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American Mathematics Comes of Age: 1875–1900

KAREN HUNGER PARSHALL* AND DAVID E. ROWE†

Within the history of science in general and the history of mathematics in particular, issues such as the beginning of American research mathematics and the subsequent founding of a mathematical community have been conspicuously ignored. In the last fifteen years, historians of American science have generated quite a number of new books on the subject of the development of science in America. With few exceptions, however, none of them

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Congress of Mathematicians, World's Columbian Exposition, 1893. Bottom row, left to right, James E. Oliver and William E. Story; second row, William B. Smith, Henry S. White, Felix Klein, Harry W. Tyler, and Thomas F. Holgate; third row, Arthur G. Webster, C. A. Waldo, E. Study, J. M. Van Vleck, H. T. Eddy, J. B. Shaw, James McMahon, and Professor of Mathematics at Hope College (John Kleinheksel); top row, E. M. Blake, H. G. Keppel, Frank Loud, Henry Taber, Oskar Bolza, E. H. Moore, and Heinrich Maschke.

deals with the period between 1875 and 1900, and none of them deals, to any extent, with mathematics.[1]

It is surely true that the years from roughly 1800 to 1875 witnessed a steady organization of the American scientific community and an increase in the overall level of scientific research being pursued by Americans. This certainly justifies a concentration on the first three quarters of the nineteenth century. Yet, it was during the last quarter of that century and through the first quarter of the twentieth century that the seeds of this earlier developmental period bore fruit. Furthermore, as John Servos has recently pointed out, mathematics, both as the handmaiden of the sciences and as an independent intellectual endeavor in its own right, was at the heart of advances, first in physics and later in chemistry and biology.[2] Since the developments of the other sciences hinged on the development of mathematics, it thus becomes crucial to the understanding of the entire history of American science to come to terms with the emergence of a mathematical research community in the United States.

To say that American mathematics came of age between 1875 and 1900 implies that it did not spring up *ex nihilo*. As an integral part of the curriculum at all levels, mathematics had come to America with the first educational institutions. Until the latter part of the nineteenth century, though, instruction had remained woefully elementary.[3] Prior to the 1820s, the curriculum of America's colleges had followed the eighteenth-century English model, concentrating primarily on Latin, Greek, philosophy, the rudiments of Newtonian mechanics, a little trigonometry and a bit of mathematics from Euclid, that is, arithmetic, elementary algebra, and some geometry. The War of 1812, however, symbolized a shift in focus from things English to things French, and Americans in higher education saw a country in which science and mathematics were highly respected and flourishing. As a result, many American colleges had established professorships in science by the 1820s to complement their extant mathematical chairs, and French texts in translation had

[1]See, for example, Stanley M. Guralnick, *Science and the Antebellum American College* (Philadelphia: American Philosophical Society, 1975); Sally Gregory Kohlstedt, *The Formation of the American Scientific Community: The Association for the Advancement of Science 1848-60* (Urbana: University of Illinois Press, 1976); Nathan Reingold, ed., *The Sciences in the American Context: New Perspectives* (Washington, D.C.: Smithsonian Institution Press, 1976); Charles E. Rosenberg, *No Other Gods: On Science and American Social Thought* (Baltimore: Johns Hopkins University Press, 1976); Daniel J. Kevles, *The Physicists: The History of a Scientific Community in Modern America* (New York: Alfred A. Knopf, 1978); John C. Greene, *American Science in the Age of Jefferson* (Ames: The Iowa State University Press, 1984); and Robert V. Bruce, *The Launching of Modern American Science, 1846–1876* (New York: Alfred A. Knopf, 1987).

[2]John Servos, "Mathematics and the Physical Sciences in America, 1880–1930," *Isis* 77 (1986): 611–629.

[3]On early nineteenth-century American mathematics education, see Florian Cajori, *The Teaching and History of Mathematics in the United States* (Washington, D.C.: Government Printing Office, 1890).

come to set new standards for scientific and mathematical learning.[4] Thus, in mathematics the stakes were raised, and calculus was introduced into a curriculum which became more and more science-oriented. By mid-century, in fact, the number of science professors amounted to almost half of many faculties, and a third of the courses which students took were scientific or mathematical in nature.

Of importance to the present discussion, however, is the fact that this rise in science teaching did not imply an increase in basic scientific research. Prior to 1875, although research was considered prestigious within the growing scientific community, there were no institutional mandates and few institutional facilities for research.[5] Furthermore, since there was little training in science beyond the undergraduate level, anyway, few people were able to reach the research level in their chosen discipline. Virtually only those who chose to study abroad, although there were notable exceptions to this, could get the extra training they needed to become productive researchers.[6] All of this began to change after 1875 with the founding of the Johns Hopkins University.

What made Johns Hopkins, as conceived and implemented by its first president, Daniel Coit Gilman, so different?[7] Unlike the presidents of long extant colleges and universities such as Harvard, Yale, and Princeton, Gilman labored under neither an unbending tradition nor a firmly entrenched philosophy of education. He realized that for his new university to survive and prosper, it had to offer something different within the context of American education. As a result of his observations abroad, Gilman recognized that the United States trailed far behind the European countries in offering advanced training in the theoretical as well as in the practical sciences. Thus, in contrast to the pre-1875 American college and university, Johns Hopkins stressed graduate education, but not at the expense of undergraduate studies, and it made research and publication institutionally sanctioned and supported activities. One of its goals was to make the United States competitive with Europe at the research level. In mathematics, it achieved this goal by appointing the then sixty-one-year-old British mathematician, James Joseph Sylvester.

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[4]*Ibid.*, and Stanley M. Guralnick, "The American Scientist in Higher Education, 1820–1910," in Nathan Reingold, ed., *The Sciences in the American Context: New Perspectives*, pp. 99–141. The figures which follow come from Guralnick, op. cit., pp. 107–108.

[5]Rosenberg, p. 146.

[6]Among these exceptions were Benjamin Peirce and Josiah Willard Gibbs.

[7]On the history of the Johns Hopkins University, see John C. French, *A History of the University Founded by Johns Hopkins* (Baltimore: The Johns Hopkins University Press, 1946), Hugh Hawkins, *Pioneer: A History of The Johns Hopkins University 1874-1889* (Ithaca: Cornell University Press, 1960), and Francesco Cordasco, *Daniel Coit Gilman and the Protean Ph.D.: The Shaping of American Graduate Education* (Leiden: E. J. Brill, 1960).

Born into a Jewish family in London in 1814, Sylvester went to St. John's College, Cambridge in 1831.[8] There, in spite of a second place finish in 1837 on the prestigious Mathematical Tripos, his failure to submit to the Thirty-Nine Articles of the Church of England prevented him from taking his British degree. In 1841, he did earn his B.A. and M.A., but from Trinity College, Dublin. By 1846, he had met Arthur Cayley, while both were studying for the Bar, and had struck up a friendship and mathematical association which would end only with Cayley's death in 1895. Together, these two mathematicians launched the field of invariant theory, one of the most active research areas of nineteenth-century mathematics, and made far-reaching contributions to higher geometry, to the theory of matrices, and to combinatorics.[9] Once again, though, Sylvester's reputation and prodigious research proved inconsequential in the broader sphere. The longstanding Tests Act denied him, on religious grounds, the sort of prestigious university position he merited, and so, from 1855 to 1870, he was professor of mathematics at the Royal Military Academy at Woolwich. He finally left academe in 1870 in the wake of a pension dispute with the Academy and remained unemployed until 1876.

Knowing of this sad state of affairs, the Harvard mathematician Benjamin Peirce greeted the news of the new university to be founded in Baltimore as a potential godsend both for his British friend and for American mathematics. In the most eloquent of terms, he urged Gilman to choose Sylvester for the mathematics professorship and assured him of the wisdom of such a choice. On September 18, 1875, Peirce wrote:

> Hearing that you are in England, I take the liberty to write you concerning an appointment in your new university, which I think would be greatly for the benefit of our country and of American science if you could make it. It is that of one of the two greatest geometers of England, J. J. Sylvester. If you enquire about him, you will hear his genius universally recognized but his power of teaching will probably be said to be quite deficient. Now there is no man living who is more luminary in his language, to those who have the capacity to comprehend him than Sylvester, provided the

[8]There are many short, biographical sources on Sylvester. See, for example, H. F. Baker's notice in *The Collected Mathematical Papers of James Joseph Sylvester*, H. F. Baker, ed., 4 vols. (Cambridge: University Press, 1904–1912; reprint ed., New York: Chelsea Publishing Co., 1973), 4:xv–xxxvii (hereinafter cited as *Math. Papers J.J.S.*).

[9]On the mathematics of Cayley and Sylvester, see Tony Crilly, "The Rise of Cayley's Invariant Theory (1841–1862)," *Historia Mathematica* **13** (1986): 241–254; Tony Crilly, "The Decline of Cayley's Invariant Theory (1863–1895)," *Historia Mathematica* **15** (1988): 332–347; Karen Hunger Parshall, "America's First School of Mathematical Research: James Joseph Sylvester at The Johns Hopkins University 1876–1883," *Archive for History of Exact Sciences* **38** (1988): 153–196; and Karen Hunger Parshall, "Toward a History of Nineteenth-Century Invariant Theory," in David E. Rowe and John McCleary, eds., *The History of Modern Mathematics*, 2 vols. (Boston: Academic Press, 1989), 1: to appear.

> hearer is in a lucid interval. But as the barn yard fowl cannot understand the flight of the eagle, so it is the eaglet only who will be nourished by his instruction .... Among your pupils, sooner or later, there must be one, who has a genius for geometry. He will be Sylvester's special pupil—the one pupil who will derive from his master, knowledge and enthusiasm—and that one pupil will give more reputation to your institution than the ten thousand, who will complain of the obscurity of Sylvester, and for whom you will provide another class of teachers .... I hope that you will find it in your heart to do for Sylvester—what his own country has failed to do—place him where he belongs—and the time will come, when all the world will applaud the wisdom of your selection.[10]

Sylvester was indeed appointed and officially assumed his duties in the fall of 1876. He began by choosing the first class of graduate fellows in mathematics, a class of two: George Bruce Halsted, who would become a controversial professor at the University of Texas at Austin, and Thomas Craig, who would eventually succeed Sylvester at Johns Hopkins. Later that summer, Sylvester stole Hopkins' first associate for undergraduate teaching, William E. Story, from Harvard. By 1878, he had founded the *American Journal of Mathematics*; he had brought out its first number with the help of Story, whom he had enlisted as managing editor; he had published over twenty papers on invariant theory; and he had gathered around him almost a dozen graduate students and assistants. By 1881, he and his assembled students and associates, Craig, Story, Fabian Franklin, Christine Ladd Franklin, William P. Durfee, and Charles S. Peirce, among others, were realizing Gilman's goal.[11]

Doctoral Dissertations Written under Sylvester at Johns Hopkins

1. Thomas Craig, "The representation of one surface upon another, and some points in the theory of the curvature of surfaces," 1878.
2. George Bruce Halsted, "Basis for a dual logic," 1879.
3. Fabian Franklin, "Bipunctual coordinates," 1880.
4. Washington Irving Stringham, "Regular figures in n-dimensional space," 1880.

[10]Benjamin Peirce to Daniel C. Gilman, September 18, 1875, Daniel C. Gilman Papers, Ms. 1, Special Collections Division, Milton S. Eisenhower Library, The Johns Hopkins University (hereinafter cited as Gilman Papers). As quoted in Parshall, "America's First School of Mathematical Research," pp. 167–168. We thank the Johns Hopkins University for permission to quote from its archives.

[11]Parshall, "America's First School of Mathematical Research," pp. 165–172. Although women were not formally admitted to Johns Hopkins in the 1870s and 1880s, Christine Ladd (later Mrs. Franklin) asked for and got permission to attend Sylvester's lectures. Sylvester even persuaded Gilman to grant her a fellowship.

5. Oscar Howard Mitchell, "Some theorems in numbers," 1882.
6. William Pitt Durfee, "Symmetric functions," 1883.
7. George Stetson Ely, "Bernouilli's numbers," 1883.
8. Ellery William Davis, "Parametric representations of curves," 1884.

Sylvester had indeed founded a mathematical school engaged in, and even competing with one another in, producing and publishing results which were recognized as significant and original both in England and on the Continent. This school centered around what Sylvester called his "Mathematical Seminarium." With Sylvester as its director and the students as his assistants, the mathematical seminarium operated as a sort of laboratory for the production of new mathematics. Basing his lectures on whatever research problems engaged him at the moment, the director offered his assistants the opportunity to join with him in creating mathematics. By posing open questions and suggesting possible attacks on difficult points, he coaxed his students into proving new results. Sylvester captured well the cooperative spirit of this mathematical laboratory in his farewell address to the Johns Hopkins University on December 20, 1883. In his words:

> I have written a great deal, and almost every paper I have written in the course of the last seven years, has originated either in the work of the Lecture room, or in private communication with my own pupils; and there are few papers in which their names do not appear. Now I remember a considerable Memoir, which you may say I have the bad taste to entitle "A Constructive Theory of Partitions"—there is no fault to be found in that part of the title, but now comes the objectionable part,— "arranged in three Acts, an Interact and an Exodion".... That paper, extending over 85 pages of the *American Journal of Mathematics*, originated with one of my students .... Mr. Durfee, in response to a question I propounded to him, brought me an answer, in less than 24 hours, founded upon a principle, vast and fertile, due to a method discovered more than 30 years ago, but which remained sterile and abortive until the discovery of Durfee gave it vitality and energy. Except for that method and the improvement made by Durfee, this long paper in three acts, an interact and an exodium [sic] would never have been written.[12]

In this paper which appeared in 1883, the Sylvester school proved that eager American mathematical neophytes responded very favorably to Sylvester's

[12]Remarks of Professor Sylvester, at the Farewell Reception tendered to him by the Johns Hopkins University, December 20, 1883 reported by Arthur S. Hathaway, typescript, p. 20, Gilman Papers. The paper referred to is James Joseph Sylvester, "A Constructive Theory of Partitions, Arranged in Three Acts, an Interact, and an Exodion," *American Journal of Mathematics* 5 (1882): 251–330, or *Math. Papers J.J.S.*, 4: 1–83.

idiosyncratic teaching techniques. In fact, this joint effort presents what George Andrews has termed "monumental" contributions to combinatorics.[13]

As Sylvester's correspondence reveals, he had taken as his conscious goal the task of creating a successful, research-level school of mathematics in America. On May 12, 1881, he wrote to his friend Cayley in England: "I firmly believe that there is a better opportunity for creating a great mathematical school here than exists in England and the young men of the Country are fired with the love of science and seem to me to be especially gifted with a genius for Mathematics which has never before now had a chance of showing itself."[14] With their results, particularly of 1883, Sylvester and his school succeeded in putting America on the "mathematical map." Of Sylvester's students, Thomas Craig did important work on the theory of differential equations culminating in his book, *A Treatise on Linear Differential Equations*; George Bruce Halsted distinguished himself in non-Euclidean geometry as well as in the history of mathematics; Fabian Franklin published a new proof of Euler's pentagonal number theorem in addition to his work on invariant theory; W. Irving Stringham advanced the theory of elliptic and theta functions; William Durfee worked on the theory of symmetric functions and its connections with invariant theory; and George Ely and Oscar Mitchell excelled in number theory. These students put forth their ideas, not only in journals published in the United States, but also in such foreign journals as the *Comptes rendus* of the French Académie des Sciences and the *Journal für die reine und angewandte Mathematik* (*Crelle*). Furthermore, the *American Journal of Mathematics*, where they published most frequently, was widely subscribed to abroad. As Sylvester's correspondence shows, the mathematical results which he and his students published there served to awaken Europe to America's growing mathematical sophistication.[15]

With Sylvester's accomplishments clearly in evidence, the following question naturally arises: how had the direction of American mathematics changed by December 1883 when Sylvester left Johns Hopkins to assume the Savilian Chair of Mathematics at New College, Oxford? The establishment at Hopkins of a graduate school which engaged in properly graduate education, that is, in the training of future researchers, forced other institutions which saw more advanced education as part of their mission, to establish similar

[13]George P. Andrews, *The Theory of Partitions*, Encyclopedia of Mathematics and its Applications, vol. 2 (Reading: Addison Wesley Publishing Co., 1976), p. 14.

[14]James Joseph Sylvester to Arthur Cayley, May 12, 1881, Sylvester Papers, St. John's College, Cambridge, Box 11. We thank the Master and Fellows of St. John's for permission to quote from their archives.

[15]For excerpts from letters to Sylvester from Charles Hermite testifying to this growing esteem, see Parshall, "America's First School of Mathematical Research," pp. 189–190.

schools.[16] With the increase in the number of graduate schools, the level of mathematical research in the United States gradually rose, and American students no longer had to look abroad for training. But American mathematical output increased only gradually. By and large, Sylvester's students failed to transport their research ethic directly to other institutions of higher education around the country. Still, the guiding philosophy of the Johns Hopkins, with its emphasis on graduate training and research, was transferred to extant universities like Harvard, Princeton, and Yale and to newly forming ones such as Clark and Chicago.[17] By the 1890s, a dozen or more American universities could boast able research mathematicians, and by 1910, several of these schools had native-son professors who enjoyed, or would soon enjoy, sustained international reputations.

The ten years that immediately followed Sylvester's departure from Johns Hopkins, however, marked a brief interlude in the training of American mathematicians on American soil. During this period, many students opted to pursue their graduate education abroad. Indeed, they were largely compelled to do so, in view of the fact that the leading universities in the United States were still in a state of transition and not yet equipped to prepare a first generation of productive research mathematicians. The quality and number of American aspirants studying overseas from 1884 to 1894 clearly reflected the mathematical coming of age underway on this side of the Atlantic. As their mentors at this crucial stage of the maturation process, the young itinerants favored the mathematicians of Germany.

In 1904, Thomas Fiske, the founder of the American Mathematical Society, estimated that about twenty percent of the Society's membership had undertaken doctoral or post-doctoral studies in Germany.[18] Impressive as this figure may seem, among the elite mathematicians of the country the percentage of those who studied at one or more of the German universities was even higher than this. Some of these prominent Americans went to Berlin to hear the lectures of Weierstrass, Kronecker, and Fuchs. Others studied with Sophus Lie in Leipzig. Several were drawn to Hilbert in Göttingen, especially after 1900, the year in which he delivered his famous Paris lecture. During the critical period from 1880 to 1895, however, the most popular and influential teacher of American mathematicians was Felix Klein. Klein attracted a handful of prominent Americans while he was in Leipzig from 1880 to

[16]See, for example, Charles Eliot's remarks on behalf of Harvard University in *Johns Hopkins University Celebration of the Twenty-Fifth Anniversary of the Founding of the University and Inauguration of Ira Remsen, LL.D. As President of the University* (Baltimore: The Johns Hopkins Press, 1902).

