



# Four Goals for Instructors Using Inquiry-Based Learning

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*Communicated by Benjamin Braun*

*Note: The opinions expressed here are not necessarily those of Notices.  
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In a recent *Notices* article [1], Ernst, Hodge, and Yoshinobu describe two pillars of inquiry-based learning (IBL): deep engagement in mathematics and opportunities to collaborate [2]. These two pillars identify key aspects of what students do in an IBL classroom. What, however, is the *instructor's* role in a successful IBL classroom? This question leads to a third pillar of IBL: instructor interest in and use of student thinking. In a stereotypical math classroom, instructor interest is often expressed with prompts such as: Does anyone have a question? What is the inverse of this linear transformation? How do you simplify this expression? These types of questions tend to have an expected response and lead to what is referred to as an Initiation-Response-Evaluation pattern of instructor and student talk. The instructor initiates a discussion by asking a question, a student responds, and the instructor indicates whether the response is correct or not (or asks

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DOI: <http://dx.doi.org/10.1090/noti1597>

a follow-up question directed toward the expected response). While such talk patterns can be useful, they do not usually promote the pillars of deep engagement in the mathematics and collaboration that characterize student experiences with IBL.

In this article, we discuss ways in which instructors can achieve four research-based goals in IBL classrooms. These goals and corresponding instructor actions are grounded in research conducted in multiple inquiry-based classrooms, including differential equations and linear algebra [3], [4], [5]:

- (1) get students to share their thinking,
- (2) help students to orient to and engage in others' thinking,
- (3) help students deepen their thinking, and
- (4) build on and extend student ideas.

As anyone who has tried to implement IBL can attest, realizing these goals takes time, patience, occasional failures, and the willingness to try out new patterns of interaction with students (often called “talk moves”). While these four goals are not an exclusive list of instructor responsibilities, they serve as a solid foundation for successful teaching in IBL classrooms.

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*What is the  
instructor's role  
in a successful  
IBL classroom?*

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## #1: Get Students to Share Their Thinking

In an IBL classroom, students spend their time actively working on problems (often with their peers) that are novel and/or challenging to them. Instructors strive to bring forth student ideas and make them public, even if the answers are incorrect or the students are tentative about their work. Creating a classroom environment where students feel safe to express their thinking is a challenge and not always easy to achieve, but research grounded in practice has revealed various prompts that can help realize this goal:

- “Dave, I know you haven’t finished the problem, but tell us your initial thinking.”
- “Take your time; we’re not in a rush.”
- “Can you say more about that?”
- “That’s an important point. Keisha, can you say that again so that everyone can hear?”
- “Learning from mistakes is an important part of doing mathematics. Who can share with us an initial approach that didn’t work out?”

These and similar talk moves (Figure 1) are productive for eliciting student thinking because they clearly communicate the expectation that students share their ideas rather than be completely correct.



Figure 1. “Talk moves” elicit student thinking.

But getting students to talk is, of course, just the start. More often than not, students’ initial thinking is muddled or difficult to interpret. Rather than evaluating the response or asking a leading follow-up question, an instructor might choose to repeat back what the student said, with or without an interpretation or rephrasing, and then check back with the speaker for accuracy. This type of talk move is referred to as *revoicing* and is a highly effective tool for initiating classroom discussions, in part because it conveys no evaluation of students’ initial contributions.

## #2: Help Students Orient To and Engage With Others’ Thinking

Once one or more students have shared their initial thinking, the role of the IBL instructor is to promote cross-talk among students where they attend to and make sense of other students’ thinking. Instructor prompts that can help achieve this goal often make use of questions that

we do not normally use in everyday conversation, yet talk moves such as the ones below can be quite effective in promoting student discourse:

- “Who can repeat in their own words what Juan just said?”
- “Do you agree with what Darrel just said?”
- “Say that again please so that everyone can hear.”
- “Can someone rephrase Diane’s explanation in their own words?”
- “So, Debbie, is Minsoo saying that...?”
- “Jesika, what do you think about what Ding just said?”

