is it the case that "we will never be able to encompass everything by rationality alone, and this is a necessary and beautiful aspect of human existence"? I think this passage is aimed at the lay part of Cheng's audience, but mathematicians will recognize the references to the programs of Hilbert and Russell

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Cheng's book uses a 4-by-4 Battenberg cake as a fanciful way to explain Latin squares, which are arrays of objects in which each object appears exactly once in each row and exactly once in each column.

to formalize and systematize mathematics and then to Gödel's incompleteness theorems.

Cheng's exposition of category theory in the second part of the book is generally excellent. However, I think she frequently overstates its foundational role within mathematics. A lay reader may receive the impression that category theory is responsible for such concepts as equivalence relations (p. 192) and representing a partially ordered set by drawing its covering relations (pp. 202–3). Equivalence relations certainly predate category theory and can be understood without it, and Hasse diagrams are Combinatorics 101. Page 196 features a triangle with vertices labeled "algebra," "geometry," and "logic," and these provocative words: "There's a theory…that all mathematicians are located on some edge of this triangle, [b]ut category theory seems to combine all three of those things." As a geometric combinatorialist, I am pretty sure I am in the interior of that triangle, and experts in algebraic topology or differential equations, among other fields, would surely say the same. Likewise, the excellent chapter "Sameness" explains how the word "same" can have different meanings (characteristically, using vivid analogies: is a toilet brush the same thing as a toothbrush?), but the writing gives the impression that category theory is *necessary* to distinguish between different kinds of sameness. I disagree. Working mathematicians can distinguish between homeomorphism and homotopy equivalence without explicit knowledge of category theory. I don't dispute Cheng's characterization of category theory as "the mathematics of mathematics"; it is just not the only tool that mathematicians use to do what we do.

In the same vein, an early chapter quotes category theory expert John Baez as saying, "But if you don't like abstraction, why in the world are you doing mathematics? Maybe you should be in finance, where the numbers all have dollar signs in front of them." Stripped of context, these lines are combative, if not offensive — how much abstraction must one like in order to qualify as a "real" mathematician, and who gets to decide? I think the author should have included the rest of what Baez wrote: "Indeed, what we often take as 'greater abstraction' is really unfamiliarity. The real answer to the problem of people thinking categories are 'too abstract' is to keep explaining category theory and how it's useful in a wide variety of problems."

These criticisms aside, the book succeeds broadly in its aim of explaining a difficult subject to a wide audience. After all, "mathematics is there to make difficult things easier," writes Cheng (p. 144), and she argues convincingly that category theory can make mathematics easier. I am reminded of a line from Eisenbud and Harris's *The Geometry of Schemes*, another successful exposition of a notoriously difficult and technical subject: "The theory of schemes is widely regarded as a horribly abstract algebraic tool that hides the appeal of geometry to promote an overwhelming and often unnecessary generality. By contrast, experts know that schemes make things simpler."

What about the recipes? I did not try the suggestion (p. 223) of dipping Brussels sprouts in dark chocolate, but the chocolate prune pudding was not bad. Cheng's fruit crisp is very simple: flour, sugar, butter, and fruit. My fruit crisp topping includes pecans and a pinch of salt, and I toss the fruit (peaches or pears) with some cornstarch to add body, and I like to add chopped crystallized ginger. Perhaps it is no surprise that the category theorist's recipe is the more elemental of the two.

Credit

Photo of cake, courtesy of Jeff Cottenden.

ABOUT THE REVIEWER

Jeremy Martin is professor at the University of Kansas working in algebraic and geometric combinatorics. He likes both Brussels sprouts and dark chocolate, just not in the same mouthful. And as the photo shows, he bakes a pretty good pi himself.

Jeremy Martin