[17]Laurence R. Veysey, *The Emergence of the American University* (Chicago: University of Chicago Press, 1965), pp. 95–96.

[18]Thomas S. Fiske, "Mathematical Progress in America," *Bulletin of the American Mathematical Society* **11** (1905): 238–246; in Peter Duren et. al., eds. *A Century of Mathematics in America, Part I* (Providence: American Mathematical Society, 1988), pp. 3–12, on p. 5.

1885, but at Göttingen in the following decade, they came to him in droves. No less than six of these former students went on to become presidents of the American Mathematical Society, and thirteen served as its vice president.[19]

Students of Klein Who Served as President of the American Mathematical Society

1. W. F. Osgood (1905–1906)	4. H. B. Fine (1911–1912)
2. H. S. White (1907–1908	5. E. B. Van Vleck (1913–1914)
3. M. Bôcher (1909–1910)	6. V. Snyder (1927–1928)

Students of Klein Who Served as Vice President of the American Mathematical Society

1. H. B. Fine (1892–1893)	8. H. Maschke (1907)
2. H. S. White (1901)	9. E. B. Van Vleck (1909)
3. M. Bôcher (1902)	10. M. W. Haskell (1913)
4. W. F. Osgood (1903)	11. V. Snyder (1916)
5. A. Ziwet (1903)	12. F. N. Cole (1921)
6. O. Bolza (1904)	13. H. W. Tyler (1923)
7. I. Stringham (1906)	

Klein first began to take a serious interest in American mathematics late in 1883 when he was offered Sylvester's chair at Johns Hopkins. In retrospect, it seems likely that Klein would have actually made the move to Baltimore if President Gilman had extended a sufficiently attractive offer.[20] When negotiations failed, however, Gilman lost the chance to secure Klein as Sylvester's successor, and Hopkins soon fell from its preeminent position among American universities in the field of mathematics. In 1889, it also lost the services of the highly respected William Story, a Leipzig Ph.D. and one of the first American mathematicians to study abroad. Story went to newly founded Clark University , whose original faculty included Henry Seeley White, Oskar Bolza, and Henry Taber.[21] Clark not only had more depth than Hopkins, its mathematics faculty was, for a brief time, the strongest in the country.

[19]Here the phrase "Klein's students" means those who studied with him at Leipzig and Göttingen whether or not they wrote their doctoral dissertation under him. For example, Osgood took his Ph.D. under Max Noether at Erlangen, and Cole returned to Harvard for his doctorate. A complete list of the American Mathematical Society presidents and vice presidents up to 1938 can be found in Raymond Clare Archibald, ed., *A Semicentennial History of the American Mathematical Society, 1888–1938*, 2 vols. (New York: American Mathematical Society, 1938) 1: 106–107.

[20]See the documents in Klein Nachlass XXII L:7, "Berufung nach Baltimore," Niedersächsische Staats- und Universitätsbibliothek, Göttingen (hereinafter abbreviated NSUB); and Constance Reid, "The Road Not Taken," *Mathematical Intelligencer* **1** (1978): 21–23.

[21]Roger Cooke and V. Frederick Rickey discuss the Clark University mathematics department in detail in "W. E. Story of Hopkins and Clark," in this volume, pp. 29–76.

Another school that rose to prominence in mathematics during the 1890s was Harvard University. Felix Klein's profound influence on Harvard mathematics can be traced back to his student Frank Nelson Cole, who came to Leipzig on a Parker Fellowship in 1883. When Cole returned to Harvard to complete his degree he took the "new math," that is, group theory and Riemann surfaces, with him. As one of his auditors, William Fogg Osgood, later recalled:

> [Cole] had just returned from Germany and was aglow with the enthusiasm which Felix Klein inspired in his students. Cole was not the first to give formal lectures at Harvard on the theory of functions of a complex variable, Professor James Mills Peirce having lectured on this subject in the seventies. That presentation was, however, solely from the Cauchy standpoint, being founded on the treatise of Briot and Bouquet *Fonctions Elliptiques*. Cole brought home with him the geometric treatment which Klein had given in his noted Leipsic [sic] lectures of the winter of 1881–1882. Cole also gave a course in Modern Higher Algebra, with its applications to geometry. The enthusiasm which he felt for his subject was contagious. Interesting as were the other courses I have mentioned, they stood as the Old over against the New and of the latter Cole was the apostle. The students felt that he had seen a great light. Nearly all the members of the Department attended his lectures. It was the beginning of a new era in graduate education at Harvard, and mathematics has been taught here in that spirit ever since.[22]

About this latter point, Osgood was certainly in a position to know, for as Garrett Birkhoff once remarked, Osgood's "... course on functions of a complex variable remained the key course for Harvard graduate students until World War II."[23] Clearly struck by Cole's lectures, Osgood decided to journey to Göttingen and seek out for himself the "great light" that Cole had seen. One year later, his future Harvard colleague, Maxime Bôcher followed his lead.

The German mathematical experience left lasting impressions on both of these young men. Osgood, who took his inspiration from Klein's approach to function theory, also infused his work with a precision reminiscent of Weierstrass' school. This combination yielded impressive results in 1900 when

[22]William F. Osgood on Cole in Thomas S. Fiske, "Frank Nelson Cole," *Bulletin of the American Mathematical Society* **33** (1927): 773–777 on pp. 773–774.

[23]Garrett Birkhoff, "Some Leaders in American Mathematics: 1891–1941," in Dalton Tarwater, ed., *The Bicentennial Tribute to American Mathematics, 1776–1976* (n.p.: Mathematical Association of America, 1977), pp. 25–78 on p. 34. Birkhoff's emphasis.

Osgood published the first truly rigorous proof of the Riemann mapping theorem. By 1907, he had written over a dozen research papers, a lengthy survey article on function theory for Klein's *Encyklopädie der mathematischen Wissenschaften*, and his own *Lehrbuch der Funktionentheorie*, a work which eventually went through five editions. Like his friend Osgood, Maxime Bôcher also distinguished himself as a mathematician. Bôcher earned his Göttingen Ph.D. in 1891 with a prize-winning dissertation in which he developed certain ideas on Lamé functions presented by Klein during the course of his lectures on the subject. Returning to the United States and a teaching position at Harvard, Bôcher expanded his thesis into the classic volume *Ueber die Reihenentwicklungen der Potentialtheorie* in 1894.[24]

As for Cole, he took his Ph.D. at Harvard in 1886 and stayed on as a lecturer before leaving for an instructorship at the University of Michigan in 1888. While at Michigan, he introduced George A. Miller to what had become his own primary field of expertise, the theory of finite groups.[25] Miller went on to study with the two leading group theorists of the era, Sophus Lie in Leipzig and Camille Jordan in Paris, and during a long career at the University of Illinois, he published over 400 papers on the theory of finite groups.[26] His teacher, Cole, finally left Michigan in 1895 for the professorship at Columbia he would hold until his death in 1926. For twenty-five of his thirty years at Columbia, Cole also served faithfully as the secretary of the American Mathematical Society.

As evidenced by the succession of mathematicians, Cole, Osgood, Bôcher, Harvard clearly functioned as an important focal point for Klein's influence on American mathematics. Another such focus was Princeton. During the summer of 1884, the young Henry Burchard Fine made his way to Leipzig, where Klein had just completed the first half of a two-semester course on elliptic and hyperelliptic functions. In spite of the fact that Fine had missed the first part of the course, Klein advised him to attend the second half even if he could not follow it completely.[27] This was just contrary to the advice he normally gave his new American students. Given their usually woeful state of readiness for advanced mathematics, Klein tended to urge them to start at the beginning and to build from there. But Klein also had a very keen eye for talent, and he sensed in Fine a student equal to the work. Fine did enroll in the course and, restudying his notes after a few weeks had passed,

[24]On Osgood, see Archibald, *Semicentennial of the AMS*, **1**: 153–158; on Bôcher, see William F. Osgood, "The Life and Services of Maxime Bôcher," *Bulletin of the American Mathematical Society* **25** (1919): 337–350.

[25]Archibald, *Semicentennial of the AMS*, **1**: 100–103.

[26]See George A. Miller, *The Collected Works of George Abram Miller*, 3 vols. (Urbana, Ill.: University of Illinois Press, 1935, 1938, 1946).

[27]On Fine, see Oswald Veblen, "Henry Burchard Fine—In Memoriam," *Bulletin of the American Mathematical Society* **35** (1929): 726–730; and Archibald, *Semicentennial of the AMS*, **1**: 167–170.

found that the entire subject was perfectly clear. Fine's beautifully written lecture notes, housed today in the Princeton Archives, testify to the acuteness of Klein's sixth sense. Furthermore, by the end of the 1885–1886 academic year, Fine had completed his doctoral thesis under Klein's supervision on a topic suggested by Eduard Study. This was the first of nine dissertations written by American students under Klein's direction.[28]

AMERICAN DOCTORAL DISSERTATIONS WRITTEN UNDER KLEIN

1. H. B. Fine, "On the singularities of curves of double curvature," Leipzig, 1886.
2. M. W. Haskell, "Ueber die zu der Kurve $\mu^3\nu + \nu^3\lambda + \lambda^3\mu = 0$ im projektiven Sinne gehörende mehrfache Ueberdeckung der Ebene," Göttingen, 1890.
3. M. Bôcher, "Ueber die Reihenentwicklungen der Potentialtheorie," Göttingen, 1891.
4. H. S. White, "Abelsche Integrale auf singularitätenfreien einfach überdeckten, vollständigen Schnittkurven eines beliebig ausgedehnten Raumes," Göttingen, 1891.
5. H. D. Thompson, "Hyperelliptische Schnittsysteme und Zusammenordnung der algebraischen und transzendenten Thetacharakteristiken," Göttingen, 1892.
6. E. B. van Vleck, "Zur Kettenbruchentwicklung Lamścher und ähnlicher Integrale," Göttingen, 1893.
7. F. S. Woods, "Ueber Pseudominimalflächen," Göttingen, 1895.
8. V. Snyder, "Ueber die linearen Komplexe der Lieschen Kugelgeometrie," Göttingen, 1895.
9. M. F. Winston, "Ueber den Hermiteschen Fall der Laméschen Differentialgleichung," Göttingen, 1897.

It was Fine's prompting that brought another Princeton graduate, Henry Dallas Thompson, to Felix Klein in Göttingen. Thompson spent six semesters there and finished with a dissertation dealing with a topic in hyperelliptic functions. He joined the Princeton faculty in 1888 and taught there for over thirty years. Both Thompson and Fine were present in 1896 when Klein was awarded an honorary doctorate at the Princeton sesquicentennial celebration. On this occasion, their former mentor also delivered a series of four lectures on the mathematical analysis of a spinning top.[29]

For many years, Fine guided science at Princeton from his position as Dean of the Science Faculty. Like Cole, he was not a top-flight research

[28]A (nearly) complete list of Klein's Ph.D. students can be found in Felix Klein, *Gesammelte Mathematische Abhandlungen*, 3 vols. (Berlin: Springer-Verlag, 1923), 1: pp. 11–13 (hereinafter cited as *Klein G.M.A.*).

[29]Felix Klein, "The Mathematical Theory of the Top," *Klein G.M.A.*, 2: 618–654.

mathematician, but he nevertheless played an important role in directing this country's mathematical development. During the academic year 1911–1912 he served as president of the American Mathematican Society, and his popular textbooks on algebra and the calculus were considered unsurpassed for their clarity of exposition. Fine made his most lasting accomplishments in his role as an administrator, however. Not only was he an excellent fund-raiser, but he also succeeded in attracting figures like Luther P. Eisenhart, Oswald Veblen, Gilbert A. Bliss, George D. Birkhoff, and Joseph H. M. Wedderburn to Princeton. Largely as a result of his appointments, Princeton became, after 1900, one of the three leading centers for mathematics in the United States, alongside Chicago and Harvard.[30]

These elite institutions did not provide the only sources of mathematical talent in turn-of-the-century America, though. At Wesleyan College, the astronomer and later American Mathematical Society vice president, John Monroe Van Vleck sent three of his undergraduates on to Göttingen: Henry Seeley White, Frederick Shenstone Woods, and his own son, Edward Burr Van Vleck.[31] Each of these students wrote a doctoral dissertation under Felix Klein before returning to teach mathematics in the United States. White, who had originally gone to Leipzig to study under Lie and Study, left there for Göttingen after one semester. On earning his degree, he took a position first at Clark, next at Northwestern, and finally at Vassar. Van Vleck brought his Göttingen degree back to the University of Wisconsin in 1893. Moving on to Wesleyan from 1895 to 1906, he returned to Wisconsin in 1906 and remained there for essentially the rest of his career. He succeeded White, Bôcher, and Fine as president of the American Mathematical Society in 1913. Woods came home to play an important role in upgrading mathematics instruction at his own institution, the Massachusetts Institute of Technology, as well as other technical schools through the widely adopted Woods and Bailey calculus text. His MIT colleague, Harry W. Tyler, who served as an American Mathematical Society vice president, also studied with Klein from 1887–1888 before going on to take his doctorate under Paul Gordan at Erlangen.[32]

Klein's first prominent American student, however, was Washington Irving Stringham, who came to Leipzig in 1880 immediately after taking his doctorate under Sylvester at Hopkins. He arrived at a most opportune time,

[30]In recognition of Fine's many contributions to his alma mater, Fine Hall, the present-day home of Princeton mathematics, was named in his honor. The building at Princeton presently called Fine Hall, however, was constructed many years after Fine's death.

[31]Robert A. Rosenbaum, "There were Giants in those Days: Van Vleck and his Boys," *Wesleyan University Alumnus* (Nov. 1956): 2–3.

[32]On White and Van Vleck, see Archibald, *Semicentennial of the AMS*, **1**: 158–161, 170–173; G. D. Birkhoff, "Edward Burr Van Vleck in Memoriam," *Bulletin of the American Mathematical Society* **50** (1944): 37–41. On Woods, Dirk Struik wrote an unpublished memoir that can be found in the archives of Wesleyan University in Middletown, Connecticut.

as Klein was just beginning a two-semester course on "Funktionentheorie in geometrischer Behandlungsweise." Klein's lectures from the second semester of this course formed the basis for his famous booklet *Ueber Riemann's Theorie der algebraischen Funktionen und ihrer Integrale.* Stringham left Leipzig in 1882 to accept a position at the University of California where he remained for the rest of his career. In 1890, he was joined at Berkeley by another Klein pupil, Mellon Woodman Haskell, who spent more time in Göttingen than any of Klein's other American students. On returning to the United States, Haskell prepared an English translation of Klein's "Erlangen Program," which was published in the second volume of the newly founded *Bulletin of the New York Mathematical Society.* One of Klein's last American students was Virgil Snyder, who went on to become a leading figure in algebraic geometry at Cornell University. Snyder was a fixture at Cornell, where he taught for more than forty years producing thirty-nine doctoral students along the way.[33]

Obviously, Klein could not have enjoyed such striking success with these Americans had he not possessed certain extraordinary qualities as a teacher. Among these were an unusual breadth of knowledge and a quick eye for fertile new ideas, characteristics that made him an unusually effective *Doktorvater.* One need only consider the diverse themes chosen by his students for their doctoral theses, many of which were undertaken as an elaboration of ideas presented by Klein in his lectures. During the late 1880s and early 1890s, Klein focused both on mathematical physics and on a geometric approach to elliptic, hyperelliptic, and Abelian integrals and functions. Since his lectures were highly informal compared to those of most German mathematics professors, the assistants charged with the task of writing them up for circulation in the *Lesezimmer* ended up burning a lot of midnight oil. For Klein's three-semester course on Abelian functions, several Americans lost sleep in a collaborative effort to produce the *Ausarbeitung.*[34]

As Fritz König has pointed out, Klein preferred to illustrate the key motivating principles of a given theory by choosing representative examples rather than by developing a comprehensive presentation of the theory itself. Furthermore, Klein peppered his lectures with numerous references to great nineteenth-century figures whose work was otherwise difficult or impossible for students to understand. He often colored his remarks on Cayley, Lie, Riemann, Plücker, Clebsch, Kronecker, Weierstrass, and others with personal assessments of the individuals and their work. Since such pronouncements were rarely heard in conventional mathematics lectures, those with a thirst for

[33]On Snyder, see Archibald, *Semicentennial of the AMS,* 1: 218–223.

[34]Six students attended the first semester of this course, and five of them, Haskell, Osgood, Thompson, Tyler, and White, were Americans.

a broad, semi-historical approach to mathematical ideas knew where to go. For similar reasons, Klein's seminars also drew respectably sized audiences.[35]

Beginning with the winter semester of 1893–1894, several women also came to Göttingen to study with Felix Klein. The first were Mary "May" Winston, a student from the University of Chicago, and an Englishwoman named Grace Chisholm. Both went on to complete dissertations under Klein, effectively opening the door for foreign women to attend the Prussian universities. (Ironically enough, German women had to wait another fifteen years for this privilege.) Grace Chisholm, who later married the English analyst W. H. Young, was said to have been Klein's favorite student. The letters she wrote home during this time vividly conveyed the excitement she felt as one of the first women to attend classes at a German university and under a great German professor. Consider, for example, her account of the first day of classes:

> Klein had his first lecture on the hypergeometric functions.... Miss Winston and I made for the Sanctum and found Klein there working till lecture time. Klein, instead of beginning with his usual "Gentlemen!" began "Listeners!" ["Meine Zuhörer"] with a quaint smile; he forgot once or twice and dropped into "Gentlemen!" again, but afterwards he corrected himself with another smile. He has the frankest, pleasantest smile and his whole face lights up with it. He spoke very slowly and distinctly and used the blackboard very judiciously. Mr. Woods said he never heard anyone lecture so well and neither have I. I found my notes afterwards perfectly clear though queerly spelt; but I understood as well as at an English lecture.[36]

The following semester, Chisholm described her lecture before Klein's seminar, a daunting experience for any aspiring doctoral student:

> The lecture came off yesterday, and if it is a success to interest one twelfth of one's audience I may be said to have achieved one. As to the other eleven I do not know what they thought about it, but May Winston says they were all quite wide awake, which is something that cannot be said for all the preceding lectures .... It took a little over an hour to deliver and there were a good many interruptions, which is always a good sign. Once... Professor Klein asked for an explanation of certain facts, a thing he is very fond of doing. I had been more frightened than anything of his questions,

[35]Fritz König's remarks "zum didaktischen Vorgehen Kleins" are in Felix Klein, *Funktionentheorie in geometrischer Behandlungsweise*, Teubner-Archiv zur Mathematik, vol. 7 (Leipzig: B. G. Teubner, 1987), pp. 255–256.

