Shiing-Shen Chern was born and educated in China, except for a D.Sc. from the University of Hamburg in 1936. He then returned to Tsing Hua University and Academia Sinica, but came to the University of Chicago in 1949. Since 1960 he has been at the University of California, Berkeley. The strong school of differential geometry in the U.S. is largely due to his influence. Recently he has become the director of a new mathematics research institute at Nankai University in Tianjin.

American Differential Geometry —Some Personal Notes

SHIING-SHEN CHERN

1. Projective Differential Geometry

I got into geometry largely through my college professor Dr. Li-Fu Chiang. Dr. Chiang received his Ph.D. in 1919 from Harvard under Julian Coolidge. On his return to China he organized a "one-man" department at the newly founded Nankai University in Tientsin, a private University having in my time an enrollment of about 300 students. There were five mathematics majors in my class. I gave seminar reports on material from the Coolidge books *Non-euclidean Geometry, Geometry of the Circle and the Sphere*, and other sources.

When I graduated from Nankai University in 1930, perhaps the only mathematician doing research in China was Dr. Dan Sun, then professor at Tsing Hua University in Peiping. Dr. Sun received his Ph.D. in 1928 from the University of Chicago under E. P. Lane. In order to work with Dr. Sun I became an assistant of Tsing Hua University and a year later a graduate student. I began to know that there was an "American school" of projective differential geometry founded by E. J. Wilczynski of Chicago. Wilczynski's first paper on the subject appeared in 1901. He gave the New Haven Colloquium Lectures of the AMS in 1906. The subject flourished till the thirties. A bibliography on projective differential geometry, compiled by Pauline Sperry (1931), contained more than 200 items. I particularly enjoyed reading the papers of G. M. Green of Harvard.

There was also an "Italian school" of projective differential geometry founded by G. Fubini and E. Cech in 1918. The American school takes as analytic basis systems of partial differential equations and uses the Lie theory to generate invariants, while the Italian school takes the differential forms to be the analytic basis.

It may be interesting to note that when, in 1949, I joined the faculty of the University of Chicago, I was essentially the successor to E. P. Lane, a typical gentleman. I spent some of my best years on the subject, but left it for greener pastures when I went to Germany.

2. Geometry of Paths

Another active group on differential geometry was in Princeton, whose representatives were L. P. Eisenhart, O. Veblen, and T. Y. Thomas. Eisenhart's *Riemanman Geometry* was a standard book and a source of information. Veblen had a wide mathematical interest, including the foundations of geometry, algebraic topology, differential geometry, and mathematical physics. He played a vital role in developing Princeton into a world center of mathematics.

My first contact with Veblen took place when he wrote to Elie Cartan in 1936 about projective normal coordinates. These are essential to the Princeton school of the "geometry of paths," because they are used to define the "normal extensions" of tensors. Projective normal coordinates can be given different definitions, each with some disadvantages. Following Veblen's letter I proposed a definition based on Cartan's geometrical approach. This result was communicated by Veblen and published in the *Annals of Mathematics* in 1938. After my return to China in 1937 he communicated several other papers of mine and my students to the *Annals* and other journals. In 1942 he invited me to visit the Institute for Advanced Study. The years 1943–1945 were among the most productive in my life. Veblen had great vision of modern mathematics and American mathematics. He contributed greatly both in his own research and in the development of mathematics in America. After his death in 1960 the American Mathematical Society set up a "Veblen prize" in geometry in his memory. I think it was Wallace Givens and myself who started this idea.

3. Developments in Topology

At the beginning of the century global differential geometry began to draw attention. For example, the first volumes of the *Transactions of the AMS* contain papers by Hilbert and Poincaré, both dealing with surface theory, in which Hilbert gave his famous proof of Liebmann's rigidity theorem of the sphere and Poincaré gave a proof of the existence of closed geodesics on a convex surface. (I believe these papers were by invitation and represented their support of the new journal.)

Perhaps the most important American contributions to differential geometry came from topology. Marston Morse's critical point theory has its origin in the calculus of variations. It became an appropriate and indispensable tool in Riemanman geometry. The study of geodesics is even now an active topic.

Another important contribution was Hassler Whitney's theory of sphere bundles, which led to general fiber bundles, a fundamental concept in differential geometry. Whitney saw the importance of cohomology in the applications of topology and introduced the characteristic classes in the topological context.