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The editorial offices will be located at Brown University, where extraordinary library resources will be available both for editorial use and for microfilm and photoprint reproduction. The editors will be Professors O. Neugebauer and J. D. Tamarkin, both of whom have had extensive editorial experience in connection with other mathematical publications. They will be supported by a full-time resident editorial staff, and by a large corps of reviewers. While this reviewing staff is not yet complete, and its personnel will change from time to time as circumstances demand, it consists at present of 300 persons, of whom about two thirds are located in America and one third abroad. Outstanding competence for the reviewing of all special fields is definitely assured.

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The publication of MATHEMATICAL REVIEWS was an undertaking beyond the present resources of the American Mathematical Society. Its inauguration has been made possible by generous subventions from the Carnegie Corporation of New York and The Rockefeller Foundation. Brown University has provided adequate space for the editorial offices, and has granted the editors unrestricted use of all its library facilities. The learned societies which are at present contributing to its financial support, and are therefore known as sponsoring societies, are the Mathematical Association of America and the American Mathematical Society. Other learned societies are expected shortly to become sponsors.

Format

MATHEMATICAL REVIEWS will be issued once a month. In normal times it will run to approximately 600 large, double-column pages and will contain several thousand reviews annually. It is impossible to forecast accurately the effect of wartime conditions upon these estimates; but since fewer papers will appear, the annual volume of MATHEMATICAL REVIEWS will be considerably smaller. Every effort will be made, however, to obtain mathematical literature on a world-wide basis, and to publish reviews as promptly as the delays due to disorganized communications will permit.

The appearance of the journal and the character of the reviews will be apparent from the sample pages which follow. The first of these is a replica of the front cover. The other two reproduce reviews which will appear in the first number of the journal.

Mathematical Reviews

Sponsored by the American Mathematical Society and the Mathematical Association of America

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where m and n are co-prime integers and $a_0, a_1, \cdots, a_m$ are rational, by reducing it to the form $Y^m=Z^n$. This last equation is a particular case of an equation previously discussed by the author [Amer. J. Math. **55**, 67–76 (1933); **59**, 921–926 (1937)]. The case $m=2$, $n=3$, $a_0=a$, $a_1=0$, $a_2=-b$, $a_3=a_4=\cdots=a_m=0$ leads to an equation solved by Fogels by the method of algebraic numbers [Amer. J. Math. **60**, 734–736 (1938)]. *I. A. Barnett* (Cincinnati, Ohio).

Pillai, S. S. On normal numbers. Proc. Indian Acad. Sci., Sect. A. **10**, 13–15 (1939).

The author considers, among others, the number $.123\cdots$ (in the scale r) formed by writing the positive integers (in the scale r) in succession. He gives a proof that these numbers are simply normal, that is, each digit from 0 to $r-1$ appears with the asymptotic frequency $1/r$. The proof of the stronger statement that these numbers are normal is inadequate. *H. S. Zuckerman* (Seattle, Wash.).

Davenport, H. and Erdös, P. On sums of positive integral kth powers. Ann. of Math. **40**, 533–536 (1939).

