2017 Leroy P. Steele Prizes

Dusa McDuff

Dietmar Salamon

Leon Simon

James Arthur

The 2017 Leroy P. Steele Prizes were presented at the 123rd Annual Meeting of the AMS in Atlanta, Georgia, in January 2017. The Steele Prizes were awarded to Dusa McDuff and Dietmar Salamon for Mathematical Exposition, to Leon Simon for Seminal Contribution to Research, and to James Arthur for Lifetime Achievement.

Citation for Mathematical Exposition: Dusa McDuff and Dietmar Salamon

Dusa McDuff and Dietmar Salamon are awarded the Steele Prize for Mathematical Exposition for their book *J-Holomorphic Curves and Symplectic Topology*, American Mathematical Society Colloquium Publications, 52, 2004.

The field of symplectic topology went through a rapid phase of development following Gromov's 1985 paper that introduced *J*-holomorphic curves into symplectic topology and intertwined this field with algebraic geometry and string theory. Techniques revolving around *J*-holomorphic curves have been a basic ingredient in the solution of many classical and crucial questions in symplectic topology, as well as in the discovery of new structures. More than thirty years after its publication, the influence of Gromov's paper in the rapidly developing field of symplectic topology is as strong as in the beginning, and many of the most exciting research topics in the field (like, for example, mirror symmetry) involve in one way or another the notion of a *J*-holomorphic map.

The book *J -Holomorphic Curves and Symplectic Topology* is a comprehensive introduction to Gromov's theory of *J*-holomorphic curves, explaining from the beginning and in detail the essential notions and results, as well as many of its spectacular applications in symplectic topology. While being among the main contributors to this development, McDuff and Salamon spent nearly a decade assembling the foundations of this subject into a mammoth 700-page book. It has since served as the most standard and undisputed reference in the field and as the main textbook for graduate students and others entering the field. The use of the abbreviation M-S in the context of *J*-holomorphic curves and symplectic topology has now become routine and causes no confusion.

This book begins with a sixteen-page overview of the subject of symplectic topology, the theory of *J*-holomorphic curves, and its applications to symplectic topology, algebraic geometry, and mirror symmetry. This overview is informative to those outside the field who are just curious and serves as a guide to the book. Each chapter begins with its own very informative introduction. The chapters and sections are structured so that the main statements are formulated as early as possible while their proofs are delayed.

In some ways, the McDuff and Salamon book on *J*-holomorphic curves is the symplectic analogue of Lazarsfeld's *Positivity in Algebraic Geometry* and Griffiths and Harris's *Principles of Algebraic Geometry*. This book, together with McDuff and Salamon's *Introduction to Symplectic Topology* and their many other contributions, has been a great help to both junior and senior symplectic geometers.

Biographical Sketch: Dusa McDuff

Dusa McDuff was born in London in 1945, grew up in Edinburgh, and in 1971 received her PhD from Cambridge University under the direction of George Reid. She spent six months in Moscow in 1969–1970 as a student of I. M. Gelfand, who had a profound influence on her mathematics. After working on topics in topology and foliation theory (often in collaboration with Graeme Segal), she was moving into the area of symplectic geometry just as Gromov published his pioneering paper and has remained there ever since. After holding positions at York, Warwick, and Stony Brook Universities, she is currently Helen Lyttle Kimmel '42 Professor of Mathematics at Barnard College, Columbia University. She received the Ruth Lyttle Satter Prize (1991), gave a plenary address to the ICM (1998), and was AWM Noether Lecturer (1998) and AMS Colloquium Lecturer (2014). She is a fellow of the Royal Society of London (1994), a member of the US Academy of Sciences (1999) and of the American Philosophical Society (2013), and has honorary degrees from the universities of York, Edinburgh, St. Andrews, Strasbourg, and Pierre and Marie Curie, Paris.

Biographical Sketch: Dietmar Salamon

Dietmar Salamon was born in Bremen in 1953 and completed his PhD at the University of Bremen in 1982 under the direction of Diederich Hinrichsen. After postdoctoral positions at the University of Madison–Wisconsin and at ETH Zurich, he took up a lectureship at the University of Warwick in 1986, where he became professor in 1994. In 1998 he moved to ETH Zurich to take up a professorship. His field of research is symplectic topology and related subjects.

He was an invited speaker at the ECM 1992 in Paris, at the ICM 1994 in Zurich, and at the ECM 2000 in Barcelona. He delivered the Andrejewski Lectures in Goettingen (1998) and at the Humboldt University Berlin (2005) and the Xth Lisbon Summer Lectures in Geometry (2009). He is a member of the Academia Europaea and a fellow of the AMS.