[36]As quoted by Ivor Grattan-Guinness in "A Mathematical Union: William Henry and Grace Chisholm Young," *Annals of Science* **29** (1972): 105–183 on p. 123.

> it is so difficult to think on an occasion like that, and although the same thing happens to nearly every one I always think it looks foolish not to be able to answer. The Gods willed on this occasion that my brain should work, and I gave the explanation to my own astonishment, and I fancy, to his too.[37]

May Winston also lectured in Klein's seminar on two separate occasions. In 1894, she spoke on "Die Kugelfunktion als spezielle Fälle der hypergeometrischen Funktion," and the following semester she lectured in a seminar on the foundations of real analysis. Most of the Americans who studied in Göttingen made at least one presentation in Klein's seminar, which was clearly one of the focal points of his teaching activity. Unlike Sylvester's highly improvised laboratory for concocting new ideas, however, Klein preferred a tightly structured setting for exploring a wide variety of mathematical subjects, many of which were far removed from his own research interests. This proved a useful vehicle for introducing students to the vast body of literature that poured from journals like Klein's own *Mathematische Annalen*.[38]

Yet, despite Klein's unprecedented influence on American mathematics, none of his American students developed into a close mathematical disciple by carrying on the distinctively Kleinian geometric approach to function theory and other branches of mathematics. For example, none compares in this regard with his German students, Robert Fricke, Walter von Dyck, Ferdinand Lindemann, or even Arnold Sommerfeld. While Klein's *Gedankenwelt* undeniably inspired nearly all of the dissertations written by his American students, its impact on these young mathematicians proved short-lived. Even where its influence was most striking, as in the cases of Bôcher, White, Snyder, and Haskell, the Americans wandered from their mentor's path after returning to the United States. Relative to his transatlantic students, Klein's influence simply lay more in his ability to train them as research mathematicians than in the specific ideas they researched under his supervision. Indeed, to many Americans, Klein served as an emissary for and a symbol of the rich expanse of mathematical culture, something they very much wanted to transplant to their own country.

Among Klein's many outstanding German students, two actually played a decisive role both in this transplantation and in the emergence of American mathematics. Oskar Bolza and Heinrich Maschke, both former Gymnasium teachers, came to the United States because neither had reasonable prospects for breaking into the German system of higher education. Bolza, whose

[37]*Ibid.*, pp. 123–124.

[38]After a student gave a lecture before the seminar, he or she entered a synopsis of the presentation in a protocol book which Klein kept for each of his seminars over a period of more than forty years. Today, these protocol books may be found in the so-called "Giftschrank" (the "poison cabinet") in the library of the Mathematics Institute in Göttingen.

principal research interests lay in function theory and in the calculus of variations, had a substantial background in physics as well, having studied with Kirchhoff and Helmholtz in Berlin. There, he also came under the influence of Weierstrass, but eventually took his degree in Göttingen under Klein in 1886 with a dissertation on the reduction of hyperelliptic to elliptic integrals. His friend, Maschke, known primarily as a geometer, was actually a very versatile mathematician conversant with practically all major fields of research. Studying first with Koenigsberger in Heidelberg, Maschke spent three years in Berlin before taking his Göttingen doctorate in 1880.[39]

During the academic year 1886–1887, the two friends studied together privately with Felix Klein, who met with them weekly in his home. According to Bolza, "Maschke, ... whose gifts were more in line with Klein's approach, won great and lasting rewards from this year with Klein."[40] As for Bolza himself, the experience proved a near catastrophe. From his point of view, "... Klein's brilliant genius, supported by a wonderful capability for geometric visualization that enabled him to divine the results and his sovereign command of almost every area of mathematics, which provided him with the richest abundance of techniques for handling any task" clashed with his own "... purely analytic gifts, deficient of all fantasy and lying in an entirely different direction."[41] The result was a nearly total breakdown in his confidence. Ironically enough, Klein had gone through just this same sort of crisis three years earlier when he found himself stranded in the wake of Poincaré's genius.[42]

Neither Bolza nor Maschke relished the idea of spending his life teaching mathematics in the secondary schools, but it was Bolza who took the first leap. In so doing, he had Klein's support and the encouragement of his American students, Cole and Haskell. Thus, in April of 1888, Bolza arrived in Baltimore with nothing more than a letter of introduction from Felix Klein to Simon Newcomb. Unlike Sylvester and Cayley, Newcomb was often rather pessimistic about the future of mathematics in the United States. As he wrote Klein, "I never advise a foreign scientific investigator to come to this country, but always tell him that the difficulties in the way of immediate success are the same that a foreigner would encounter in any other country."[43] He went on to say that there was little opportunity to teach higher mathematics: "We have indeed several hundred so-called colleges; but I doubt that ... one half of the professors of mathematics in them could tell what a determinant is. All they want in their professors is an elementary knowledge of the branches

[39]See Bolza's autobiography, *Aus meinem Leben* (München: Verlag Ernst Reinhardt, 1936); on Maschke, see O. Bolza, "Heinrich Maschke: His Life and Work," *Bulletin of the American Mathematical Society* **15** (1908): 85–95.

[40]Bolza, *Aus meinem Leben*, p. 18.

[41]*Ibid.*

[42]See Jeremy Gray, *Linear Differential Equations and Group Theory from Riemann to Poincaré* (Boston: Birkhäuser, 1986), pp. 273–309.

[43]Simon Newcomb to Felix Klein, April 23, 1888, Klein Nachlass XI, NSUB.

they teach and the practical ability to manage a class of boys, among whom many will be unruly."[44] Thus, Bolza counted himself lucky when Newcomb supported his appointment as "Reader in Mathematics" at Johns Hopkins. In January 1889, he taught a one-semester course there on substitution theory, relying on notes from one of Klein's lectures on the subject. He followed his short stint at Hopkins with a three-year associateship at Clark University, which was about to open for instruction in the fall of 1889.

Unfortunately, the situation at Clark rapidly deteriorated during the three years Bolza spent there, a circumstance he attributed primarily to politics rather than to financial difficulties. In a letter to Klein, he described how President G. Stanley Hall had embittered the faculty with his "endless lies."[45] Yet, in all fairness, Hall was in an impossible situation. At this time, presidents at most other leading universities held nearly complete control over the procurement and disbursement of their institution's funds. In his role as benefactor, however, Jonas Clark made sure that he had Hall's hands tied relative to finances. Unfortunately, Mr. Clark apparently thought that running a university was little different from running a business firm. In the end, his frugal business sense more than Hall's incompetence, caused the university's undoing. In January 1892, all but two of the school's faculty members signed a document in which they collectively tendered their resignation. Although this was eventually withdrawn, discontent continued to rule the campus. Seizing this opportunity, William Rainey Harper, president of the newly founded University of Chicago, raided the Clark campus and offered its faculty the chance to abandon their sinking ship for his new luxury liner backed by Rockefeller money. Not surprisingly, his pitch worked, and he eventually walked away with most of Clark's outstanding scholars, including the physicist A. A. Michelson, the anthropologist Franz Boas, and the mathematician Oskar Bolza.[46]

Like Hall at Clark, Harper was also interested in hiring prominent German scholars whenever he could. Shortly before the job of putting together a faculty at Chicago began, Heinrich Maschke had finally followed his friend, Bolza, to the United States and had taken a job as an electrician for the Weston Electrical Company in Newark. Prior to Maschke's departure, Klein had predicted that, like Odysseus, after many wanderings he would end up in Ithaca (i.e., at Cornell).[47] As it turned out, Maschke did even better. Bolza managed to negotiate a position for them both at Chicago. Thus, when the University of Chicago opened its doors in the fall 1892, two of its three mathematicians were students of Felix Klein.

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[44] *Ibid.*

[45] Oskar Bolza to Felix Klein, May 15, 1892, Klein Nachlass VIII, NSUB.

[46] The relationship between Hall and Clark is chronicled in Orwin Rush, ed., *Letters of G. Stanley Hall to Jonas Gilman Clark* (Worcester, Mass.: Clark University Library, 1948).

[47] Heinrich Maschke to Felix Klein, July 5, 1892, Klein Nachlass X, NSUB.

Even with the founding of universities like Clark and Chicago, American mathematicians continued to study in Göttingen. Although they went in ever decreasing numbers through the 1920s, Americans such as Earle R. Hedrick, Max Mason, Charles Noble, and William D. Cairns went to Göttingen to study not under Klein but under the then reigning star, David Hilbert. Hilbert's arrival there in 1895 allowed Klein a free hand to pursue the various organizational and administrative projects he had long had in view. In fact, during his visit to Chicago in 1893, Klein already sensed that American mathematics was about to enter a new era. In the closing remarks of his Evanston Colloquium lectures he suggested that it was time for him to relinquish his role as the premier teacher of American mathematicians:

> ... I do not regard it as at all desirable that all students should confine their mathematical studies to my courses or even to Göttingen. On the contrary, it seems to me far preferable that the majority of the students attach themselves to other mathematicians for certain special lines of work. My lectures may then serve to form the wider background on which these special lectures are projected. It is in this way, I believe, that my lectures will prove of the greatest benefit.[48]

Even as Klein spoke, Eliakim Hastings Moore and his colleagues at the newly founded University of Chicago stood ready to assume the responsibility of educating American mathematicians.

E. H. Moore was born in 1862 in Marietta, Ohio, a small town on the Ohio-West Virginia border.[49] At the age of seventeen, he had and took the opportunity to go to Yale where he fell under the influence of the mathematician-astronomer, Hubert Anson Newton. In 1883, the year Sylvester left Hopkins, Moore received the A.B. degree as valedictorian of his class and earned his Ph.D. in mathematics two years later under Newton for a thesis on the algebra of n-dimensional geometry.[50] Realizing that his student had advanced as far as an American education at the time allowed, Newton encouraged Moore to continue his studies in Germany.

[48]Felix Klein, *The Evanston Colloquium: Lectures on Mathematics* (New York: Macmillan, 1894), p. 98.

[49]On the details of E. H. Moore's life, see Gilbert A. Bliss, "Eliakim Hastings Moore," *Bulletin of the American Mathematical Society*, 2d. ser., **39** (1933): 831–838. On E. H. Moore at the University of Chicago, see Karen Hunger Parshall, "Eliakim Hastings Moore and the Founding of a Mathematical Community in America, 1892–1902," *Annals of Science* **41** (1984): 313–333; reprinted in Peter Duren et al., eds., *A Century of Mathematics in America, Part II* (Providence: American Mathematical Society, 1988), pp. 155–175.

[50]E. H. Moore, "Extensions of Certain Theorems of Clifford and Cayley in the Geometry of n Dimensions," *Transactions of the Connecticut Academy of Arts and Sciences* 7 (1885): 9–26.

Traveling to Göttingen in the summer of 1885, Moore spent one semester there studying German and mathematics before moving on to Berlin for the winter of 1886. In Berlin, he fell under the influence not only of Karl Weierstrass but also of Leopold Kronecker before returning to the United States to begin his career at the end of the summer. After serving first as a high school instructor and then as a tutor at his alma mater, Moore held his first permanent university job at Northwestern in 1888. Soon thereafter, though, he had a much more attractive option to consider.

Harper, the president-elect of the University of Chicago, approached Moore with an offer of a full professorship and the acting headship of the Department of Mathematics at his new university.[51] After relatively painless negotiations, Moore accepted the position and made the short move from Evanston to Hyde Park. As with the choice of Sylvester at Hopkins, the selection of Moore at Chicago benefited the university as well as American mathematics. During his forty years on the faculty there, Moore not only succeeded in building a first rate department but also proved instrumental in organizing a self-sustaining American mathematical community.

When the University of Chicago opened in the fall of 1892, E. H. Moore and his two colleagues, Oskar Bolza and Heinrich Maschke, began their instruction of mathematics at both the graduate and undergraduate levels. A priori, it was not at all clear that these three men would be able to work together as a like-minded, mathematical team. Reflecting back on the situation many years later, Bolza explained that Moore ". . . was almost five years younger that I, even more than eight years younger than Maschke and was at that time little known. In addition to that, Maschke and I were foreigners who for many years had been close friends and who had lived in the absolute freedom of the German university. All of these were factors which could have risked the inner peace of the department."[52] Could have, but did not risk that all-important inner peace, for by all accounts, these three mathematicians complemented one another perfectly as teachers and as scholars.[53] In fact, the first evidence of their ability to work together successfully came very early on in their association and centered on the World's Columbian Exposition.

Held in Chicago in 1893 to commemorate the four-hundredth anniversary of the discovery of America, the Columbian Exposition involved, in addition to the displays, amusements, and cultural activities associated with a world's

[51]Richard J. Storr has chronicled the founding of the University of Chicago in *A History of the University of Chicago: Harper's University The Beginnings* (Chicago: University of Chicago Press, 1966).

[52]Bolza, *Aus Meinem Leben*, p. 26.

[53]See, for example, Bliss' remarks in "Eliakim Hastings Moore," p. 833.

fair, a series of congresses which reflected the then current intellectual endeavors of the world.[54] Relative to mathematics, Moore organized a committee consisting of Bolza, Maschke, Henry Seeley White, and himself which extended invitations on behalf of the Congress to mathematicians from the United States and Europe. The venture proved quite successful, attracting forty-five mathematicians from Austria, Germany, Italy, and nineteen states of the Union, as well as contributed papers from mathematical representatives of France, Russia, and Switzerland. Furthermore, Felix Klein, who had longed to lecture in the United States ever since the negotiations over Sylvester's chair at Hopkins failed, readily accepted the invitation to participate in the Congress as the keynote speaker. Considering the fact that three of the members of the organizing committee, Bolza, Maschke, and White, had studied under Klein in Germany, this was an obvious choice. Yet, it also underscored the enormous debt that American mathematics owed to Germany. The American participation at all levels of the Congress proved, however, that mathematics in this country was beginning to stand on its own two feet.

After the formal close of the Congress, Moore and his Chicago colleagues took further advantage of Klein's presence in the United States by attending the Evanston Colloquium lectures. Hosted by Henry Seeley White, by then at Northwestern, Klein gave a two-week-long series of special lectures to roughly two dozen auditors before returning to Germany. These lectures, which appeared in print in 1894, served as the prototype for what would become the *American Mathematical Society Colloquium Publications.*[55]

With their organizational and mathematical appetites whetted by the success of both the Congress and the colloquium, Moore and his friends next approached the New York Mathematical Society for money toward the publication of the papers read at the Congress. Writing almost fifty years later, Raymond C. Archibald viewed this as a "... major publication enterprise, transcending local considerations and sentiment [which] quickened the desire of the Society for a name indicative of its national or continental character."[56] Owing largely to the promptings of E. H. Moore and his colleagues, the New York Mathematical Society met as the *American* Mathematical Society on July 1, 1894.

Despite its nominal nationalization, though, the Society continued to meet monthly in New York to the virtual exclusion of all but those living in the Northeast. By 1896, Moore and his associates at Chicago had figured out a

[54]On the history of the Chicago's World's Fair, see Reid Badger, *The Great American Fair: The World's Columbian Exposition and American Culture* (Chicago: University of Chicago Press, 1979).

[55]See footnote 48 above.

[56]Archibald, ed., *Semicentennial of the AMS,* **1**: 7.

way to insure the mathematical vitality of the Midwest region as well. In December of that year, Moore mailed an invitation to mathematicians as far west as Kansas and Nebraska and as far east as Ohio to come to Chicago on December 31, 1896 to discuss the possible formation of a "Chicago Section" of the Society. As conceived by Moore, a formally sanctioned Chicago Section would provide not only a vehicle for the official and regular involvement of Midwesterners in the activities of the Society but also an alternative power base in Chicago for the organization. Taking the enthusiastic response to his call to Chicago before the Society early in 1897, Moore succeeded in winning approval for his idea, and the Chicago Section convened for the first time on April 24, 1897.[57]

With this goal achieved, Moore next turned his attentions to the improvement of the printed dissemination of mathematics. Like Sylvester before him, he became involved in the movement to found a new mathematics journal. Prior to 1899, the American mathematical community already supported the *American Journal of Mathematics*, the *Annals of Mathematics* (founded by the astronomer, Ormond Stone at the University of Virginia in 1884), and the *Bulletin of the American Mathematical Society* (begun in 1891 as the *Bulletin of the New York Mathematical Society*). Yet Moore and others sensed the need for a periodical which stressed not only research at a high level but also the work of American contributors. In short, they wanted a journal which showcased *American* mathematics.[58] In 1899, this goal also became a reality when the American Mathematical Society founded its *Transactions* and appointed Moore as the editor-in-chief.[59]

Moore's ascension to the editorship of the *Transactions* underscored his growing political influence within American mathematics. In 1899, he was already serving out a two-year term as vice president of the Society, and in 1900, the membership elected him to its presidency.[60] Moore used his national post to champion the cause of mathematics education at all levels of the curriculum. Like his colleague, John Dewey, he argued for a more active, hands-on approach to mathematics teaching and tried to implement

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[57] For the history of the Chicago Section, see Arnold Dresden, "A Report on the Scientific Work of the Chicago Section, 1897-1922," *Bulletin of the American Mathematical Society* **28** (1922): 303-307.

[58] According to Moore and many of his contemporaries, the *American Journal*, under the editorship first of Sylvester and then of Simon Newcomb, favored contributions from mathematicians abroad to the exclusion of papers by Americans. The *Annals of Mathematics* had too much of a popular, non-research-oriented flavor, and the *Bulletin* targeted expository and historical work as opposed to research-level mathematics.

[59] On the controversy surrounding the establishment of the *Transactions*, see Archibald, ed., *Semicentennial of the AMS*, 1: 56-59.

[60] Of the first six presidents of the Society, Moore was the only one based in the Midwest and not in the Northeast.

such ideas in his own department at Chicago. One manifestation of this educational progressivism was the Mathematical Club founded in 1892.[61]

Unlike Sylvester's "Mathematical Seminarium," the Mathematical Club functioned as a forum for the presentation of completed research. Graduate students and faculty alike lectured on their current work before the group and answered both questions and criticisms. As Gilbert A. Bliss, one of the early students at Chicago, described it:

> Those of us who were students in those early years remember well the tensely alert interest of these three men [Moore, Bolza, and Maschke] in the papers which they themselves and others read before the Club. They were enthusiasts devoted to the study of mathematics, and aggressively acquainted with the activities of the mathematicians in a wide variety of domains. The speaker before the Club knew well that the excellence of his paper would be fully appreciated, but also that its weaknesses would be discovered and thoroughly discussed. Mathematics, as accurate as our powers of logic permit us to make it, came first in the minds of these leaders in the youthful department at Chicago,....[62]

With its goal of encouraging and promoting the highest standards of research and exposition, the club served as the training and proving ground of a second generation of American mathematicians.