These types of talk moves encourage other students to revoice the thinking of their classmates. Sometimes student revoicing leads to a different conclusion or justification and thus creates an opportunity for a broader debate among multiple students. A student misinterpretation may come to the surface, encouraging the original speaker to use more precise mathematical language or explain their idea in a different way. All of these outcomes are mathematically productive.

## #3: Help Students Deepen Their Thinking

Even if students express their thoughts and listen to others’ ideas, the discussion can still fail to be mathematically productive if it does not include solid and sustained reasoning. Most students are not skilled at pushing to deepen their own reasoning or that of their classmates. Therefore, a key role of the instructor is to continuously and skillfully press students for reasoning, explanation, and justification. Instructors can help achieve this by using the following prompts:

- “How can we check to make sure this is correct?”
- “What is the reasoning that allows you to make that conclusion?”
- “Do you agree or disagree and why?”
- “How does that relate to what we learned yesterday?”
- “Can anyone come up with a different way to explain that?”
- “OK, I hear what you are saying, but what about this counterexample?”
- “We now have three different solutions and not all three can be correct. Work with your partner(s) to decide, with justification, whether each solution is correct or not.”

As these questions suggest, IBL instructors must take a proactive role in pressing students to develop their ideas and to explain and justify their thinking, navigating between students’ ways of reasoning and conventions of the broader mathematical community (Figure 2).

## #4: Build On and Extend Students’ Ideas

IBL classroom discussions are opportunities for students to report on the progress they have made working (individually, in pairs, or in small groups) on challenging problems. Such occasions are also opportunities for the instructor to build on student work to advance their mathematical agenda, to make connections to more formal or conventional mathematics, and/or to help students see and appreciate that they are actually *doing* mathematics. Using the following instructor moves are some ways in which instructors can build on and extend student work:



**Figure 2.** After being prompted to consider a classmate's claim, students work in small groups to decide if they agree or disagree and why.

- Restating what students said or did in more conventional or formal terms.
- Introducing a new but related concept, definition, representation, or procedure that extends what students did.
- Restating a student's explanation and attributing authorship to the student or students, i.e., creating the sense that mathematics is arising out of students' own work.
- Restating student ideas in ways that connect to established mathematical culture.

Benefits of such instructor moves include highlighting how student ideas fit into the larger mathematical trajectory; providing scaffolding for students to clarify, to elaborate, and to extend their mathematical positions; representing mathematics as something that can develop from students' own work; and supporting students' enculturation into the discipline of mathematics.

## Conclusion

Effectively supporting students' deep engagement in mathematics and peer-to-peer collaboration is an omnipresent challenge for instructors. The four goals discussed here provide some insight into what an instructor can do to realize the student-focused pillars of deep engagement in mathematics and collaboration. Just as a stool with three legs is sturdier than one with only two, IBL classrooms are on a stronger foundation when all three pillars are taken into account: the two focused on the activity of students and the one focused on the activity of the instructor.

## References

- [1] D. C. ERNST, A. HODGE, and S. YOSHINOBU. What is inquiry-based learning? *Notices Amer. Math. Soc.* **64** (2017), no. 6, 570–574.
- [2] S. LAUREN, M-L. HASSI, M. KOGAN, A-B. HUNTER, and T. WESTON, *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*, [www.colorado.edu/eer/research/documents/IBLmathReportALL\\_050211.pdf](http://www.colorado.edu/eer/research/documents/IBLmathReportALL_050211.pdf)
- [3] K. MARRONGELLE and C. RASMUSSEN. Meeting new teaching challenges: Teaching strategies that mediate between all lecture and all student discovery. In M. Carlson and C. Rasmussen (eds.), *Making the Connection: Research and Teaching in Undergraduate Mathematics Education*, Math. Assn. Amer., 2008, pp. 167–178. Washington, DC.
- [4] C. RASMUSSEN and O. KWON. An inquiry oriented approach to undergraduate mathematics, *J. Math. Behavior* **26** (2007), 189–194.
- [5] M. WAWRO, M. ZANDIEH, C. RASMUSSEN, and C. ANDREWS-LARSEN. *The Inquiry Oriented Linear Algebra Project*, 2017, [io1a.math.vt.edu](http://io1a.math.vt.edu)

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