He is the author of several books, including the joint monographs *Introduction to Symplectic Topology* and *J-Holomorphic Curves and Symplectic Topology* with Dusa McDuff, and of over seventy research papers. He has supervised twenty PhD students.

Response from Dusa McDuff and Dietmar Salamon

It is a great honor to receive the Leroy P. Steele Prize for Mathematical Exposition for our book *J-Holomorphic Curves and Symplectic Topology*.

Our collaboration started in 1990 at a conference at the University of Warwick. During the preceding year we had both given lecture courses on symplectic topology and decided to put our acts together to write a monograph about that newly emerging subject, not imagining how much effort would go into this in the course of the following quarter of a century. In December 1993 at a conference in Tel Aviv—coincidentally in honor of Misha Gromov's fiftieth birthday—we decided that the theory of *J*-holomorphic curves, together with the vast amount of introductory material, would be too much for a single volume.

So one book turned into two, and the one on *J*-holomorphic curves "bubbled off." It was initially conceived as a fairly brief introduction, together with a proof of the gluing theorem, and appeared less than a year later, even before our *Introduction to Symplectic Topology*, in the AMS University Lecture Series under the title *J-Holomorphic Curves and Quantum Cohomology*. About a decade later we decided to correct errors and include more details and applications, after which the manuscript tripled in size to almost 700 pages and was published in 2004 under the title *J-Holomorphic Curves and Symplectic Topology* in the AMS Colloquium Publications series. An updated and corrected second edition appeared in 2012.

Our work on both manuscripts required a certain amount of compromise, as well as extensive arguing, as often we were approaching the subject from rather different points of view, which one might characterize as more geometric (Dusa) versus more analytic (Dietmar). However, we both found this process stimulating, and in the end it led to a much better result than either of us could have achieved on our own.

We learnt a great deal from each other, as well as from many other researchers in the field, to whom we wish to express our deep gratitude. We are very happy that our books helped many others to study this beautiful subject and also are deeply honored that our efforts have been recognized with the award of this prize.

Citation for Seminal Contribution to Research: Leon Simon

The Steele Prize for Seminal Contribution to Research is awarded to Leon Simon for his fundamental contributions to geometric analysis and in particular for his 1983 paper "Asymptotics for a class of nonlinear evolution equations, with applications to geometric problems," published in the *Annals of Mathematics*.

In this groundbreaking paper, Simon addressed the basic question of what a minimal variety must look like near a singularity. This is a question of fundamental importance, since singularities in minimal varieties (as well as solutions to many other nonlinear problems) are generally unavoidable. Once one knows that singularities occur, one naturally wonders what they are like. The first answer, already known to Federer and Fleming in 1959, is that they weakly resemble cones. Unfortunately, the simple proof leaves open the possibility that a minimal variety looked at under a microscope will resemble one blowup, but under higher magnification, it might (as far as anyone knows) resemble a completely different blowup. Whether this ever happens is one of the most fundamental questions about singularities.

Simon recasts this as a question of long-time behavior of solutions to gradient flows. He then uses this formulation to prove the uniqueness of the tangent cone at a singularity where there is a tangent cone with an isolated singularity at the vertex. Simon also obtains, in the same paper, a similar uniqueness result for other variational problems. A couple of years earlier, in a landmark paper, Allard and Almgren had proven uniqueness under an additional integrability assumption on the cross-section by entirely different methods.

In the 1960s Łojasiewicz showed that, for an analytic function on Euclidean space, any limit point of its gradient flow is in fact a limit. Simon realized in a brilliant way that many fundamental problems in analysis and geometry can be recast as infinite-dimensional Łojasiewicz theorems and, by ingenious analytic arguments, deduced from the finite-dimensional Łojasiewicz theorem.

The significance of Simon's pioneering paper extends well beyond these results. In fact, Simon obtained these results as an application of a strikingly original and general method that he developed in the paper, based on the Łojasiewicz inequality in real algebraic geometry, known now as the Łojasiewicz–Simon inequality. The basic analytic ingredient that Simon developed to carry out this method has proven to be an extraordinarily powerful tool of far-reaching impact. It has since been applied or adapted in uniqueness and related questions in a very large number of contexts, ranging from differential geometry to fluid dynamics and superconductivity.

Simon himself subsequently used this inequality, together with a host of further new ideas, to show regularity of the singular set of a minimal variety.

Leon Simon's paper has had extraordinary impact on analysis, geometry, and applied mathematics. Hundreds of papers have been written either directly applying the Łojasiewicz–Simon inequality or based upon the insights contained in this paper. Without a doubt Simon's ideas will continue to be applied and further developed in future work.