Among this second generation, thirty students earned their Ph.D.'s under Moore's guidance. During Chicago's first fifteen years, Moore's mathematical interests ranged from group theory to the foundations of geometry to the foundations of analysis, and his students' work reflected not only this diversity but also their mentor's insights. Between 1896 and 1907, in fact, the list of Moore's students reads like a *Who's Who* in early twentieth-century mathematics.[63] The algebraist Leonard E. Dickson, the geometer Oswald Veblen, the analyst George D. Birkhoff, and the topologist Robert L. Moore, each grew up on E. H. Moore's brand of mathematical thinking and matured into independent-minded mathematicians who made seminal contributions to their respective fields as well as to the body politic.[64] Together, these four mathematicians published thirty books and over six hundred papers in addition to directing the research of almost two hundred Ph.D.'s. They each

[61]The logbooks of the Mathematical Club from its beginnings through the 1950s are housed in the Department of Special Collections, Joseph Regenstein Library, University of Chicago. In the earlier volumes (prior to 1900), the speaker's name as well as the date and title of his or her talk are accompanied by a short synopsis of the results presented.

[62]Bliss, p. 833. This was also quoted in Parshall, "E. H. Moore and the Founding of a Mathematical Community in America," pp. 329–330.

[63]For a complete list of Moore's students, see Bliss, p. 834.

[64]The statistics which follow were originally presented in Parshall, "E. H. Moore and the Founding of a Mathematical Community in America," pp. 330–332.

also edited major journals, served as Society president, and won election to the National Academy of Sciences. Finally, like their mathematical father, they built or maintained premier departments at their respective institutions with Dickson at Chicago, Veblen at Princeton and later at the Institute for Advanced Study, Birkhoff at Harvard, and R. L. Moore at the University of Texas at Austin.

Doctoral Dissertations Written under Moore at Chicago 1896–1907

1. Leonard Eugene Dickson, "The analytic representation of substitutions on a power of a prime number of letters; with a discussion of the linear group," 1896.
2. Herbert Ellsworth Slaught, "The cross ratio group of 120 quadratic Cremona transformations of the plane," 1898.
3. Derrick Norman Lehmer, "Asymptotic evaluation of certain totient sums," 1900.
4. William Findlay, "The Sylow subgroups of the symmetric group on *K* letters," 1901.
5. Oswald Veblen, "A system of axioms for geometry," 1903.
6. Thomas Emory McKinney, "Concerning a certain type of continued fractions depending upon a variable parameter," 1905.
7. Robert Lee Moore, "Sets of metrical hypotheses for geometry," 1905 (under the direction of E. H. Moore and O. Veblen).
8. George David Birkhoff, "Asymptotic properties of certain ordinary differential equations with applications to boundary value and expansion problems," 1907.
9. Nels J. Lennes, "Curves in non-metrical analysis situs, with applications to the calculus of variations and differential equations," 1907.

Why did Moore's students succeed where Sylvester's students had failed? While Sylvester proved that American students had the talent to extend the frontiers of at least certain areas of mathematical research, his idiosyncratic teaching style forced them into narrowly focused topics which soon ran dry mathematically. Furthermore, there was no well-established mathematical community in the America of the early 1880s to support their continued development. Without both this broader community and the strong personality of Sylvester to sustain it, Sylvester's school collapsed. Unable to go out and set up graduate-level programs, his students failed to maintain a tradition of training American mathematicians on American soil.

With no viable options for them at home, Americans turned to Europe, and particularly to Felix Klein in Germany, for their mathematical inspiration between 1884 and 1894. During these ten years, Klein willingly accepted the responsibility for the mathematical future of the United States but came to sense that he was playing only an interim role.

By the nineteenth century's close, American universities had made definite, serious, and long-term commitments to graduate education and to the fostering of basic research. With mathematics as the case in point, there were jobs for new Ph.D.'s at institutions which encouraged and nurtured their further growth as mathematicians. Furthermore, through the organizational efforts of Moore, Klein's students, and others, the small enclaves of mathematical research growing in scattered locations like Chicago, New York, Boston, Princeton, Baltimore, Berkeley, and Austin, were unified under the aegis of an even broader support system, the American Mathematical Society. To a large extent, the spectacular developments which took place from Sylvester's arrival in Baltimore in 1876, through Klein's tutelage in the 1880s and 1890s, to Moore's dominance at Chicago by 1900, paved the way for the mathematical preeminence America would come to enjoy in the twentieth century.

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W. E. Story of Hopkins and Clark

ROGER COOKE AND V. FREDERICK RICKEY

INTRODUCTION

The career of W. E. Story (1850–1930) is intimately bound up with the first period (1875–1920) of institutionalized American mathematical research. Until after the Civil War, professors of mathematics in America generally attempted only to understand and transmit to their students the mathematics of previous generations. They rarely engaged in mathematical research, partly because their universities did not foster such activity. It was only during the general cultural expansion immediately following the Civil War that a few Americans began to study mathematics at European universities and some American universities began to offer graduate degrees in mathematics. The establishment of graduate programs at Hopkins, Clark, and Chicago is the clearest sign of a mathematical awakening in America. Although the program at Clark is the least known of these three, it was the leading light of institutionalized American mathematical research in the early 1890s. It also formed a transition between the program at Hopkins, which blossomed during J. J. Sylvester's tenure from 1876 to 1883, and that at Chicago, which developed rapidly in the mid-1890s.

An important figure in America's late-nineteenth-century emergence from the mathematical backwaters was William Edward Story. He graduated from Harvard, earned a Ph.D. in Germany, conducted mathematical research as a faculty member at Hopkins, and developed the graduate program at Clark. Thus not only was Story a central actor in the development of American mathematics, but also his career was a microcosm of the new mathematical activity. These are some of the reasons his biography provides an ideal basis for discussing the mathematical climate of the time. To emphasize the changes in that climate, it is appropriate to begin with his intellectual forbearers, who represent an earlier, less institutionalized phase of mathematical activity.[1]

1. Story's Intellectual Background

Benjamin Peirce (1809–1880), the first great American mathematician, was professor of mathematics at Harvard for nearly fifty years, from 1831 until his death in 1880, but there were only two periods when he had many advanced mathematical students. The first was during the 1850s and early 1860s when the *American Ephemeris and Nautical Almanac* office was located in Cambridge (1849–1867). One member of this group was Charles W. Eliot (1834–1926), who earned his A.B. in 1856 and A.M. in 1858 and then stayed on for three years as tutor in mathematics. In this capacity he helped Peirce introduce written final exams. One of the objections to this reasonable sounding proposal was faculty concern about the students: "more than half of them can barely write; of course they can't pass written examinations" [Flexner 1930a, p. 86]. Eliot also taught chemistry at Harvard (1858–1863) and MIT (1865–1869) before becoming president of Harvard in 1869. Up to this time, most colleges had a lock-step curriculum, but Eliot instituted the free elective system. This system allowed weak students to avoid mathematics and strong students to take as much as they wanted. Peirce was a strong advocate of this system, for it allowed him to devote his energy to the good students. This policy brought Peirce another group of advanced students in the last decade of his life [Anonymous 1911a, p. 7].

The Harvard class of 1871 consisted of 158 graduates, three of whom became mathematicians. Henry Nathan Wheeler (1850–1905) was the author

[1]The authors would like to express their sincere thanks to Dr. Stuart Campbell, University Archivist at Clark University, for his most gracious help with using the archives. Both he and University Historian, Dr. William A. Koelsch, were generous in sharing their knowledge of the early history of Clark University. We would also like to thank Mrs. Cynthia Requardt of Johns Hopkins University for archival assistance. We thank the Milton S. Eisenhower Library at The Johns Hopkins University for permission to publish the letters quoted in §§2–3 below from the Daniel Coit Gilman papers, MS. 1, Special Collections, and cited as "Gilman papers." Finally, we thank the Clark University Archives, Clark University, Worcester, MA, for permission to publish the letters and documents quoted in §§5–10 and for permission to publish the photographs which appear herein.

of half a dozen elementary mathematics texts. He served as proctor and instructor at Harvard until 1882 when he took charge of the educational department at Houghton Mifflin Publishing Company [Anonymous 1921a, pp. 187–188]. William Elwood Byerly (1849–1935) continued his education at Harvard, earning his Ph.D. in 1873 (Harvard's first two Ph.D.s were in this year) with a dissertation on "The Heat of the Sun." After teaching at Cornell for three years, he returned to Harvard where he taught until his retirement in 1913. Byerly was an exceptional teacher and administrator and an early advocate of higher education for women. Of his six textbooks, those on the calculus are noteworthy for initiating the long lists of exercises that are so common today. From 1899 until 1911, Byerly was one of the editors of the *Annals of Mathematics*, which had been founded by Ormond Stone (1847–1933) at the University of Virginia in 1884.

The third mathematician from the class of 1871 was William Edward Story (1850–1930), the main character of our story. Born in Boston on 29 April 1850, Story was the eldest son of Isaac and Elizabeth Bowen Woodberry Story and a descendant of Elisha Story, who came from England about 1700 and settled in Boston. His ancestors included Dr. Elisha Story of Bunker Hill, one of the "Indians" at the Boston Tea Party, and a great uncle, Joseph Story, who was a Supreme Court justice. He was "fitted for College" at the high school in Somerville Massachusetts, where his father was a lawyer [Anonymous 1921a, p. 163].

In the fall of 1867, he entered Harvard College, where he took advantage of Eliot's new elective program and "took all the courses in mathematics then given" including one on elliptic functions and another on the *Theoria Motus* of Gauss (Story to Gilman, 29 July 1876; Gilman papers). Story graduated "with Honors in Mathematics, (being the only graduate who has as yet complied with the requisites for those honors since their establishment in 1870–71)" (J. M. Peirce to Gilman, 4 July 1876; Gilman papers).

In contrast to his classmate Byerly, Story chose to go to Germany for further study. Although a thin stream of students had been going abroad for sixty years, he was ahead of the flood. For the next two and one-half years (September 1871–January 1874), Story studied mathematics and physics at Berlin and Leipzig. He returned home for the spring and summer of 1874 before going back to Germany on a Parker fellowship in October 1874. These fellowships, offered only to Harvard graduates, were designed to encourage study abroad.

Story was one of the first American mathematicians to take a degree at a German university, receiving his Ph.D. at Leipzig on 31 July 1875 for a dissertation entitled *On the Algebraic Relations Existing Between the Polars of a Binary Quantic.* James Mills Peirce (1834–1906) called this "a most masterly treatment, involving considerable originality, of a very abstruse & important subject of the modern 'Higher Algebra' or Theory of Quantics" (to

William E. Story at his desk, 1892–1893. (Clark University Archives)

Gilman, 4 July 1876; Gilman papers). Remembering that J. M. Peirce was a great teacher but not a creative scholar, one might not put much stock in this evaluation; however, Peirce remarked that his father, Benjamin, concurred in the judgment.

According to the Vita in his Ph.D. dissertation, Story attended the lectures of Weierstrass, Kummer, Helmholtz, and Dove in Berlin, and Neumann, Bruhns, Mayer, Von der Mühll, and Engelmann in Leipzig. We have been unable to determine who directed his dissertation, but suspect it was Karl Neumann (1832–1925). It was not Felix Klein as Reid [1978a, p. 21] surmises; we have no evidence of any contact between Story and Klein during Story's student days.

After earning his degree, Story returned to Harvard, where he served as tutor from September 1875 until July 1876.

2. The Johns Hopkins University

On the day before Christmas in 1873, the Baltimore bachelor financier Johns Hopkins died, leaving his entire fortune of seven million dollars to found a university and hospital. While planning the university, the trustees sought, and received, considerable guidance from three university presidents, all of whom had been trained as scientists: Charles William Eliot, president of Harvard from 1869 to 1909, James Burrill Angell, president of Michigan from 1871 to 1909, and Andrew Dixon White, president of Cornell from 1866 to 1885 [Hawkins 1960a, p. 9]. It was decided to interview Daniel Coit Gilman (1831–1908), who was then serving as the first president of the University of California. Gilman quickly made it clear that he wanted to found a university of national scope which promoted advanced scholarship and the training of graduate students [Hawkins 1960a, p. 22]. Since the trustees were already inclined in this direction, they agreed with him and offered him the position. Gilman quickly accepted the invitation to be the first president of Johns Hopkins University.

Gilman told the trustees that if they could hire a great classicist and an outstanding mathematician, everything else would take care of itself [Flexner 1946a, p. 29]. He hired classicist Basil Gildersleeve (1831–1924) and mathematician James Joseph Sylvester (1814–1897), and things did take care of themselves. Sylvester did not come easily or cheaply, but once the complicated negotiations were completed, he was most enthusiastic about "our university." It took $6,000 in gold to get him, a handsome salary considering that Yale's highest salary was then $3,500, Harvard's $4,000, and these were unusually high [Hawkins 1960a, pp. 42–43].

Sylvester arrived in Baltimore in May of 1876, but left again almost immediately for New York City to look for "his most precious box—the one containing his life's work in manuscripts" [Hawkins 1960a, p. 44], which had been lost in transit. As he had found the heat unbearable, he continued North to Harvard to visit his old friend Benjamin Peirce, who had been his host in 1842 and 1843 after Sylvester spent a few months at the University of Virginia. Perhaps it should be added—to quell persistent rumors—that Sylvester did not quit that post because he killed a student [Feuer 1984a].

The Peirces—both Benjamin and his son James Mills—independently recommended that Sylvester hire Story as an assistant professor. Story's dissertation impressed Sylvester, and so he promised to try to meet him. Sylvester had also asked about Story's classmate, Byerly, so J. M. Peirce wrote to President Gilman of Hopkins that Byerly

> is a man of great ability & character, a good mathematician, an assiduous worker, & would be an accession to any university in the country. I told Mr. Sylvester however that I thought Dr. Story would be an even better man for you ... [4 July 1876; Gilman papers]

After describing Story's background, calling him a "mathematician of great promise," indicating that they would hate to lose him yet felt they could not hold him back, and singing his praises for several pages, J. M. Peirce adds:

> My Father wishes me to say that he fully concurs in my opinion of Dr. Story.... We both think him the most promising mathematician that has been produced here for many years, & likely to hold a distinguished position among the Scientific men of America. He is by no means a mere teacher. [Gilman papers]

But Sylvester continued to complain of the heat and "depression"—it plagued him every summer in America—and so decided to return to England for a holiday before classes began at Hopkins. Sylvester's departure left it up to Gilman to negotiate with Story for the position as Sylvester's assistant. Gilman telegraphed an offer to Story, adding:

> If you desire light work and a good place in which to study I think you will find the place of an Associate ... honorable and advantageous. [Hawkins 1960a, p. 44]

Not surprisingly, Story found this a bit condescending, and so Gilman quickly learned that younger mathematicians can be difficult to deal with too. Story replied "as distinctly as possible" that he wished

> to devote my leisure time to original work as a mathematician, not merely as a student. I do not therefore lay so much stress upon having much leisure, but the high character of the work which seems to be demanded at Baltimore is a greater object with me.... I know what work is, and have no objection to it. [Story to Gilman, 29 July 1876; Gilman papers]

Story ended the letter by expressing the desire for an interview. He wanted to explain to Gilman his plans for a mathematical journal and a student mathematical society. He also tried for a better position at Harvard—remember he was only a tutor—but when nothing materialized he accepted the position at Hopkins [Hawkins 1960a, p. 45].

In the fall of 1876, William Story became "associate" at Hopkins. He was the only other faculty member in mathematics at Hopkins besides Sylvester. The title was equivalent to that of assistant professor elsewhere although the rank was not created at Hopkins until 1945. In 1883, when the new rank of "Associate Professor" was created at Hopkins, Story was promoted to that rank.

There is evidence that Story succeeded in founding his student mathematical society. *The Johns Hopkins University Circulars*, which are a rich source of information about the university, contain titles and reports of the talks given at the monthly meetings of the "Mathematical Society." From one of these we learn that when Lord Kelvin lectured at Hopkins in 1884, he spoke to a group of mathematicians who called themselves "the coefficients" [Gilman 1906a, p. 75].

3. The American Journal of Mathematics

On 3 November 1876, only a few weeks after classes began at Hopkins, President Gilman held a dinner in honor of Sylvester. Probably Gilman saw to it that Story's idea of a mathematics journal "emerged," for on 8 November 1876 a crudely duplicated letter was sent out proposing "The American Journal of Pure and Applied Mathematics" (see [French 1946a, pp. 51–52] for the text). The proposed title was doubtless influenced by the British *Quarterly Journal of Pure and Applied Mathematics*, which Sylvester had edited since he and Ferrers founded it in 1855 to replace the *Cambridge and Dublin Mathematical Journal*. The letter was signed by Sylvester, Story, Rowland, and Newcomb.[2] It elicited more than forty responses, all but one favorable. Most promised to subscribe and many offered suggestions. The suggestion of Joseph Henry that the journal be an instrument for education as well as research was, fortunately, ignored. The proposed new journal also aroused interest in the popular press.

[2]The physicist Henry A. Rowland (1848–1901) was the first faculty member and full professor hired by Gilman, whose interest had been piqued when he learned that the *American Journal of Science* had thrice rejected Rowland's papers because of his youth. Today Rowland is remembered for work he did in the 1880s: the invention and ruling of concave spectral gratings to accurately measure wavelengths of light. The mathematical astronomer Simon Newcomb (1835–1909) was associated with Hopkins from its beginnings, first as a visiting lecturer and later as Sylvester's replacement. Although essentially self-educated, Newcomb did study with Benjamin Peirce, getting a degree at the Lawrence Scientific School at Harvard in 1858.

As might be expected of any new journal, the *American Journal of Mathematics* had its initial difficulties. Gilman could not find a publisher to assume ownership, and the trustees of Hopkins refused to take on the burden, although they did provide $500 per volume, or about a fifth of the cost [Hawkins 1960a, p. 75]. This explains why the title page of the new journal proclaimed that it was "issued under the auspices of Johns Hopkins." Sylvester was wise enough to realize that the financial and managerial details of the journal were not his forte and would take time away from his research, so Story was appointed "associate editor in charge."