In the application of the Hardy-Littlewood method to Waring's problem [Landau: Vorlesungen über Zahlentheorie, Bd. 1, Teil VI, Kap. 4] use is made of inequalities of the form $N_s^{(k)}(n)>n^{\alpha-\epsilon}$, $\alpha=\alpha(k, s)$. Here $N_s^{(k)}(n)$ denotes the number of integers $m\leqq n$ for which the equation $m=h_1^k+h_2^k+\cdots+h_s^k$ has a solution in integers $h_i\geqq 0$. The larger α can be made the better the final result. In a previous paper, Davenport [Proc. roy. Soc. London, Ser. A. **170**, 293–299 (1939)] defined admissible exponents as follows: The real numbers $\lambda_1, \lambda_2, \cdots, \lambda_s$ satisfying $\lambda_1\geqq\lambda_2\geqq\cdots\geqq\lambda_s>0$ are called admissible exponents for kth powers if the number of solutions of $x_1^k+\cdots+x_s^k=y_1^k+\cdots+y_s^k$ in integers $x_1, \cdots, x_s, y_1, \cdots, y_s$, subject to the conditions $P^{\lambda_i}<x_i<2P^{\lambda_i}$, $P^{\lambda_i}<y_i<2P^{\lambda_i}$, is $O(P^{\lambda_1+\cdots+\lambda_s+\epsilon})$ as $P\to\infty$ for any $\epsilon>0$. He then showed that if $\lambda_1, \cdots, \lambda_s$ are admissible exponents and $\alpha=(\lambda_1+\cdots+\lambda_s)/k\lambda_1$, then $N_s^{(k)}(n)>n^{\alpha-\epsilon}$, for any $\epsilon>0$ and $n>n_0(\epsilon)$. The present paper is concerned with the construction of admissible exponents for $k\geqq 3$ by the method of Erdös, with modifications due to Davenport. The following results are established: Theorem 1. Let $\theta=1-k^{-1}$. Then $1, \lambda, \lambda\theta, \cdots, \lambda\theta^{s-2}$ are admissible exponents for kth powers, provided λ satisfies $k\lambda-(k-1)\leqq\lambda\theta^{s-2}$. The proof is by induction on s. Theorem 2. Admissible exponents for kth powers are $1, 1-k^{-2}, 1-k^{-1}-k^{-2}$. The proof employs a device similar to the one used by Davenport for the case $k=4$ [C. R. Acad. Sci. Paris **207**, 1366 (1938)].

R. D. James (Saskatoon, Sask.).

ANALYSIS

Theory of Sets, Theory of Functions of Real Variables, Theory of Measure and Integration

Kempisty, Stefan. Sur les fonctions à variation bornée au sens de Tonelli. Bull. Sém. math. Univ. Wilno **2**, 13–21 (1939).

Following Radó, the area of a surface $z=F(x, y)$ can be expressed in terms of a Burkill integral, the integrand of which is formed by means of Geöcze's functions of intervals $G_i(F, R)$, $i=1, 2$ [cf. Saks: Theory of the Integral, (1937), chap. V]. Since the G_i are defined by means of a Riemann integral, this implies two passages to the limit. To avoid this, the author introduces the functions of intervals

$$\Lambda_1(F, R)=k\cdot\min_{b\leqq y\leqq b+k}|F(a+h, y)-F(a, y)|,$$
$$\Lambda_2(F, R)=h\cdot\min_{a\leqq x\leqq a+h}|F(x, b+k)-F(x, b)|,$$

where R stands for the interval $a\leqq x\leqq a+h$, $b\leqq y\leqq b+k$. It is shown that the upper Burkill integral of Λ_i coincides with the Burkill integral of G_i and that, starting from Λ_i, the area of $z=F(x, y)$ can be expressed as a Burkill integral. Using a known theorem [Saks, p. 179], it follows, furthermore, that a necessary and sufficient condition that $F(x, y)$ be of bounded variation in Tonelli's sense is that Λ_1 and Λ_2 be of finite variation. *W. Feller* (Providence, R. I.).

Marcinkiewicz, Józef and Zygmund, Antoni. Sur la dérivée seconde généralisée. Bull. Sém. math. Univ. Wilno **2**, 35–40 (1939).

A function $f(x)$ is said to possess at the point $x=x_0$ a generalized second derivative with the (finite) value s, in the sense of de la Vallée Poussin, or, respectively, of Schwarz, if the condition

$$(1)\qquad \lim_{t\to 0} 2[f(x_0+t)-f(x_0)-at]/t^2=s(x_0)$$

or

$$(2)\qquad \lim_{t\to 0} [f(x_0+t)+f(x_0-t)-2f(x_0)]/t^2=s(x_0)$$

holds. Condition (1) implies (2) but not conversely. The authors have proved [Fund. math. **26**, 1–43 (1936)] that if (2) holds for $x_0\epsilon E$, (1) holds almost everywhere in E.