Biographical Sketch: Leon Simon

Leon Simon is emeritus professor of mathematics at Stanford University. Born July 6, 1945, in Adelaide, South Australia, he received his bachelor's degree at the University of Adelaide in 1967 and his PhD, written under the direction of James Michael, from the same institution in 1971. After briefly holding a lectureship at Flinders University in Adelaide, he took a postdoctoral assistant professorship at Stanford University from 1973 to 1976. After holding professorships at Minneapolis, Melbourne University, and the Australian National University in Canberra, he returned to Stanford as professor of mathematics in 1986. He was chair of mathematics at Stanford for the period 1998–2001.

Simon's main research interests are in geometric measure theory and partial differential equations, in particular the theory of minimal surfaces and related problems in the geometric calculus of variations.

He was elected Fellow of the Australian Academy of Sciences in 1983, the American Academy of Arts and Sciences in 1994, and the Royal Society in 2003. He was awarded a Sloan Fellowship in 1975, an Australian Mathematical Society Medal in 1983, the Bôcher Prize of the American Mathematical Society in 1994, and a Humboldt Award in 2005. He gave an invited talk at the ICM in 1983 and is an AMS Fellow. In the course of his career, he has supervised the thesis work of eighteen graduate students.

Response from Leon Simon

I am very honored to be chosen for this award. The cited work was carried out during my time at the Australian National University in Canberra, and I owe a great debt to a number of people, including Robert Bartnik, John Hutchinson, Peter Price, and Neil Trudinger, who were responsible for the congenial and very active research environment during that time. I am of course also indebted to those who provided me with inspiration and support in the period prior to that, including James Michael (1920–2001), who was an inspiring undergraduate teacher and who supervised my PhD work, and David Gilbarg (1918–2001), Rick Schoen, and S.-T. Yau during my postdoctoral period at Stanford. I'm also greatly indebted to Robert Hardt, who acquainted me with many of the finer points of geometric measure theory during our collaborations at the University of Minnesota and the University of Melbourne in 1977–1979.

Citation for Lifetime Achievement: James Arthur

The 2017 Steele Prize for Lifetime Achievement is awarded to James Arthur for his fundamental contributions to number theory and harmonic analysis, and in particular for his proof of the Arthur–Selberg trace formula.

Introduction of *L*-functions into the theory of automorphic forms began with a conjecture of Ramanujan, its proof by Mordell, and the exploitation of Mordell's ideas by Hecke, who had already had experience with Euler products in the context of Dedekind ζ-functions and related *L*-functions. Later Selberg introduced methods from the spectral theory of second-order differential equations on a half-line, as well as a form of the Frobenius reciprocity theorem, familiar from the representation theory of finite groups. In the context of discrete subgroups of Lie groups it became known as the Selberg trace formula. For groups with compact quotient, it is hardly more difficult than the Frobenius theorem itself. For groups with quotients of finite volume but not compact, not only its formulation but also its proof required ingenuity and a good deal of skill in the use of the spectral theory.

The first trace formula for general groups was established by Arthur in the 1970s in a series of three papers. Starting from the particular case of SL(2) which had been established by Selberg in 1956, Arthur has built a whole mathematical framework and introduced many major tools in noncommutative harmonic analysis in order to prove the trace formula for a general reductive group. The final result is now called the Arthur–Selberg trace formula. The proof in itself takes sixteen long and difficult papers that Arthur published between 1974 and 1988. This is considered to be a major achievement in mathematics.

As Langlands suggested at the end of the 1960s, the trace formula is a powerful tool for proving the Langlands principle of functoriality, especially in the so-called endoscopic case. For this purpose, one first needs to stabilize the Arthur–Selberg trace formula. Arthur published eight papers between 1997 and 2003 on the stabilization process. Using the stable trace formula and the Fundamental Lemma proved in 2008 by Ngô Bảo Châu, Arthur has recently been able to establish the Langlands functoriality for the standard representations of the classical groups (symplectic, orthogonal, and unitary).

As a consequence, he has obtained explicit formulas for the multiplicities in the automorphic discrete spectrum for those classical groups. The Arthur–Selberg trace formula is a central tool in Lafforgue's proof of the Langlands correspondence for function fields.

Arthur's contribution to mathematics is fundamental. His work already has had, is having, and will have an enormous impact on several branches of mathematics. But his service to the mathematical community is also very impressive. Arthur played an important role in shaping the work of several important national and international committees and organizations. All this culminated when he served as President of the American Mathematical Society.

