

2018 Who Wants to Be a Mathematician Round One Qualifying Test

1. Find the y-intercept (y-coordinate only) of the line whose equation is $2x + 5y = 7$. 7/5

2. $2 + 4 + 6 + 8 + \dots + 2018 =$ (write your answer as a number, not a product) 1,019,090

3. $\left(\cos\frac{\pi}{12} + \sin\frac{\pi}{12}\right)\left(\cos\frac{\pi}{12} - \sin\frac{\pi}{12}\right) =$ (simplify so that your answer contains no trig functions) $\frac{\sqrt{3}}{2}$

4. What ancient Greek mathematician had a "sieve," used for finding prime numbers, named after him? (circle one) a. Archimedes b. Eratosthenes c. Euclid d. Pythagoras b.

5. Write $\frac{\log_5 9}{\log_{25} 3}$ as a rational number. 4

6. For which one of the following choices for m are the base m numbers 25_m and 27_m both primes? (circle one) a. 8 b. 9 c. 11 d. 12 e. 13 d.

7. You and two friends decide to pay a restaurant bill by having each person flip a fair, two-sided coin. If all three coins show heads or all show tails, you will split the bill three ways. Otherwise, the person whose coin landed differently from the other two will pay the entire bill. What is the probability that you will not have to pay anything? 1/2

8. Suppose that m and n are positive integers such that $5m + 3n = 41$. What is the smallest possible value for $|m^2 - n^2|$? 33

9. In rectangle ABCD at right, DP is perpendicular to PC. Find x, the distance from A to P, where P is between A and the midpoint of segment AB. a.

a. $3 - 2\sqrt{2}$ b. $3 - \sqrt{2}$

b. $3 - \sqrt{7}$ d. $3 - \sqrt{7}/2$

The diagram shows a rectangle ABCD with vertices A at the top-left, B at the top-right, C at the bottom-right, and D at the bottom-left. The height of the rectangle is labeled as 1, and the width is labeled as 6. Point P is located on the top side AB. A line segment DP is drawn from vertex D to point P, and another line segment PC is drawn from point P to vertex C. The two line segments DP and PC are perpendicular to each other. The distance from vertex A to point P along side AB is labeled as x.

10. What is the largest prime factor of $2^{14} + 1$? 113

Thanks for participating.