On 17 March 1878, Sylvester invited Benjamin Peirce to Baltimore "to dine with us and some of the supporters of the Mathematical Journal to celebrate its birth which is now daily expected and which you have done much to promote" [Archibald 1936a, p. 139]. Although the first issue, dated "January 1878" did not appear until at least March of that year [Archibald 1936a, p. 136], Sylvester realized that "Story is a most careful managing editor and a most valuable man to the University in all respects and an honor to the University and its teachers from whom he received his initiation" [Archibald 1936a, p. 139]. Publication deadlines are the scourge of all editors, and Sylvester was no exception. Two years later, on 25 March 1880, he wrote Mrs. Benjamin Peirce that "Our December number of the Journal [vol. 2, no. 4] still tarries in coming out," but, rather than being disturbed by the delay, he is delighted with the issue itself. He continues:

> It will be a glorious number and two contributions from [your son] Charles [Sanders Peirce (1839–1914)] ... will form not the least interesting part of its contents. It opens with Tables of Invariants and concludes with two dissertations on the 15 puzzle [of Kirkman]. So you see we take a wide range. But I tell Dr. Story that the 15 puzzle will be the gem of the number and help to make the other matter go down. [Archibald 1936a, pp. 144–145]

These papers, by W. W. Johnson and Story, were the first to show the impossibility of certain arrangements of the sliding blocks in this puzzle which "was engrossing the minds of millions of people" and is still familiar today to Macintosh users. This paper became part of Story's popular fame. As his obituary states:

> Dr. Story was deeply interested in all kinds of puzzles. His mathematical mind and profound knowledge combined with practice to make him a great expert. Few problems of this description baffled him, no matter how difficult they might be. [*Worcester Evening Gazette*, 10 April 1930]

While there had been some early editorial disagreements between Sylvester and Story [Archibald 1936a, p. 137], matters came to a head with the January

1880 number of Volume 3. Sylvester sailed for England in the late spring, as was his custom throughout his stay at Hopkins, and as he had done the previous year, he left Story in charge, with instructions about how he wanted the issue put together.

In early June, Sylvester wrote Gilman inquiring why he had not received an acknowledgment of a paper he sent Story [Fisch and Cope 1952a, p. 358]. Then, on 22 July 1880, Sylvester sent Gilman an eight-page letter in which his indignation is clearly manifested by his heavy, nearly illegible penstrokes:

> I have sent off a telegram to you this morning requesting to be informed when "Journal did or will appear." A telegram sent to Story a week or two ago has met with no response. His answer by letter to my message through you was utterly unsatisfactory.
>
> He gave no explanation worthy of the name why I had to wait for 8 or 9 weeks before receiving an acknowledgement which I had requested of a communication for the Journal sent from [illegible] on my arrival there. If he treats me in this way how is he likely to act towards other contributors?
>
> He informs me that he has allowed Rowland to exceed the limits of the Journal by 20 pages in flat disobedience to my directions and without referring the matter to me for my opinion and in the face of the fact known to him that I had risked giving offense [to] C. S. Peirce by requesting him (which he complied with) to abridge his most valuable memoir in order that the proper limits might not be exceeded and above all that the publication of the number that was due might not be delayed.
>
> It ought to have appeared (as all the matter had been sent in before my departure) during the month of May or very early in June at latest. It is now the end of July and I am kept by Story on this as on all other matters connected with the Journal (since I left) completely in the dark and am unable to give any reply if asked when it will appear. It is 7 months after time. Every one (persons of the highest position that I can name) says that this delay and irregularity are doing immense injury to the Journal.
>
> When I consider Story's conduct since my absence this year and couple it with the fact of his disobeying my directives when I was absent last year and the inexcusable want of right [?] feelings not to say mala fidés exhibited by him in his [illegible] of Mr. Kempe's valuable memoir, I have come to the conclusion that it is inexpedient that we should continue to act together in carrying on the Journal and as I am primarily responsible to the Public, to the Trustees, and the World of Science for its success, I formally request that arrangements may be made for dissolving the present

> connexion of Story with the Journal and myself as I can no longer work satisfactorily with or feel any confidence in him—for I consider that his conduct has proved him to be wanting in loyalty and trustworthiness—I shall be willing to return to America at any moment when requested and shall be prepared to take upon myself in future any additional amount of labor in connexion with the Journal and will undertake unaided to carry it on satisfactorily and in a businesslike manner. I could and of course would, take means to provide myself with some useful subordinate in whom I could place confidence and would undertake that under no circumstances should the funds of the University be called upon for assistance beyond that stipulated for under the existing arrangements. I feel the deepest and (as mature reflexion and consultation with others who are dispassionate enable me to affirm) well founded displeasure with Dr. Story and no explanation that he might assume to offer can remove this feeling or ever again induce me to place confidence in him—I do not write this under any seal of confidence.
>
> He is at liberty to know of my opinion of his conduct and the wish I have expressed to be released from all further connexion with him in the conduct of the Journal on the ground that I can no longer place any confidence in him.
>
> I am willing to return at the shortest possible notice if in your opinion the interests of the Journal render it desirable that I should do so. [Gilman papers]

We have no information about what blunder Story made in handling A. B. Kempe's now famous paper "On the Geographical Problem of the Four Colours" [*American Journal of Mathematics* **2** (1879), 193–200]. He did follow it with his own "Note on the preceding paper" [*American Journal of Mathematics* **2** (1879), 201–204]. Perhaps Sylvester did not consider this appropriate.

Before continuing the discussion of the contents of Sylvester's letter, we should let Story tell his side of it, as he did in a letter to Gilman of 26 July 1880. It should be kept in mind, however, that he is reacting not to the above letter of Sylvester but to the telegram and letters he received from Sylvester as well as to a note from Gilman of July 24. It was not until 7 August 1880 that Gilman could write Story that he had received Sylvester's letters of July 22 and 24. He did not show them to Story as Sylvester allowed, but notes

> I think a frank explanation to him of the serious difficulties you have encountered and an apology for any delay on your part to answer his telegram and letter would not be amiss... I should be truly sorry to have you lose his confidence and good will. I think they are possessions which you will not lightly forfeit. [Gilman papers]

Here is Story's letter to Gilman in full (from the Gilman papers). He obviously had anticipated the request for a frank explanation.

Catonsville, Baltimore Co., Md.
26. July. 1880.

My Dear Sir:

The first number of vol. III of the "Journal of Mathematics" is not yet out, although all the articles are nearly or quite ready for the press. It has been a very hard number for me. Every page of Stringham's and [C. S.] Peirce's articles has been worked over by me, and I have read Sylvester's and Rowland's as carefully as I could without working all the formulae out. Franklin also read Sylvester's, and he took no little time about it, during which I had to wait. Just before he left Sylvester gave orders to replace Craig's article of 14 pages (the first 14 in the number) by Stringham's: "On regular figures in n-dimensional space", which Stringham had not then in any kind of form. I worked this paper out very carefully with Stringham, giving him constantly suggestions and criticisms, and it was only the day before he sailed that he put the finishing touches to it. This paper was a great cause of delay. There is now no particular reason why the number should not appear as soon as the sheets can be worked off. I shall explain all this to Sylvester in a letter of same date as this. He is very hard to satisfy, especially when away from the field of operations. He writes me that he greatly disapproves of my course in inserting the whole of Rowland's article in this number, thereby causing the number to run over the regular limit by 16 pages. But Rowland insisted on the insertion of his paper entire, although I explained to him that Sylvester expressly desired that the number should not exceed the regular limits. R. said "Cut down Sylvester!" However it is not too late now to change and I shall cut off R.'s paper at the usual end of a number, running on the latter half in the next number. So there are nearly or quite ready for the press 27 pages of Number 2. I cannot please all parties. I have not yet found time for any original work this vacation, but in the necessary pauses in this unremunerative [?] editing have been rusticating a little.

I understand that much dissatisfaction is felt at Harvard on account of the appointment of L. & others over men who have been there some time and who thought they had a right to some consideration.

Very Truly Yours
William E. Story

The closing comment about "L" being appointed at Harvard is enigmatic, but the rest is straightforward. Sylvester ordered Story to replace Thomas Craig's "Orthomorphic projections of an ellipsoid upon a sphere" by a not yet finished paper of W. I. Stringham. As both of them were Hopkins Ph.D.s under Sylvester (1878 and 1880, respectively) and both were teaching at Hopkins, we must presume that Sylvester felt Stringham's enumeration of n-dimensional polyhedra was of more interest—remember that higher dimensional geometry was then very much in vogue—than Craig's continuation of Gauss's work on the projection of an ellipsoid on a sphere. Nowadays we would consider Craig's paper more interesting; Sylvester's editorial decision is probably only a reflection of his interest in pure mathematics. Story must have had to work hard to force Stringham's paper into the same fourteen pages that Craig's was to occupy. Probably the two plates that were sewn in took extra time in printing. His only reward was Stringham's "grateful acknowledgement ... especially to Dr. Story, for valuable suggestion[s]" [*American Journal of Mathematics* **3** (1880), 14].

It is understandable that Rowland, one of only four professors at Hopkins, insisted that his paper not be cut into two parts. He did get his way, but it was not by cutting down Sylvester. Story must have decided—or perhaps he was told—to keep the issue to the prescribed size by putting off Rowland's paper to the second number of Volume 3. Thus Number 1 must have consisted of three papers occupying eighty-eight pages: Stringham; C. S. Peirce, "On the algebra of logic;" and Sylvester, "On certain ternary cubic-form equations." Number 2 began with Rowland's paper and was followed by Craig's.

Although Sylvester wanted to lay all the blame for the delay on Story, it appears that a good deal of it was caused by the changes Sylvester initiated. We have already noted the substitution of Stringham's paper for Craig's, and a note that the printer sent Gilman (received 27 July 1880) says that

> We have just learned that Prof. Sylvester's article was only returned to us on Saturday last, and that it was *dreadfully cut up*, and that another proof of it has to go out. [Fisch and Cope 1952a, p. 358]

Perhaps this proof was the one read by Fabian Franklin, but more likely it was Sylvester's own. We do not know when this issue finally appeared, or whether Sylvester persisted in his demand that Story be fired immediately, but we do know that on 7 August 1880 C. S. Peirce wrote Gilman:

> I have received from Sylvester an account of his difficulty with Story. I have written what I could of a mollifying kind, but it really seems to me that Sylvester's complaint is just. I don't think

> Story appreciates the greatness of Sylvester, and I think he has undertaken to get the *Journal* into his own control in an unjustifiable degree. I think that we all in Baltimore owe so much to Sylvester that he should be supported in any reasonable position with energy; I hope the matter may not go to the length of displacing Story because I think he is admirably fitted for it in other respects than those complained of. But Sylvester ought to be the judge of that. It is no pleasure to me to intermeddle in any dispute but I feel bound to say that Sylvester has done so much for the University that no one ought to dispute his authority in the management of his department. [Fisch and Cope 1952a, p. 297]

This attempt at mediation did not succeed for long, if at all, for Story's name last appears as "Associate Editor in Charge" on the title page of Volume 3 which is nominally 1880. The title page, contents and errata would have been the last part of the volume printed, sometime before the spring of 1882 (for C. S. Peirce was then working on his father's celebrated paper on linear associative algebras which appeared in Volume 4 ("1881"), Number 2 [Fisch and Cope 1952a, p. 299]). We suspect that Volume 3 was printed in early 1882, for that is consistent with the fact that throughout the records at Clark, Story listed his term as 1878–1882 (see, e.g., [*Decennial*, p. 546]). It would be interesting to know precisely which issues of the journal Story edited. We also do not know whether he was fired or resigned under duress. Fisch and Cope claim that Story took a "quasi-proprietary interest" in the journal [1952a, p. 358]. We find this too strong a judgment, though perhaps his outlook played a role in his demise as editor.

Story was replaced by Thomas Craig (1855–1900), who was, like C. S. Peirce, dividing his time between the U. S. Coast Survey (Craig was assistant in the Tidal Division) and half-time teaching at Hopkins. Craig, who was Sylvester's first Ph.D. (1878) at Hopkins, had been teaching there since he arrived when the university opened in 1876 (except for the spring of 1878). Whether Sylvester wanted Craig full time at Hopkins so that he could replace Story as associate editor-in-charge we do not know, but from a letter that he wrote to Gilman on 28 March 1881, we learn that he did get him:

> Allow me to express the great satisfaction I feel in the interest of the University at the measures adopted by the Trustees to secure the continuance of Craig and Peirce. We now form a corps of no less than eight working mathematicians—actual producers and investigators—real working men [sic]: Story, Craig, Sylvester, Franklin, Mitchell, [Christine] Ladd [Franklin], Rowland, Peirce; which I think all the world must admit to be a pretty strong team. [Fisch and Cope 1952a, p. 297]

While we are not sure when Craig replaced Story as associate editor of the journal, his name did not appear on the title page until Volume 6, dated 1884, where he is listed as "Thomas Craig, Ph.D., Assistant Editor." Story's name last appears on Volume 3 (1880). Sylvester's alone appears on Volumes 4 and 5. Newcomb becomes the chief editor beginning with Volume 7, with Craig being "associated." This state of affairs continues until Volume 16 (1894), when the journal is "Edited by Thomas Craig with the Co-operation of Simon Newcomb." The same is true the next year, but Craig's name does not appear at all on Volume 21 (1899), when Craig had to resign due to poor health. He died in 1900.

Before concluding this section, we want to go back and consider the question of who founded the journal. On 20 December 1883 at a banquet in honor of Sylvester, who was about to leave Hopkins to take up the position of Savilian Professor of Geometry at Oxford, Sylvester was explicitly given credit by Gilman for founding the journal. Sylvester's reply was as follows:

> You have spoken about our *Mathematical Journal.* Who is the founder? Mr. Gilman is continually telling people that I founded it. That is one of my claims to recognition which I strongly deny. I assert that he is the founder. Almost the first day that I landed in Baltimore, when I dined with him in the presence of Reverend Johnson and Judge Brown, I think, from the first moment he began to plague me to found a *Mathematical Journal* on this side of the water something similar to the *Quarterly Journal of Pure and Applied Mathematics* with which my name was connected as nominal editor. I said it was useless, there were no materials for it. Again and again he returned to the charge, and again and again I threw all the cold water I could on the scheme, and nothing but the most obstinate persistence and perseverance brought his views to prevail. To him and to him alone, therefore, is really due whatever importance attaches to the foundation of the *American Journal of Mathematics* ... [Cordasco 1960a, p. 107]

Sylvester's reluctance because of lack of material had already been countered by Rowland in an article decrying the state of American science and the need for scholarly journals. Regardless of how modest Sylvester might have been on his departure, there is no doubt that President Gilman deserves a very large share of the credit for introducing scholarly journals in this country and especially the series of American journals that he began at Hopkins. This is also indicated in his reminiscence titled *The Launching of a University*:

> When Sylvester agreed to come to Baltimore, he was requested to bring along with him the *Mathematical Journal* of which he had been one of the editors, but this was not practicable. His American colleague, Dr. W. E. Story, independently proposed the

> establishment of an *American Journal of Mathematics*, and, after a good deal of correspondence, it was decided to begin such a journal, in a quarterly form, and to ask the concurrent editorial aid of professors in other universities. It was intended that the *Journal* should be open freely to contributors in any part of the country. [Gilman 1906a, pp. 116–117]

Thus we see that Story independently had the idea of founding the journal. Story's involvement is also evident from his entry in Poggendorff: "Gründete 1878 & edirte bis 1881 Vol. 1–3 mit Sylvester d. 'Amer. J. of Math'." [Vol. 3, p. 1303], although we do not know whether this was written by Story himself or by Poggendorff.

We have provided all of this detail partly because of its inherent interest, but also to contradict the common myth that Sylvester founded the *American Journal of Mathematics*. This appraisal is too simplistic. There is no doubt that his international reputation and connections played a vital role in the development of the journal. But Gilman deserves credit for seeing that the publication of scholarly journals was absolutely vital to the development of his university, and William Story deserves credit for independently seeing the need for and conceiving of a journal of mathematics. Story most likely was also instrumental in encouraging Gilman to get Sylvester involved. But however it began, Story did not get his own journal at Hopkins.

4. Story's Best Student at Hopkins, Henry Taber

While Sylvester was at Hopkins, Story taught a variety of subjects ranging from quaternions, elliptic functions, and invariant theory to mathematical astronomy and the mathematical theory of elasticity. But he seemed to favor higher plane curves and solid analytic geometry, subjects which for him included the general theory of curves and surfaces. In the fall of 1884—Sylvester had left the previous January—Story began giving an "Introductory Course for Graduates" which consisted of short sequences of lectures on the leading branches of mathematics and which was designed to give the beginning graduate student an overview of mathematics [Cajori 1890a, p. 276]. Story's care in the redesign of the curriculum is alluded to by Fabian Franklin (1853–1939), writing after Sylvester's death:

> It would never in the world have done to have a whole faculty of Sylvesters; anything like a systematic programme would have been out of the question, ... the presence of *one* Sylvester was of absolutely incalculable value. Not only did he fire the zeal of the young men who came for mathematics, but the contagion of his intellectual ardor was felt in every department of the university, and did more than any one thing to quicken that spirit of idealistic

> devotion to the pursuit of truth and the enlargement of knowledge which is, after all, the very soul of a university. [*The Nation*, 22 October 1908, p. 381]

An essential part of the student's education was the "mathematical seminary," to use their quaint sounding phrase. Sylvester presided during his tenure, but when Newcomb replaced him there were three such seminaries. One was run by Craig, one by Newcomb, and one by Story. The purpose of these seminaries was to get students involved in research [Cajori 1890a, p. 276].

Space does not permit an excursion into the details of the Hopkins curriculum. Instead, we shall trace the studies of one student, whose future career forms an important part of the story we are telling. Fortunately, the *Johns Hopkins University Circulars* make it possible to trace each student in minute detail, since the *Circulars* provide a list of those enrolled in every course, as well as a wealth of other information about the academic program. The student we are interested in is perhaps not typical, since he started in philosophy and switched to mathematics. Also his subsequent career was more distinguished than that of most Ph.D.s, despite the chronic health problems which retarded somewhat his academic progress. Nevertheless, his biography gives insight into the state of American mathematics at both Hopkins and Clark during the period of the current study.

Henry Taber (1860–1936) was born at Staten Island, New York, on 10 June 1860. He entered Yale's Sheffield Scientific School to study mechanical engineering in 1877, but had to leave temporarily because of illness. When he found himself unsuited for engineering, he was allowed to substitute a special course in mathematics for part of his work. Taber finally graduated with a Ph.B. in 1882.