In this note the condition (2) is still further generalized. Let $a<b<c$ be fixed; the generalized second derivative is said to exist and be equal to s, if

$$(3)\quad \lim_{t\to 0} [A\cdot f(x_0+at)+B\cdot f(x_0+bt)+C\cdot f(x_0+ct)]/t^2=s(x_0),$$

where A, B, C are uniquely determined by linear relations involving a, b, c. Again (1) implies (3); the principal result is that, as in the case of (2), condition (1) must hold almost everywhere on any set E where (3) holds. *J. A. Clarkson.*

Kershner, Richard. The number of circles covering a set. Amer. J. Math. **61**, 665–671 (1939).

The author proves in a very simple manner that if M denotes a bounded plane point set, $\bar{M}$ its closure, and $N(\epsilon)$ the minimum number of circles of radius ϵ which can cover M, then

$$\lim_{\epsilon\to 0} \pi\epsilon^2 N(\epsilon)=(2\pi/3\sqrt{3}) \text{ meas. } \bar{M}.$$

P. Hartman (New York, N. Y.).

Krzyżański, Mirosław. Sur l'extension de l'opération intégrale de Denjoy aux fonctions de deux variables. Bull. Sém. math. Univ. Wilno **2**, 41–51 (1939).

The main purpose of this paper is to extend to two dimensions the so-called descriptive definition of the Denjoy integral which is given in terms of the integral function. Any rectangle will be tacitly assumed to have sides parallel to the axes. Let $F(R)$ be an additive rectangle function defined on a rectangle R_0, let E be a subset of R_0 and let R_E denote the smallest rectangle containing E. The function $F(R)$ is said to be absolutely continuous on E if for each $\epsilon>0$ there exists a $\delta>0$ such that $\sum_{i=1}^n|F(R_i)|<\epsilon$ for each finite set of non-overlapping rectangles $\{R_i\}$ with $\sum_{i=1}^n$ meas $(R_i)<\delta$, provided further that each R_i is in R_E and contains a point of E. If $F(R)$ is absolutely continuous on E,

Vincent, J. J. The mathematical theory of shuttle projection. J. Textile Inst. 30, T103–T126 (1939).

The author studies the exact motion of a loom shuttle when the elasticity of the picking mechanism is taken into account, but its mass is neglected. Under various mathematical assumptions he arrives at differential equations of the type $\ddot{x}+\lambda x=F(t)$. Seven different cases of possible motions are discussed. *W. Feller* (Providence, R. I.).

Heinrich, Gerhard. Resonanzschwingungen eines Systems bei vektorieller Überlagerung der erregenden Impulse. Z. angew. Math. Mech. 19, 216–223 (1939).

If a rotating system of natural frequency ω is acted on by a periodic torque of frequency ω_0, a necessary condition for resonance is that $\omega=k\omega_0$, where k is an integer. This is shown to be sufficient unless certain additional relations, which are explicitly found, hold. The results are extended to damped oscillations. *P. Franklin* (Cambridge, Mass.).

Høiland, Einar. On the interpretation and application of the circulation theorems of V. Bjerknes. Arch. Math. og Naturvid. 42, no. 5, 69 pp. (1939).

This paper is based on the circulation theorems for a perfect fluid due to V. Bjerknes, the more useful being $\int a_T dm=\int g_T dm$; the integrals are taken round a thin tube of constant cross-section; a_T, g_T are tangential components of acceleration and body force, respectively; dm is the element of mass. Using this theorem the author claims to establish necessary and sufficient conditions for stability in the case of a fluid at rest or in rotation as a vortex under the action of gravity. In general the fluid is either homogeneous or heterogeneous and incompressible. Flow between parallel planes is also discussed. The arguments are not mathematically convincing. *J. L. Synge* (Toronto, Ont.).

Molière, G. Berechnung verallgemeinerter Gitterpotentiale. Z. Kristallogr., Mineral. u. Petrogr. Abt. A. 101, 383–388 (1939).