In 1992 Arthur was elected a Fellow of the Royal Society. He was elected a Foreign Honorary Member of the American Academy of Arts and Sciences in 2003 and a foreign member of the National Academy of Sciences in 2014. In 2015 he was awarded the Wolf Prize in Mathematics.

Biographical Sketch: James Arthur

James Arthur is a university professor and holds the Ted Mossman Chair in Mathematics at the University of Toronto. He was born in Hamilton, Ontario, in 1944 and received a BSc from the University of Toronto in 1966, an MSc from the University of Toronto in 1967, and a PhD from Yale University in 1970. He then held positions in mathematics at Princeton University, Yale University, and Duke University before returning to the University of Toronto in 1979.

Arthur is a fellow of the Royal Society of Canada, a fellow of the Royal Society of London, a Foreign Honorary Member of the American Academy of Arts and Sciences and a Foreign Associate of the National Academy of Sciences. His various honors and awards include an honorary doctorate at the University of Ottawa in 2002, the Canada Gold Medal in Science and Engineering in 1999, and the Wolf Prize in Mathematics in 2015. He has given several addresses at International Congresses of Mathematicians, including a Plenary Lecture at the congress in Seoul, Korea, in 2014, and he gave a Plenary Lecture at the first Mathematical Congress of the Americas in Guanajuato, Mexico, in 2013. He is presently working on Beyond Endoscopy, a proposal by Robert Langlands for using the trace formula to study the general principle of functoriality.

Arthur has served mathematics in several senior administrative roles. He was a member of the Executive Committee of the International Mathematics Union from 1991 to 1998 and the Academic Trustee for Mathematics on the Board of Trustees of the Institute for Advanced Study from 1997 to 2007. He also served as president of the AMS from 2005 to 2007. He lives in Toronto with his wife, Penny. They have two sons: James, a poet in the creative writing program at Johns Hopkins University; and David, a computer engineer at Google, in Mountain View, California.

Response from James Arthur

I am thrilled and honored to receive the Steele Prize for Lifetime Achievement. It is a cliché, but true nonetheless, for me to say that I feel humbled to look down the list of past winners. I would like to thank the Steele Prize Committee for selecting me. I would also like to thank the AMS and the many mathematical colleagues in particular who donate their time to serve on prize committees and to participate in the many other activities that do so much to help our subject thrive.

I was not a prodigy in mathematics as a child. As a matter of fact, I am quite happy that my record for the Putnam exams was not available to the Prize Committee. But I do remember being fascinated even as a child by what was said to be the magic and power of mathematics. These feelings have remained with me throughout my professional life, and they have motivated me more than any specific theorem or result.

I am very grateful to Robert Langlands for his encouragement, both during my time as a graduate student and since then. I am also grateful to him personally and as a member of the larger community for what he has given to mathematics. His mathematical discoveries truly are magical and powerful. They are becoming more widely known among mathematicians today, and I have no doubt that they will bring pleasure and inspiration to many generations of mathematicians to come.

Much of my mathematical life has been connected in one way or another with what has become known as the Arthur–Selberg trace formula. It is now a very general identity that, like other things in mathematics, links geometric objects (such as closed geodesics) with spectral objects (such as eigenvalues of a Laplacian). The trace formula has many different terms, but as we are beginning to understand them now, each of these sometimes arcane quantities (either geometric or spectral) seems to have its own particular role in the larger scheme of things. I have been fortunate that the trace formula has assumed a more central role than might have been imagined earlier. I am excited to think that there is now a well-defined (if also rather imposing) strategy for using the trace formula to attack what is known as the principle of functoriality, the central tenet of the Langlands program.

About the Prizes

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–1906, and Birkhoff served in that capacity during 1925–1926. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through PhD students; (2) Mathematical Exposition: for a book or substantial survey or expository research paper; (3) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field or a model of important research. The Steele Prizes for Mathematical Exposition and Seminal Contribution to Research each carry a cash award of US$5,000; the Prize for Lifetime Achievement, a cash award of US$10,000.

The list of previous recipients of the Steele Prizes may be found on the AMS website at `www.ams.org/profession/prizes-awards/prizes`.

The members of the Committee to Select the Winner of the Steele Prize for 2017 were:

- Paul F. Baum
- Tobias H. Colding
- Simon Donaldson
- Phillip Griffiths
- Carlos E. Kenig
- Nancy J. Kopell
- Vladimir Markovic (Chair)
- Yuval Peres
- Karen Uhlenbeck

The Prize for Seminal Contribution to Research is awarded on a six-year cycle of subject areas. The 2017 prize was given in geometry/topology, the 2018 prize will be given in discrete mathematics/logic, the 2019 prize is open, the 2020 prize in analysis/probability, and the 2021 prize in algebra/number theory.

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