Taber went to Hopkins in the fall of 1882. From the *Circulars*, we know that he attended Story's higher plane curves (three hours), as well as Thomas Craig's elliptic functions (three hours) and calculus of variations (two hours). He also took Charles Sanders Peirce's elementary logic course (four hours each semester), which seemed to attract his interest. In the spring, he took Story's conic sections (three hours). In 1883–1884, Taber took Peirce's advanced logic course (two hours) and his probabilities course in the spring (two hours). Incidentally, Story also was an auditor in the probabilities course, since he is listed among the students in the *Circulars*. Taber took no courses from Sylvester in the three semesters when they overlapped. Since Peirce was not reappointed for the 1884–1885 academic year (for unknown reasons, cf. [Hawkins 1960a, p. 195]), Taber switched fields and began to take more mathematics courses. In the fall of 1884, Taber took three of the five courses (thirteen hours per week) taught by Story: the introductory course for graduates (five hours), number theory (two hours), and modern synthetic geometry (three hours). For the next year and a half Taber took no courses; we conjecture that he was ill.

In the fall of 1886, Taber took two of Story's four courses: quaternions (three hours) and advanced analytic geometry (two hours). In addition, he took Story's seminar (Story again taught thirteen hours). In the spring of 1887, he continued in these three courses and picked up the second half of Story's introductory course for graduates.

In his sixth year (actually the second half of his fourth year, taking account of the eighteen-month hiatus in his enrollment), Taber took Story's linear associative algebras (two hours) and advanced analytic geometry (three hours). Finally, the theory of functions course, taught by Craig using the books of Briot and Bouquet, and Hermite, attracted Taber's interest. In his final semester at Hopkins, Taber took only Story's seminar. It is interesting to note that in this year—1888—Story gave a course in "symbolic logic" which may well have been the first such course in a *mathematics* department; Taber, however, did not take this course.

On 14 June 1888, Henry Taber received his Ph.D. in "Mathematics and Logic" for a thesis entitled "On Clifford's n-fold algebras" [*American Journal of Mathematics* **12** (1890), pp. 337–396]. No director is listed for his thesis in *Circular* #67, but undoubtedly it was Story. The next year (1888–1889), Taber was "Assistant in Mathematics" at Hopkins, teaching analytic geometry (two hours) and trigonometry (one and one-half hours) both semesters. Alas, the only thing that has changed in the intervening century is that we now take more hours per week to do this!

In the spring of his year as assistant Taber attended Craig's abelian functions (two hours) and then a very famous course on the "Theory of Substitutions" which met five hours per week for four weeks. The latter was taught by Oskar Bolza, an 1886 Ph.D. of Felix Klein, from whom he had taken a similar course in Germany in the summer after he received his degree. This course represents the first discussion of Galois theory in this country. Ten people attended the course including Craig, Franklin, and Story, i.e., all of the faculty except Simon Newcomb, who never taught courses unrelated to astronomy, and C. Smith, who taught only solid analytic geometry [*Circular* #71]. The absence of Newcomb is rather odd, since it was he who had encouraged Bolza to lecture on the theory of substitutions and its application to algebraic equations [Bolza 1936a, p. 20].

Under ordinary circumstances, it is likely that Story and Taber would both have spent their entire careers at Hopkins, contributing a respectable amount to mathematical research, but not having great impact on the direction in which it developed. However, in 1889, an opportunity arose for Story to mold a mathematics department in his own image. That opportunity changed


Henry Taber at his desk, 1892–1893. (Clark University Archives)

the careers of both Story and Taber and had a significant impact on the development of mathematical research in America.

5. The Founding of Clark University

Jonas Gilman Clark (1815–1900) was a New England farm boy with little schooling whose mother taught him to love books and reading. He learned the wheelwright's trade and then went into business selling manufactured goods, first in New England and then in California. Through sagacious strategy, he captured a large share of the hardware and furniture trade in California for several years, but then, because of health problems, Clark was forced to sell his business. He invested his large profit conservatively and wisely and eventually became an extremely wealthy man. He traveled widely throughout Europe, acquired a large library, and took a deep interest in higher education. The founding of a university by his old California friend, Leland Stanford, and the approach of his seventy-first birthday seem to be the impetus for implementing plans to endow a university of his own. Clark wanted to begin with an undergraduate college and then develop graduate programs later. See [Koelsch 1987a] for further details.

The board of trustees that Mr. Clark appointed chose G. Stanley Hall (1846–1924), a prominent psychologist of unusual intellectual breadth and achievement, as the first president of Clark University. Hall had spent several years studying in Europe, had earned a Ph.D. at Harvard in 1878, and had been a professor of psychology at Johns Hopkins since 1881. He was reluctant to leave Hopkins until he formed the opinion that he could create a purely graduate university. He made this clear in his letter of acceptance, stating that he had no interest in "organizing another College of the old New England type, or even the attempt to duplicate those that are best among established institutions old or new" [Atwood 1937a, p. 4].

Hall's first act as president of Clark University was to take a year-long "pedagogic tour" to study European educational methods and facilities and to hire distinguished faculty if possible. On this tour, he tried "to get a clear idea from the expressed opinion of their colleagues, of the relative merits of each of the best German professors, in each of the departments we contemplate" (Hall to Clark, 22 November 1888; [Rush 1948a, p. 24]). What was "contemplated" by Hall was a "purely graduate institution," with work originally in only five areas: mathematics, physics, chemistry, biology, and psychology. It is clear that Hall set his sights on what he believed was the best. He wrote Jonas Clark on 14 November 1888:

> I have learned on all sides that Professor Klein, of whom we have often spoken as about the very best mathematician in Europe, is widely so considered here by those experts most competent to

> judge. I lately spent several hours with him talking about the possibility of his joining us at Worcester. He is inclined to come if he could have $5000 per year which was offered him at Baltimore. [Rush 1948a, p. 21]

Earlier, Hopkins had attempted to replace Sylvester with Felix Klein (1849–1925) [Reid 1978a], and now Hall was going to attempt to hire him at Clark since he was "a great man enough ... to keep our American mathematical students from going abroad to study higher mathematics" [Rush 1948a, p. 25]. One consideration that prevented Klein from going to Hopkins was still an issue, namely the question of sick pay and pensions, which were universal in Germany, but nonexistent at Clark and elsewhere in the U.S. A new issue was that Klein wished to come for only six months a year for several years, so that he could retain his position in Germany. But the most formidable obstacle was the German *Kultusministerium.* As Hall wrote in the letter quoted above,

> This ministry is very reluctant to lose its best men, and, if there is any talk of their going to America diffuses the sentiment that they love money more than science and are not patriotic. Thus they are discredited among the universities.

While there is no evidence that this was ever done, leaving a prestigious position in Germany would certainly have the same effect. In Klein's case, he was decorated by the government for not going.

Klein was interested, said Hall, since "he told me he was chiefly attracted by the opportunity of doing only very advanced work for a very few men, with whom he could carry on his researches." Mr. Clark liked Hall's plan of hiring Klein and approved of "the policy of securing several of the best men that can be obtained" (Clark to Hall, 4 December 1888). However, less than a month after his first letter, Hall wrote that Klein,

> has at length decided (after my going several times to Göttingen to see him) that his wife is so opposed to going to America (and I fancy that the certainty of his speedy call to the first chair of mathematics in Europe soon to be vacated in Berlin by Weierstrass so very well assured) that even if called he could not leave Germany. [12 December 1888; Rush 1948a, pp. 27–28]

At this point, it is tempting to consider "what if." However, we shall avoid the biographical subjunctive and leave the story as it is.

For a variety of reasons, Hall was unable to hire a European for any of the five departments he contemplated. Thus he was forced to go with native talent. The question then arose: Who was the best American mathematician? Hall made the logical and correct choice: He hired William Edward Story.

At the time, Story was forty years old and the possessor of a Ph.D. from Leipzig, which Hall regarded as the best university in Europe. He was an established and respected mathematician with eleven research papers to his credit. He was a member of the London Mathematical Society (elected March 1879), Corresponding Member of the Natural Society of the Natural and Mathematical Sciences of Cherbourg, and a Resident Fellow of the American Academy of Arts and Sciences (elected May 1876). At Hopkins, he had a reputation as a good teacher and was the senior pure mathematician (not counting the hybrid Newcomb). He was also well known and trusted by Hall, having associated with him at the best and essentially only Ph.D. granting institution on the North American continent. In short, Story was the natural choice.

It is not known whether Hall considered anyone else for the position of chairman of the mathematics department, but a glance at the 80 "starred" names in the first edition of J. McKeen Cattell's *American Men of Science. A Biographical Dictionary* (1906) shows that there was really little choice among mathematicians between the ages of thirty and fifty. In 1903, Cattell had a group of ten mathematicians rank all American mathematicians, the top eighty of which are starred in the 1906 edition. The numerical rankings were published in the 1933 edition of *American Men of Science*, p. 1269. Many of those listed had taken classes from Story at Hopkins, and the best of those ranked (E. H. Moore) had received his Ph.D. only in 1885 (at Yale). An examination of Cattell's list makes it clear that Story was the best-qualified person for the job. We are not claiming that Story was a great mathematician, for he was not, but only that he was the best available at the time.

There were many reasons why Story might have wanted to leave Hopkins. He was not a full professor there, though he had been there thirteen years.[3] He was not the editor of the *American Journal of Mathematics*, which had been one of his youthful ideas. Finally, he had come to feel that Hopkins was not the wonderful place intellectually that he thought it might and should be: "a peculiar organization of the Mathematical department of the Johns Hopkins made me feel that I was not as free to carry out my own ideas as I wished" (Story to Hall, 12 December 1912). On the positive side, there would be a lighter teaching load and that would leave more time for research. But perhaps most importantly of all, he would have the opportunity to develop a department that focused on graduate education and on research. And he could do it the way that he thought best. For all these reasons, it is likely that the opportunity to move to Clark would have attracted Story.

[3]Story was passed over in favor of Simon Newcomb. This decision is unreasonable if one feels that the department head should be a pure mathematician, rather than an astronomer. Of course many consider Newcomb a mathematician, for he did serve as president of the AMS. Today, a recreational problem posed by Newcomb is of interest in combinatorics [MR, 58 #10473].

Story was originally hired as acting head of the mathematics department. Why he did not receive the title of head is not known; perhaps Hall still hoped to hire a distinguished European. More likely, simple titles were the style of the day. Hall himself was only "temporary professor of Psychology" [*First Announcement*, May 1889, p. 8].

But once Hall had hired Story, the rest of the faculty was easy to fill in. Taber, of course, was happy to follow his mentor. Curiously though, his resignation from Hopkins was announced in the *Circulars* before that of Story. Craig suggested to Bolza that Clark would be a good place for him [Bolza 1936a]. Since Bolza had been introduced to Hall by Klein when Hall was on his pedagogical tour of Europe, he was inclined to accept also. Thus when Clark opened its doors to mathematicians in the fall of 1889, it offered Story as professor, Bolza as associate, and Taber as docent.

Oskar Bolza (1857–1942) had entered the University of Berlin in 1875. His family hoped that he would enter the family business of manufacturing printing presses, but his scholarly bent won out. His first interest was linguistics, then he studied physics under Kirchhoff and Helmholtz, but experimental work did not attract him, so he decided on mathematics in 1878. The years 1878–1881 were spent studying under Elwin B. Christoffel and Theodor Reye at Strasbourg, Hermann A. Schwarz at Göttingen, and particularly Karl Weierstrass in Berlin. "Undoubtedly, the fact that he was a student in the famous 1879 course of Weierstrass on the calculus of variations exerted a strong influence on the formation of Bolza's mathematical interests, although some twenty years elapsed before he began active research in this field, for which he was to gain world renown" [Dictionary of American Biography]. Bolza received his Ph.D. under Klein in 1886 and the following year he, and his good friend Heinrich Maschke (1853–1908), were in a private seminar with Klein. This had the curious effect of undermining Bolza's confidence. He was awed by Klein's quickness, and felt that Maschke was a better mathematician than he was. Bolza had done some practice teaching at a Gymnasium, and he found the experience too physically demanding; there was no energy left for research. His friends Maschke and Franz Schulze-Berg formed the same opinion and left for the United States in 1891. Bolza followed soon thereafter, for he realized there was little hope of obtaining a university position in Germany. Soon after his arrival, he went to Hopkins, where he presented the famous lectures on Galois theory mentioned above.

6. Three Golden Years, 1889–1892

Clark University's *First Official Announcement* in May 1889 contained almost no information about the Department of Mathematics: "Appliances for this department are also liberally ordered; the names of instructors will soon be announced" (p. 18). The *Second Register and Announcement*, which appeared in May 1890, reported that President Hall had hired three Hopkins mathematicians, William Story, Oskar Bolza, and Henry Taber to serve as faculty for the Department of Mathematics and recorded what they had taught in 1889–1890 . The first year their audience consisted of one scholar, L. P. Cravens, whose previous position was Superintendent of Schools in Carthage, Illinois, and the following five fellows in mathematics: Rollin A. Harris, an 1885 Ph.D. at Cornell; Henry Benner, an 1889 M.S. at the University of Michigan; Joseph F. McCulloch, an 1889 M.A. from Adrian (Michigan) College; William H. Metzler (1863–1943), an 1888 A.B. from the University of Toronto; and Jacob William Albert Young (1865–1948), an 1887 A.B. from Bucknell, who had studied at the University of Berlin in 1888–1889.

Fortunately, we are able to get a very detailed picture of the activities of the faculty and the types and level of mathematics studied from the *Registers*. From the *Second Register and Announcement* (pp. 27–28), we learn that Story "directed courses of reading in the following subjects, supplementing the text books by lectures five times weekly:" (1) modern higher algebra, (2) higher plane curves, (3) general theory of surfaces and twisted curves, (4) theory of numbers, (5) calculus of finite differences, (6) calculus of probabilities, (7) quaternions, and (8) modern synthetic geometry. He also gave a course of lectures twice weekly on (9) analytic mechanics. There is no information on how much time was devoted to each of these topics, but probably Story lectured on these topics sequentially. We do know that he taught seven hours per week.

Of the nine courses described in the *Register*, four no longer make up part of any curriculum (numbers (2), (3), (7), and (8)), at least in anything like the form described, though part of their subject matter is subsumed in courses that students do take nowadays. The others are more or less completely taught in the standard undergraduate curriculum of the present. Number theory dealt with what we now call elementary number theory, through quadratic reciprocity. Probability theory was elementary discrete probability, through Bernoulli trials and the study of errors of observation. The course in quaternions indicates the influence of the British school, the influence of Sylvester on Story. The lectures on mechanics must have helped to expand the rather limited offerings in physics, a department which at that time had only one fellow and one faculty member. But the latter was none other than Albert A. Michelson (1852–1931), who attained permanent glory for his experiments on the velocity of light.

Bolza, the Göttingen Ph.D. who came to Clark by way of Hopkins, represented the closest Hall was able to come to importing a German mathematician. He was just at the beginning of his career, but he was well versed in the mathematics of Göttingen and Berlin. His topics for the year were: (1) definite integrals, (2) calculus of variations, (3) elliptic functions, and (4) the theory of functions. In addition, he gave a special course twice a week on "Weierstrass' theory of elliptic functions and Riemann's theory of hyperelliptic integrals." The first topic, which included line integrals and Fourier series, seems rather elementary, but the others were sophisticated even by present day standards. All of these strongly reflect the ideas of Riemann and Weierstrass. Students who heard Bolza lecture were hearing the latest mathematics that could be said to have attained anything like a definitive form.

Taber's second year of teaching was much more exciting than the analytical geometry and trigonometry that he had taught at Hopkins the previous year and is akin to what postdoctorates do today. His course on the theory of matrices was an exposition of topics related to his dissertation, extending the ideas of Cayley, Sylvester, and Clifford. It was therefore fully in the British school, except that the ideas of Benjamin Peirce on linear associative algebras were discussed.

Harris, who was the author of three papers in the *Annals of Mathematics* and the only Ph.D. among the five fellows, gave lectures on the use of analytic function theory in the construction of maps.

The *Register* also indicates which courses were to be given in Clark's second year (1890–1891), says a bit about the facilities at Clark, lists the publications of Story (eleven), Bolza (four), Taber (one to appear), and Harris (three), and indicates the current research topics of the faculty. Story is investigating non-Euclidean geometry, and Taber is applying matrices to nonions and developing Clifford's geometrical algebras and their applications to non-Euclidean geometry.

In summary, the first year of operation at Clark produced an admirable amount of both research and instruction in the very latest topics. Although direct contact with the established European masters was lacking, the mathematicians who were present had studied with these masters and carried with them some of the zeal and ability in research which characterized this vigorous period. Of the three great centers of research, Britain, Germany, and France, the first two showed a fairly direct influence on the work at Clark. The names of Cayley, Clifford, and Sylvester show beyond any doubt the strong influence of the British school on the direction of research, while the frequent mention of Weierstrass and Riemann in Bolza's course injected a significant German influence. At this stage, only the French influence seemed to be missing; no mention was made in the *Register* of the work of Hermite and Picard, even though these two mathematicians had made enormous contributions to the subjects of linear algebra (as we now regard it) and analytic function theory, which were being taught by Story, Bolza, and Taber. This work, however, was closely related to the work of Weierstrass, and may have been mentioned at least in passing in the lectures of Bolza. Unless some lecture notes are discovered, we shall never know.