Conditionally convergent lattice sums of type

$$\sum_l{}' \pm r_l^{-n}, \qquad n=1, 2, 3,$$

are considered for simple types of ionic lattices, the index symbol $l=(l_1, l_2, l_3)$ relating to a lattice point independent of its charge. A study is made also of the generalized lattice potential

$$\phi_n(r)=\sum_l\sum_k e_k|r-r_k{}^l|^{-n}.$$

Use is made of Ewald's device in which the integration is broken up into two parts after r_l^{-n} has been expressed as an integral of Laplace's type. The generalizations of Ewald's formula involve the functions

$$G(x)=2\pi^{-\frac{1}{2}}\int_x^\infty e^{-t^2}dt,\quad D(x)=4\pi^{-\frac{1}{2}}\int_x^\infty e^{-t^2}t^2dt,$$
$$L(x)=\int_x^\infty e^{-t}dt/t.$$

H. Bateman (Pasadena, Calif.).

TOPOLOGY

Zorn, Max. Continuous groups and Schwarz' lemma. Trans. Amer. math. Soc. 46, 1–22 (1939).

This is a contribution to the theory of conformal mappings from a topological point of view. The author considers a family N of single-valued continuous mappings of a metric space S into itself; N is subjected to restrictions which describe some of the topological properties of the special case in which S is the interior of the unit circle in the complex plane and N the totality of analytic (conformal) mappings F ($F(S)\subset S$). The decisive restriction on N is that of normality: an infinite sequence $\{F_i\}$ of mappings in N either determines one or more limit mappings in N or else $\{F_i(x)\}$ has no limit point for any x. An F in N which admits a fixed point and has an inverse in N is called a rotation. The theory of rotations and "circumferences" about an arbitrary but definitely fixed point p is developed. For the definition of "circle" (circular region) further restrictions on N, S stating certain properties of separation are introduced. It then becomes possible in terms of abstract circles and circumferences to define relations of the form $|x|\leqq|y|$ which are equivalent in the case of analytic mappings to inequalities between moduli. A sequence of theorems leads to the following result: If $F(p)=p$ $(F\subset N)$, then $|F(x)|\leqq x$ for every x. This is the formulation, appropriate to the general setting under consideration, of Schwarz' lemma about analytic mappings. In the last section it is shown that every S which forms part of a system S,N must be separable, locally compact, and semicompact. *P. A. Smith* (New York, N. Y.).

Wallace, A. D. Some characterizations of interior transformations. Amer. J. Math. 61, 757–763 (1939).

It is shown that if S and S_1 are metric spaces and $T(S)=S_1$ is continuous, the following conditions are each equivalent to the property of interiority for T: (i) for any sequence of sets Y_n converging to Y in S_1, $\lim T^{-1}(Y_n)=T^{-1}(Y)$; (ii) for any set Y in S_1, $T^{-1}(\bar{Y})=\overline{T^{-1}(Y)}$; (iii) for any set X in S, $T(X^0)=T(X)^0$, where X^0 denotes the set of all interior points of X. Also a study is made of transformations, called strongly continuous, satisfying $T(X)=\lim T(X_n)$ for every sequence of sets X_n converging to X in S and of transformations, called normal, such that there exist arbitrarily small neighborhoods of any point whose boundaries transform into boundary points of the images of the neighborhoods. *G. T. Whyburn* (Charlottesville, Va.).

Whyburn, G. T. The existence of certain transformations. Duke math. J. 5, 647–655 (1939).

A transformation f of a continuum A into a continuum B is said to be non-alternating if for any two points b_1 and b_2 of B the set $f^{-1}(b_1)$ does not separate any two points of $f^{-1}(b_2)$ in A; it is said to be interior if $f(U)$ is an open subset of B for every open U in A. The chief result is to show that, in order that a non-alternating interior transformation f exist carrying a locally connected continuum A into a linear interval B, it is necessary and sufficient that A be a cyclic chain, that is, A contain two points x and z such that every point y of A lies on an arc of A with end points x and z. In proving the existence of the desired transformation f, it is shown that f can be chosen as monotone ($f^{-1}(b)$ connected for every $b\varepsilon B$) in addition if A is unicoherent or f can be light ($f^{-1}(b)$ totally disconnected) if A is 1-dimensional. Finally any connected graph can be mapped on a linear interval by an interior transformation and a tree A can be so mapped if and only if A has no infinite branch points and the set of end points of A is closed except for a finite set. *W. L. Ayres.*