

*Conjugacy Classes in Semisimple Algebraic Groups*  
(Revisions)

Subtract 4 from each page number in the Index. (This production error seems to be corrected in the paperback reprint.)

- 7** In line 3 replace  $M_{\mu-k\alpha}$  by  $M_{\mu+k\alpha}$ .
- 8** In line -11 read  $\mathfrak{sl}(r+1, K)$ .
- 8** In line -1, replace “codimension 2” by “codimension 4”.
- 9** In line 5, replace “dimension  $2r$ ” by “codimension  $2r$ ”.
- 9** In line 8, just before “The center of  $\mathfrak{g}$  is”, insert: “Let  $p = 2$ .”
- 18** The second sentence following Proposition 1.9 should conclude: “... the orbit map  $G \rightarrow \text{Cl}(x)$  induces an isomorphism  $G/C_G(x) \cong \text{Cl}(x)$ .”
- 19** Replace the period at the end of line -2 with a colon.
- 20** In line -4 read “semisimple”.
- 22** At end of line -8 read “elements of  $G$ ”. At the beginning of line -2 read: “Obviously  $C \supset C_G(X) \cap C_G(x) \dots$ ”
- 29** In the final paragraph, delete the sentence “For any abelian group  $A \dots$ ”. Rewrite the following sentence: “We want to show that  $K^\times$  is isomorphic to the subgroup  $(\mathbb{Q}/\mathbb{Z})_{p'}$  of  $\mathbb{Q}/\mathbb{Z}$  consisting of elements of order prime to  $p$ . Clearly  $(\mathbb{Q}/\mathbb{Z})_{p'} \cong \mathbb{Q}_{p'}/\mathbb{Z}$ , where  $\mathbb{Q}_{p'}$  consists of rational numbers representable as fractions  $a/b$  with  $p \nmid b$ .”
- 37** In line 12 read “ $\varepsilon_i - \varepsilon_j \in \Phi \cap \overline{A} \dots$ ” In lines 15–17, read “If precisely two such components occur,  $\overline{\Psi} - \Psi$  yields a single nonzero coset in the Klein 4-group  $\overline{A}/A$ , which is not annihilated by some character (of order 2, prime to  $p$ ).”
- In line -10, read “Let  $H$  be a connected”.
- 45** The last symbol on line 11 should be  $g_0$ .
- 57** In line 3, read “ $\lambda \in P$ ”. Replace lines 6–7 by: “Moreover, one can totally order the vector space  $E$  in a way compatible with the positive system in  $\Phi$ , so that  $\mu > 0$  whenever  $\langle \lambda, \mu \rangle > 0$ . (Use a lexicographic ordering with  $\lambda$  as first basis vector.)” Delete line 14: “Unless  $\alpha \dots$ ”

- 60** In line  $-7$ , read  $F_4, E_7$  in place of  $E_7$ . The error here in the computations of Lou [112] was corrected by Liebeck and Seitz in their 2012 book added to the list below.
- 61** In line 11, replace “which in turn” by “whose identity component”.
- 64** In line  $-12$ , replace “a positive root in  $\Phi'$ ” by “a positive root”.
- 65** Begin line  $-11$  with “The restriction to  $X$  of this map ...” In line  $-2$  read “ $\alpha_k(t) = c_k$ ”.
- 66** In line 9, replace “lies in  $U_i$ ” by “lies in  $V_i$ ”.
- 67** In line  $-5$  read: “the regular unipotent elements in  $G'$  are dense in the unipotent variety of  $G'$ ”.
- 68** Replace the sentence beginning “Now we get ...” on line 6 with: “Now use the method of 0.15 to construct a closed subset of  $G/C_G(s) \times G$  mapping onto  $F - R$ , by taking pairs  $(gC_G(s), x)$  for which  $g^{-1}xg \in I$ .” Replace line 9 by: “Most of the fibres of  $\pi$  consist of a single class (a regular semisimple class).”
- 69** In line 8, delete extra “that”.
- 71** In last line of (1), read  $\chi_1(g)$  in place of  $\chi(g)$ .
- 72** In line 25, replace  $\chi(x)$  by  $\chi_1(x)$ .
- 73** In line 4, read: “The differentials  $d\chi_i$  are independent at  $x \dots$ ” In line 15, reference should be Steinberg [185, (2)].
- 74** In lines 8 and 12, replace  $\tau_{\pm}$  by  $\tau_i^{\pm}$ . Expand the proof in the final paragraph of 4.21, giving more explicit details.
- 79** In line 4, omit the extra “an”. Read “ $\{\alpha \in \Phi^+ \mid w(\alpha) < 0\}$ .” at the end of line 15. In line  $-6$  replace “proposition” by “theorem”.
- 80** Replace the first sentence of Theorem 5.3 (iii) by: “Each element  $u \in C$  lies in only finitely many conjugates of  $V$ .”
- 81** Line 5 should begin: “ $\dim \text{Cl}_P(u) = \dim V$ .”
- 81** Replace the first two sentences in the second paragraph of the proof by: “Theorem 5.2 shows that there exist elements of  $\text{Cl}(V)$  lying in only finitely many conjugates of  $V$ , which (by the proof of Corollary 5.2)

form an open subset of  $\text{Cl}(V)$ . This set is clearly a union of conjugacy classes, so it includes the chosen open class  $\text{Cl}(u)$ .