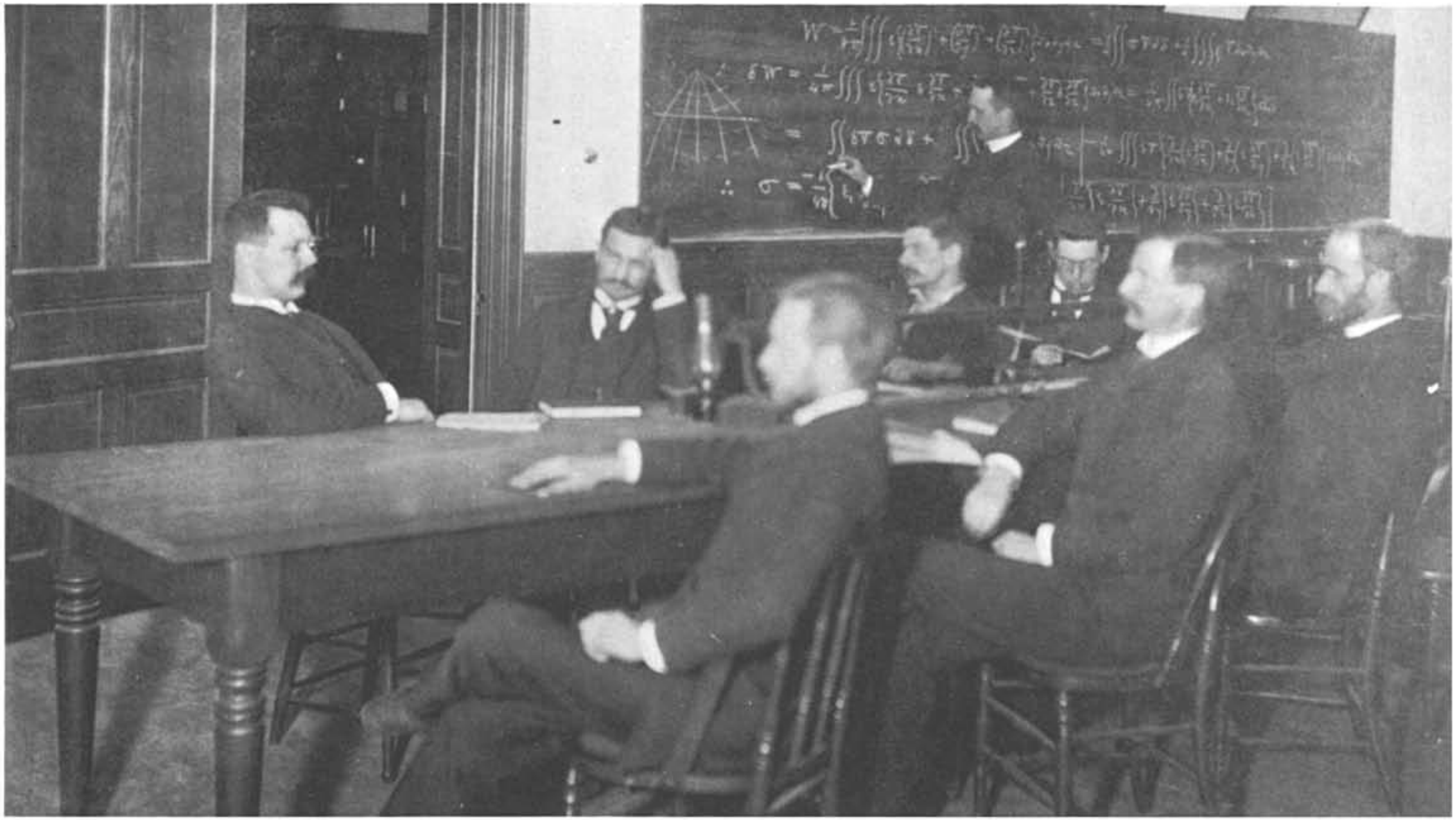
The *Third Register and Announcement* of May 1891 revealed that two new mathematicians and one new physicist had been added to the faculty for the second academic year, 1890–1891. Henry S. White joined the department as assistant, and Joseph de Perott as docent. Mathematics was also represented in the physics department by the heavily mathematical physicist Arthur Gordon Webster.

Joseph de Perott (1854–1924) was appointed docent in mathematics. He was born in St. Petersburg, raised in Thumiac, France, studied in Paris and Berlin 1887–1880 but received no degree, and was a close friend of Sonja Kovalevskaya. His interest was number theory. For more information on this most colorful of the Clark mathematicians see [Cooke and Rickey 199?a].

Henry S. White (1851–1943) was appointed assistant in mathematics in the fall of 1890. He was born in Cazenovia, NY, graduated from Wesleyan University in 1882, and then taught for several years. On the advice of close friends on the Wesleyan faculty, including Van Vleck, he decided to go to Germany for advanced study. He first went to Leipzig where he studied with Sophus Lie and Eduard Study for a year and then to Göttingen to seek out Felix Klein. Oskar Bolza, Wilhelm Maschke, and Frank Nelson Cole had left the year before White arrived. He took courses from Schwarz and Schönflies and wrote a dissertation on abelian integrals under Klein's direction.

White returned to the U.S. in March 1890 to a position in the "preparatory department" at Northwestern University. However, G. Stanley Hall, to whom Klein had introduced him during Hall's tour of Europe, offered him a position as assistant in mathematics at Clark. White accepted "Though the salary was hardly adequate for subsistence, I accepted it eagerly in spite of kind offers from Evanston and Middletown. My teaching was mainly algebraic and projective geometry, and the invariant-theory of linear transformations" [White 1946a, p. 24]. White had a productive year at Clark, writing two papers, one on ternary and quarternary [sic] linear transformations, and one giving a symbolic proof of Hilbert's method for deriving invariants and covariants of ternary forms.

Arthur G. Webster (1863–1923) was appointed docent in mathematical physics. After graduating at the head of his class with an A.B. from Harvard in 1885 with honors in mathematics and physics, Webster spent a year at Harvard as instructor in mathematics, before leaving for Europe on a Parker Fellowship. He studied at the Universities of Berlin, Paris, and Stockholm, earning a Ph.D. at Berlin in 1890. Webster became a full professor at Clark in 1900 and was elected to the National Academy of Sciences in 1903 at the early age of thirty-nine. He was noted for the heavy use of mathematics in his physics textbooks, including his *Partial Differential Equations of Mathematical Physics* (1927, which was translated into German in 1930 by none other than Gabor Szegö.



Arthur Gordon Webster lecturing on mathematical physics, 1892–1893. (Clark University Archives)

The university had taken great care to define the position of docent, and their final description had even been reported in the *New York Times*. By the charter of the university,

> The highest annual appointment is that of docent. These positions are primarily honors, and are reserved for a few men whose work has already marked a distinct advance beyond the Doctorate and who wish to engage in research.

During his first academic year of 1890–1891, Perott used the two hours per week allotted to a docent to discuss "the most elementary parts of the theory of numbers," but carried the subject far beyond what Story had done the previous year. Perott even included a sketch of Kummer's theory of ideal numbers.

White took over some of the courses given by Story the previous year and lectured on (1) higher algebra, (2) higher plane curves, (3) plain [sic] cubics and quartics, (4) abelian integrals, (5) algebraic surfaces and twisted curves, and gave an introductory course on (6) modern synthetic geometry.

For his second year at Clark, Henry Taber chose to lecture on (1) quaternions, (2) multiple algebra, and (3) logic. The first part of the logic course dealt with symbolic logic as based on the work of DeMorgan, Mitchell, and C. S. Peirce. The second part dealt with the theory of induction and especially the work of John Stuart Mill and Peirce.

The most exciting new development in the second year was that Story began a "seminary." White emphasizes in his 1946a autobiography how the Hilbert basis theorem excited the participants in Story's seminar.

Clearly, there was much more activity the second year. Not surprisingly, what occurred the first year was at too high a level for some, and only two of the six students returned, namely Metzler and Young, both of whom earned Ph.D.s from Clark. They were joined by five more auditors. Alfred T. DeLury (an 1889 B.A. from Toronto University) and Thomas F. Holgate (an 1889 M.A. from Victoria University in Canada) were the new fellows. The three new scholars were Levi L. Conant (an 1887 Dartmouth M.A.), John J. Hutchinson (an 1889 A.B. at Bates College), and Frank H. Loud (an 1873 A.B. at Amherst College).

The academic year 1891–1892 appears, in retrospect, to have been the brightest in the history of mathematics at Clark University. Story, Bolza, Perott, Taber, and White gave lectures which one would greatly wish to have heard. The catalog descriptions alone, from the *Register and Fourth Official Announcement* (April 1892), are exciting to anyone who has looked at these topics. A few samples will suffice to give the flavor. Story lectured on (1) the history of arithmetic and algebra, (2) some topics of analysis situs,

> 3. Modern Algebra; an advanced course on the covariants and invariants of systems of quantics involving any number of variables, their conditions, numbers, and syzygies. The writings of Cayley, Sylvester, and Hilbert formed the basis of this course, from which the symbolic methods of Aronhold, Gordan, and Clebsch were necessarily excluded. The lecturer presented also the results of his own recent investigations. Twice a week from January to March and weekly during the rest of the year.

and (4) algebraic plane curves of the fourth and higher orders. In addition, "Story has also conducted weekly two-hour meetings of the mathematical department." These seminar topics included "Cantor's hyperinfinite number-system" and "Models illustrating rotation in 4-fold space."

Bolza lectured on (1) definite integrals, (2) elliptic functions, (3) calculus of variations, (4) theory of functions, and

> 5. Klein's Icosahedron-Theory, finite groups of rotations, the corresponding groups of linear substitutions, rational automorphic functions. Twice a week until March 1.

Perott discussed (1) theory of numbers (advanced course), and (2) numerical computations. Taber lectured on (1) modern algebra, (2) applications of the theory of matrices to bi-partite quadratic functions, and (3) symbolic logic. White discussed (1) modern synthetic geometry, and

> 2. Higher Plane Curves (Introductory Course); use of homogeneous coordinates, ordinary singularities of algebraic curves, projection and reciprocal figures, rational curves, Pluecker's relations, envelopes, tactinvariants, configuration and reality of inflexional points on the general cubic, conjugate points on the cubic, quadric transformation and general Cremona-transformations.

He also discussed (3) algebraic surfaces and twisted cubics and (4) theta-functions of three and four variables.

Since this is a report on what was done in 1891–1892 rather than what was planned, it is hard to dispute the claim in the *Register* that "The facilities for the study of the higher mathematics offered by this University are unsurpassed in this country." With this high level of activity, it is amazing how modest the requirements for admission were:

> Differential and Integral Calculus, Plane Analytic Geometry, through Conic Sections, Solid Analytic Geometry, through Quadric

> Surfaces, Elements of the Theory of Algebraic Equations. A knowledge of the theory of Determinants and their application to the solution of linear equations, and of Differential Equations is desirable.

The published intentions of Clark University were being admirably fulfilled at this point. The very latest research from Europe was being studied and extended in a new American university. Moreover, the graduate students were being intimately involved in collaboration with the faculty.

This was a superb faculty. Both the German Bolza and the American White were students of Klein, and Bolza had a strong Weierstrassian component as well. Combined with the Berlin-Paris training of Perott, these mathematicians gave Clark a strong continental influence. This was something which was never present at Hopkins. The British and American influences were provided partly by Story and even more strongly by Taber, who had absorbed the spirit of Sylvester's work. Thus the Clark mathematicians were prepared to work in the best traditions of both the British and continental schools, and to continue the American work in logic and associative algebras begun by the Peirces.

It is no exaggeration to say that in 1892 Clark had the strongest mathematics department in the New World. Cattell's 1903 survey in *American Men of Science* is one way to evaluate the department. All five of the Clark faculty except Perott are listed among the top twenty: Bolza is fifth, White eighth, Story fifteenth, and Taber nineteenth. Also, Webster, the physicist, is listed twenty-fifth among the mathematicians, and fifth among physicists. To be sure, Chicago makes an even stronger showing in 1903, when the survey was taken, but the building of Chicago under E. H. Moore had just begun in 1892, the year we are discussing. The list also points up the relative decline of Hopkins during this period, though some account must be taken of the fact that Thomas Craig had died in 1900, just before the survey was conducted.

There is no doubt that for the years 1889–1892, Clark University was the preeminent school of mathematics in the Americas.

7. Revolt and Retrenchment

The early achievements and promise of Clark University were blighted by an unfortunate faculty rebellion that culminated during the 1891–1892 school year. Members of the faculty in several departments became disillusioned with the course of events and many of them left. The faculty members blamed President Hall, whom they felt had not kept his promises. Hall portrayed himself as caught between the faculty and the founder. It now seems that Mr. Clark also wanted an undergraduate school, and so the real disagreement was between him and President Hall. For present purposes, it is unnecessary to analyze the exact causes of the rebellion or attempt to fix the blame,[4] for the mathematicians were not active participants even though two of them, White and Bolza, did leave after that year, Bolza to go to Chicago, White to Northwestern.

In April of 1892, President William Rainey Harper (1856–1906) of the newly formed University of Chicago showed up in Worcester, having heard of the unhappiness among the faculty at Clark. Backed by the wealth of John D. Rockefeller, Harper was able to offer $7,000 to department heads that Clark had paid only $4,000. Although not the only factor, money certainly played a role. In 1891–1892, the five Clark mathematicians had a combined salary of $7,200, the same amount that Harper was offering his new department heads at Chicago. He was able to hire two of the four full professors at Clark; only Story and Hall remained. The greatest loss was the physicist A. A. Michelson. Great as this disaster was, it does show that the Clark faculty had a reputation for quality.

Refusing an offer from Johns Hopkins in 1891, White accepted one from Northwestern in 1892 for reasons which apparently had nothing to do with the general exodus of scholars from Clark in that year. "The inducements were, first a better salary with assured permanency, and second, proximity to the new University of Chicago and my highly valued friend E. Hastings Moore, its new head professor of mathematics. He indeed tried to bring me into his department but could not secure sufficient appropriation" [White 1946a, p. 24]. The higher salary was an understandable motive, since on 28 October 1890 White had been married. Apparently, White left Clark without bitterness, for he corresponded with Hall in September 1893 about Klein's itinerary in the U.S., and in 1903 he asked Hall for a letter of recommendation.

Another serious loss to the mathematics department was Oskar Bolza. One might surmise that Bolza left Clark because Hall promised to hire Maschke, and then backed out, but that does not seem to be the case. Although Bolza had no personal battle with Hall, the rebels had persuaded him to make some commitments to them, which he felt obliged to live up to [Bolza 1936a, p. 23]. In addition, like White, he was attracted by E. H. Moore.

The loss of two of its distinguished faculty reduced the department at Clark to a loyal core of three—Story, Taber, and Perott. They remained as the only faculty in mathematics until their retirements in 1921. In addition to the loss of faculty, the university was impoverished. From 1892 until his death in 1900, Mr. Clark gave no more money to the university. During this period there was only $32,000 per year to support the entire institution.

[4] As might be expected, different participants perceive these events differently. For Hall's view, see his *Life and Confessions of a Psychologist*, New York, 1923. While both Atwood [1937a] and Barnes [1925a] are Clark people, they have other views. By far the most balanced presentation is Koelsch [1987a].

Nonetheless, Story was given a salary increase for staying, and Taber was promoted to assistant professor.

One would think that the split in the department would cause lasting animosities, but there is no evidence of that. On the contrary, there are some signs of cooperation and good will. The three University of Chicago mathematicians, Moore, Bolza, and Maschke, worked with White to organize the International Mathematical Congress of 1893, which brought Felix Klein as head of the German universities exhibit. During this visit, Klein resided with his former student White in Evanston, commuting the twelve miles to Chicago every day. Klein's seminars, which were originally to have been divided between Northwestern and the University of Chicago, had to be given at Northwestern because of flooding in Chicago.

Story was elected president of this "zeroth" International Congress of Mathematicians, that is, the Columbian Exposition in Chicago in 1893. This shows that he was held in high regard by the American mathematical community. At this meeting, there were four representatives present from Clark—Story, Taber, Webster, and Keppel. Of the other American universities, only Chicago had as many, and two of those had recently been at Clark. There were only thirteen American residents who presented papers at the meeting, and two of them were from Clark. Taber gave a talk "On orthogonal substitutions," which definitely shows that Bolza had an influence on his work. In absentia, Perott contributed "A construction of Galois' group of 660 elements."

At the World's Columbian Exposition in Chicago, Clark University had 150 square feet of exhibition space wherein "each department will be represented by photographs, descriptive pamphlets, publications of the university and otherwise" [*New York Tribune*, 6 Feb. 1893]. More than 170 of these photographs survive in the Clark Archives, some 25 of which pertain to the mathematics department. Several of these are reproduced here. Most of the mathematical photographs deal with the "set of Brill's admirable models ... and Björling's thread-models of developable surfaces" which Story considered so vital to the teaching of higher geometry. A list of these models occupies eighteen pages in the *Third Annual Report of the President*, April 1893. These were not the only photographs taken, in addition:

> A graduate student at Clark University, Mr. H. G. Keppel, is taking a series of photographs of the mathematical models and portraits of mathematicians to which he has access. It will include stereoscopic views of about one hundred different models. [*Bulletin of the American Mathematical Society* **1**(1894–1895), 127]

There follows a list of thirty-five "portraits already photographed." These photographs of Herbert Govert Keppel (1866–1918) are not in the Clark Archives, and their whereabouts are a mystery.

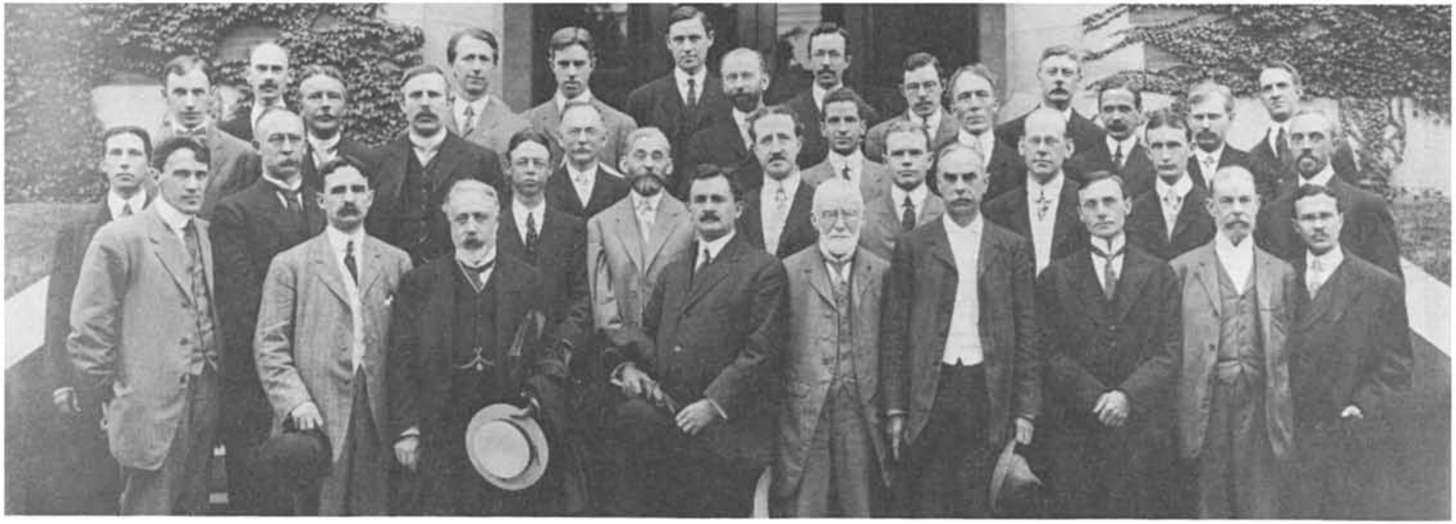
The mathematics and physics group, 1892–1893. Seated, left to right: (2) A. G. Webster, (3) H. Taber. Standing: (4) T. F. Holgate?, (5) H. G. Keppel?, (7) W. E. Story. Other mathematics students at Clark in 1892–1893 were L. W. Dowling, T. F. Nichols, F. E. Stinson, and W. J. Waggener. Perott is not in the photograph. (Clark University Archives)

Through a grant of $500 from Senator George F. Hoar, a member of the board of trustees, Story was finally able to get his mathematical journal. The first number of the *The Mathematical Review. A Bi-Monthly Journal of Mathematics in all its Forms* was published in July of 1896. The other number in this volume was published in April 1987. It was followed by part of another volume in 1897 and then quietly ceased publication. Although no records survive, it undoubtedly ceased publication because of lack of funds and competition from other journals. It primarily consists of dissertations presented at Clark (recollect that the *Transactions of the American Mathematical Society* were founded primarily to publish dissertations). See §11 for a list of the individuals who received degrees in mathematics from Clark.