**91** Add to 5.10 a discussion of Borho’s approach to the proof, with added citation.

**93** In line 8 read “ $G$  is a connected semisimple group of rank  $r$ .”

**96** In the last line, replace  $h_i$  by  $h^{(i-1)}$ .

**99** In line 9 read “the fibres  $gB$  of . . .” In line –13 replace “ $v =$ ” by “ $v \in$ ”.

**101** In line –3 follow the displayed set by “ $\rightarrow G/B^{(w)}$ ”.

**102** Replace the display in line 13 by

$$\dim C_G(u) + (\dim(C \cap U^{(w)}) - \dim U^{(w)}) = r + 2 \dim \mathfrak{B}_u.$$

**103** In line 24 replace  $\pm\alpha_j$  by  $\alpha_j$ . In line 27 replace “Each of the root systems” by “The Weyl group of each root system”.

**106** In line –6 replace  $q \in Q_\beta$  by  $q \in P_\beta$ .

**110** In line 23 replace  $V/F_{n-i}$  by  $V/V_{n-i}$ .

**111** In line –7 replace “case-by-work” with “case-by-case work”.

**112** In (ii) of the lemma, replace  $W/W'$  by  $W' \setminus W$ .

**113** In line 4, replace  $W/W'$  by  $W' \setminus W$ .

**116** Reword the Remark at the end of section 6.19 as follows. End the first sentence at: [170, §5]. Then write: “For example,  $\mathfrak{g}$  may be obtainable by base change from a Chevalley  $\mathbb{Z}$ -form as happens when  $G$  is simply connected. Otherwise  $\mathfrak{g}$  might vary: see 0.13.”

**118** In line 1 of 6.21, read “Theorem 6.20”. In lines –14 to –13, replace “a 1-dimensional additive subgroup” by “an  $r$ -dimensional affine space”.

**127** Replace the sentence on lines 3–6 by: “This equals  $d_0 + d_1$ , since each irreducible  $\mathfrak{a}$ -summand of  $\mathfrak{g}$  has a 1-dimensional intersection with  $\mathfrak{g}_0$  or  $\mathfrak{g}_1$  (but not both). Among these are  $d_0 - d_2$  trivial  $\mathfrak{a}$ -summands. To summarize:”

**128** Replace line –6 by

$$\lambda_1-1, \lambda_1-3, \dots, -(\lambda_1-3), -(\lambda_1-1), \dots, \lambda_d-1, \lambda_d-3, \dots, -(\lambda_d-3), -(\lambda_d-1)$$

In line –9 read  $\mathfrak{sl}(2, K)$ .

**131** In line –2 replace  $x_{-\beta}$  by  $y_\beta$ .

**133** In lines 17–19, read “...is needed because centralizers need not be connected. We say that a unipotent element  $u$  (or its class) is distinguished if the group  $C_G(u)^\circ$  is unipotent.”

**133** In line 26 read “distinguished”.

**138** In the first line below the table, read “come from”.

**141** In line –11 replace “fails to be normal” by “has non-normal closure”.

**148** In line 10 replace  $a \equiv bF(c)^{-1}$  by  $a \equiv cbF(c)^{-1}$ . In the last line read “upper unitriangular group”.

**149** The first symbol on the first line should be  $G$  (not  $U$ ).

**154** In line 14 read “if and only if it lies in”. In part (ii) of the theorem, read “in  $|C_U(u)/C_U(u)^\circ|$  classes.” In line –11 replace  $q^{m-r}$  by  $q^{m-r}(q-1)^r$ .

**169** Reword the second sentence of (4) in 9.6: “This map respects the gradings, and its image lies in the fixed point space ...”

**176** In line 2, cite the paper by Shi.

**188** In the reference [112], the page numbering should be 1144–1146.

## Updated references

[145 ] J.P. Serre, Sém. Bourbaki (1993–94), Exposé 783, Astérisque **227** (1995).

[195 ] D.M. Testerman, J. Algebra **177** (1995), 34–76.

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## Added references

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