Clark University was very proud of its accomplishments, and so to celebrate its tenth anniversary, Story and Hall's right-hand man, L. N. Wilson, prepared a large (vi + 566 pp.) volume entitled *Clark University, 1889–1899. Decennial Celebration*, which is a gold mine of information about the university. We learn, for example, that the "mathematical department was not modelled after that of any other institution, but was determined by the conception of what would constitute perfection in such a department" [p. 68] and that in making appointments to fellowships and scholarships, "We are on the lookout for geniuses" [p. 65]. Before the *Decennial* volume was published, a public celebration was held. As part of this, Émile Picard of the University of Paris was invited to give a series of lectures on mathematics, and Ludwig Botzmann of the University of Vienna lectured on physics. They, along with three other individuals, were granted honorary degrees from Clark University on 10 July 1899. Previously, the only degree was the earned Ph.D. The *Decennial* volume contains a long description of the individual departments and, most importantly, a list of over 500 publications by people who had been associated with Clark in its first decade.

On 10 September 1909, the twentieth anniversary of Clark was celebrated. This time, honorary degrees were given to five mathematicians: E. H. Moore of Chicago, William Fogg Osgood of Harvard, James Pierpont of Yale, Edward Burr Van Vleck of Wisconsin, and Vito Volterra of Rome. Fortunately, we have a picture of this gathering. This was perhaps the most famous meeting ever held at Clark. On the same day, Sigmund Freud of Vienna and Carl Jung of Zürich were given honorary degrees. This was the only such honor that Freud ever received.

When Mr. Clark died in 1900, the university faculty were hoping that he would rescue them from their financial plight. Instead, Clark continued with his original plan, leaving one-fourth of his estate to the University, another one-fourth to the library, and with the remainder, he did what he had wanted all along. He founded an undergraduate college, Clark College.



Photograph taken for the twentieth anniversary celebration at Clark, 1909. Front row, left to right: (1) Robert William Wood, (2) E. H. Moore, (3) Vito Volterra, (4) A. A. Michelson, (5) John Monroe Van Vleck, (6) Edwin Herbert Hall, (7) James Edmund Ives (Clark Ph.D., Physics, 1901), (8) W. E. Story, (9) Norton A. Kent. Second row: (1) Stephen Elmer Slocum (Clark Ph.D., 1900), (2) A. G. Webster, (3) Ernest Rutherford (between rows two and three), (4) unknown, (5) Edward Burr Van Vleck, (6) J. W. A. Young (Clark Ph.D., 1892), (7) Norman E. Gilbert, (8) Ernest Fox Nichols, (9) Guy G. Becknell?, (10) Henry Sedgwick?. Third row: (1) Thomas Lansing Porter, (2) Theodore William Richards, [Rutherford], (3) George D. Olds, (4) Carl Barns, (5) unknown, (6) H. Taber, (7) Albert Potter Wills (Clark Ph.D., Physics, 1897), (8) unknown, (9) Elmer Adna Harrington. Fourth row: (1) Rocket pioneer Robert Hutchings Goddard (Clark Ph.D., Physics, 1911), (2) Joseph George Coffin (Clark Ph.D., Physics, 1903), (3) Arthur W. Ewell, (4) Frank B. Williams (Clark Ph.D., 1900), (5) unknown, (6) Chester Arthur Butman, (7) unknown. William Fogg Osgood, James Pierpont, and the astronomer Percival Lowell were present, but have not been located in the photograph. Perott is not in the photograph. (Clark University Archives)

8. Story's Best Student at Clark, Solomon Lefschetz

One way of determining the quality of the mathematics department at Clark University and its role in the development of graduate mathematical education is to examine the careers of the graduate students there, especially the twenty-four students who received Ph.D.s from 1892 to 1917. We have included some information about all of them in §11 below, but we shall concentrate our attention on the most famous of the group.

In the fall of 1910, two new graduate students arrived at Clark to join the three who were already there. One was Alice Berg Hayes, the first woman to receive a degree in mathematics at Clark. Women were not allowed to be graduate students at Clark until 1900, although Leona Mae Peirce (Ph.D. Yale, 1899) studied informally with Story in the 1890s. Hayes received a master's degree in June 1911 for a thesis entitled "Reduction of Certain Power Determinants" which she wrote under Story's direction. The other new student was Solomon Lefschetz, whom Hayes married in 1913.

Solomon Lefschetz was born of Turkish parents in Moscow on 3 September 1884 and was reared in France. He was a student at the École Centrale in Paris from 1902 to 1905 when he received a degree as "ingénieur des arts et manufactures." He then came to the U.S. where he worked for a few months with the Baldwin Locomotive works, and was then on the engineering staff of Westinghouse Electric and Manufacturing Company in Pittsburgh until 1910.

> He lost both of his hands in 1907; the heroic spirit which later enabled him to overcome all but insurmountable obstacles, and to attain to his present position of eminence, must be unique in the annals of the mathematical brotherhood. [Archibald 1938a, p. 237]

Because of this accident, he soon realized that his "true path was not engineering but mathematics" [Lefschetz 1970a, p. 344]. Going back to his French roots, he read the three-volume treatises of Émile Picard (*Analysis*) and Paul Appell (*Analytical Mechanics*), both of whom were professors at the École Centrale. "I plunged into these and gave myself a self-taught graduate course. What with a strong French training in the equivalent of an undergraduate course, I was all set" [Lefschetz 1970a, p. 344].

In May of 1910, Lefschetz accepted an appointment as junior fellow at Clark, which waived fees and paid $100 in ten monthly installments. When he accepted, he added a postscript to his letter: "I ask, as a special favor, that you should forward me the catalogue of the University for 1910, with programms [sic] for 1910–1911 as I intend to do some hard digging during the summer" (Lefschetz to Hall, 9 May 1910). From this *Register*, Lefschetz learned the philosophy of the department, a philosophy which is well worth emulating today:

> The chief aim of the department is to make independent investigators of such students as have mathematical taste and ability; these naturally look forward to careers as teachers of the higher mathematics in colleges and universities, and we believe that the course of training best adapted to the development of investigators is also that which is most suitable for all who would be efficient college professors, even if they are not ambitious to engage in research. The first essential of success in either of these lines is the habit of mathematical thought, and the direct object of our instruction is the acquisition of this habit by each of our students. With this end in view, we expect every student to make himself familiar with the general methods and most salient results of a large number of different branches of mathematics, conversant with the detailed results and the literature of a few branches, and thorough master of at least one special topic to the extent of making a real contribution to our knowledge of that subject.

Since Lefschetz played such an important part in the rise of mathematics in the Americas, at the Universities of Nebraska (1911–1913), Kansas (1913–1923), Princeton (1923–1953), and Mexico (1944–1966), we shall describe the course work that was announced in the *Register* that he requested. Story planned to teach (1) analytic geometry of higher plane curves, higher surfaces, and twisted curves, five hours, (2) finite differences, two hours, (3) history of mathematics, two hours in the fall, and (4) a seminary for advanced students. Taber intended (1) theory of functions of real and imaginary variables, elliptic functions, and definite integrals, five hours, (2) theory of bilinear forms, two hours in the fall, (3) theory of integral equations, two hours in the spring, and (4) a seminary. Perott was to offer (1) theory of numbers, two hours in the fall, and (2) abelian integrals, two hours in the spring.

Unfortunately, no records survive as to which courses Lefschetz actually took in 1910–1911. We can make some conjectures from the annual report submitted by Story on 10 October 1911 dealing with what was actually taught during 1910–1911. Story did teach higher plane curves, but only three days a week. Since there were two students in this course, and since it had also been offered the previous year, Lefschetz was undoubtedly one of them. We also suspect that he was one of the three students in Story's calculus of operations, including the calculus of finite differences. Story offered no seminar that year and Lefschetz certainly did not attend Story's course in mathematics for practical purposes. Taber's theory of functions was undoubtedly familiar to Lefschetz, but he probably attended the two-hour supplementary course.

The other offerings were half-year courses: Taber's bilinear forms and Perott's advanced number theory. Undoubtedly, most of Lefschetz's time was devoted to research under Story's guidance.

Story assigned Lefschetz the problem of investigating "the largest number of cusps that a plane curve of given degree may possess" [Lefschetz 1970a, p. 344]. This resulted in his dissertation "On the Existence of Loci with Given Singularities" which was published in the *Transactions of the American Mathematical Society* **14** (1913), 23–41. There is no doubt that Lefschetz appreciated the education that he received at Clark. A few years later, when he planned a return visit, he wrote ahead asking to give a series of lectures on his recent research, adding "I know of no other place where I may expect to get an audience as surely as at Clark Univ. & none where I'd care more to have one than there" (Lefschetz to Hall, 16 May 1913).

9. The Library

Yet another measure of the quality of Clark University was its library. Lefschetz wrote:

> At Clark there was fortunately a first rate librarian, Dr. L. N. Wilson, and a well-kept mathematical library. Just two of us enjoyed it—my fellow graduate student in mathematics and future wife, and myself. I took advantage of the library to learn about a number of highly interesting new fields, notably about the superb Italian school of algebraic geometry. [Lefschetz 1970a, p. 344]

This high opinion of the library by a distinguished mathematician can be complemented by information describing the contents and quality of the library.

Jonas Clark was a self-educated man who read widely, collected books and manuscripts, and understood the importance of a good library. Consequently, he donated his personal library of some 3,200 volumes, and set up a separate endowment consisting of $100,000, the income of which was to be used for the purchase of books and the maintenance of the library. The faculty were invited to contribute lists of books that they wanted, and the library purchased whatever they requested. A few of these early lists survive.

Just before Clark University opened, Florian Cajori conducted a survey of American mathematical education. Of the 168 schools responding, 117 subscribed to no mathematics journals, 11 subscribed to only the *American Journal of Mathematics*, 12 to only the *Annals of Mathematics*, and 28 subscribed to several mathematical periodicals [1890a, p. 302]. There is no explanation of what several means, but there is no doubt that Clark was soon to be near the top.

In 1893, Clark subscribed to sixteen mathematical periodicals and to thirteen others which contained articles on mathematics. Of these sixteen, all but two consisted of complete runs. The importance of complete sets of serial publications was well understood by the librarian [*Decennial*, p. 196]. In 1900, Bryn Mawr, by contrast, subscribed to twenty-two mathematical journals. There are some fifty listed in Robert Gascoigne's *A Historical Catalogue of Scientific Periodicals, 1665–1900* (Garland Press, 1985), though, of course, a good many of those were no longer being published in 1890. By the time Lefschetz was at Clark, there were about sixty mathematical periodicals in the collection.

In the *Second Register and Announcement* we read:

> The facilities to be found here for the study of mathematics in its various branches are unexcelled in this country. The library is provided with complete sets of all the more important current mathematical periodicals and the publications of the scientific societies of the world, with the standard treatises on the subjects now particularly engaging the attention of mathematicians, the collected works of the great mathematicians, and many books illustrating and discussing the history of mathematics; to which will be added from time to time such other works as may be needed or appear desirable.

The total library holdings in 1900 of 18,000 volumes may seem meager compared to the half million at Harvard and the 90,000 at Johns Hopkins, but remember that this number represented only four fields (chemistry disbanded after the exodus), and then only with graduate-level works that some faculty member requested.

Still in the library at Clark is a *List of Books in Mathematics in the Clark University Library. Worcester Massachusetts, December 1, 1908* (Z 733 C5). This typescript of seventy-seven pages lists the books according to the classification scheme devised at Clark and gives a real indication of the riches of the library. Section "C 21 Works, complete and select." lists the collected works of more than sixty mathematicians. Our experience has been that this remains a very nice small library to work in if your interests are in late-nineteenth-century mathematics.

In 1900, when Jonas Clark died, he left one-fourth of his estate to the library (received on the death of Mrs. Clark in 1903). Thus they had $32,000 per year for the library, a sum they were never able to spend. He also left $150,000 for the construction of a library building. This was built in 1904 at a cost of only $125,000. An addition was started in 1909 at a cost of $100,000. In 1921, the college and university libraries were merged.

The library also had the admirable habit of trying to obtain a copy of every publication of everyone who had ever been associated with the university.

The records of this collection still exist in a separate card catalog in the Archives. Unfortunately, many of the unbound reprints have been destroyed. Even the collection of master's and doctor's dissertations is not complete.

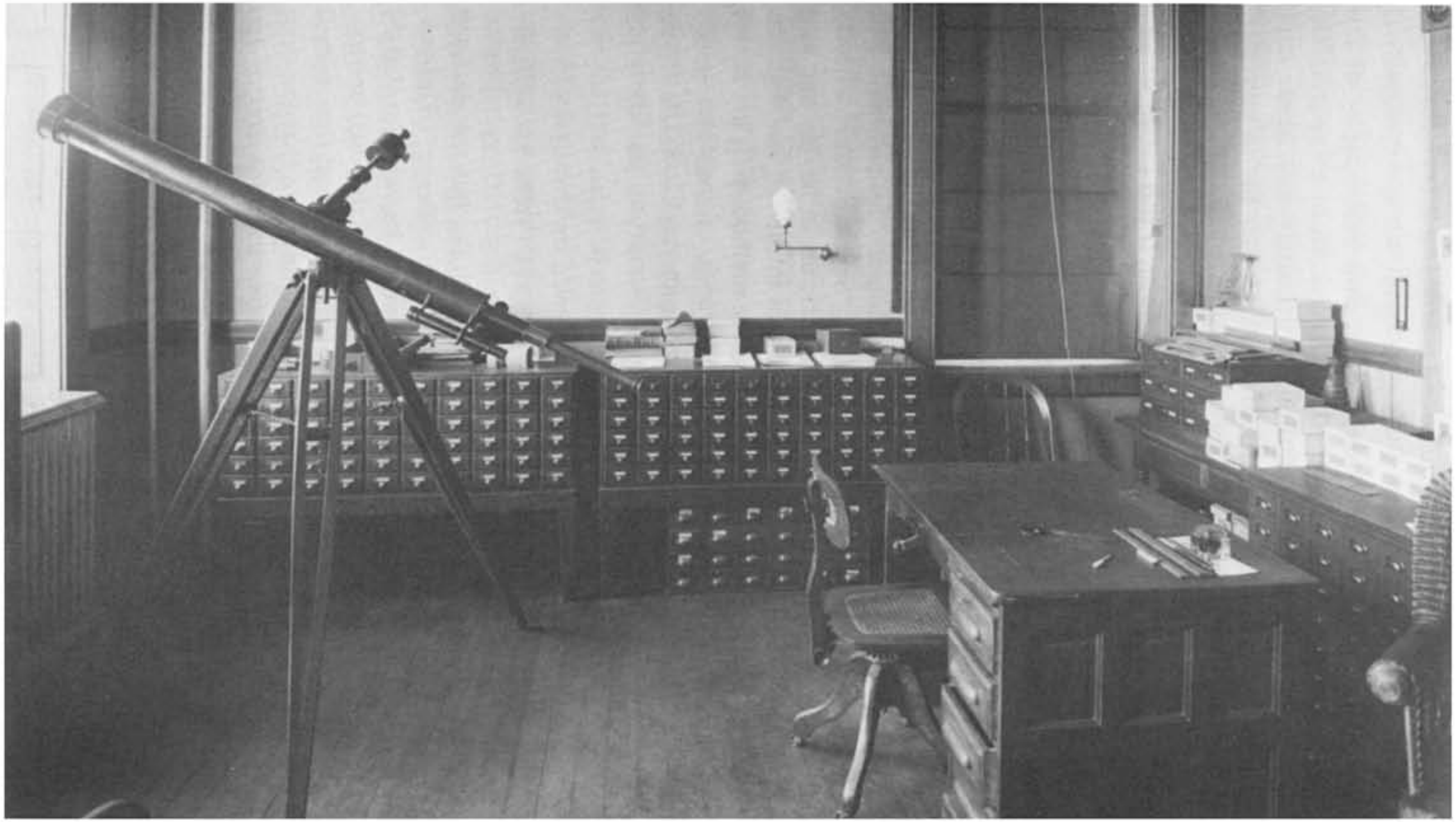
In addition to the library, Story compiled his own bibliography of the mathematical literature. In the early 1900s, it consisted of some 100,000 cards. He aimed to get it published, but unfortunately that never happened. "In 1931 through the generosity of Clark U. and the initiative of Prof. F. B. Williams the Library [of the AMS] acquired the mathematical Bibliography (156 drawers and 35 boxes of cards) of the late Prof. W. E. Story (1850–1930)" [Archibald 1938a, p. 93]. It is not known if this catalogue still exists.

10. Denouement

The penultimate student to receive a master's degree in mathematics at Clark University was Ida Louise Bullard (Pearson). She graduated from Mount Holyoke College with thirty hours of mathematics courses, although nine of them were precalculus. She received "testimonials" from Anna J. Pell (Wheeler) and Sara Effie Smith, and was appointed "Senior Scholar in Mathematics" in 1918–1919. The faculty was enthusiastic about having her, and they kept her busy. She took fifteen hours of classes in the fall and eighteen in the spring. The classes were taught by the lecture method, with her at one end of a very large table in the mathematics classroom, taking notes as fast as she could. Two boxes of her class notes survive in the Clark University Archives. In addition, she wrote a master's thesis, "Report on the Literature of Fractional Derivatives" (1919).

On 10 August 1973, Louise Pearson was interviewed by University Historian William A. Koelsch. From his notes after this interview, we learn her impressions of the faculty. Story was "a very nice, dignified, grey-headed gentleman" who encouraged her to continue for a Ph.D., but she declined because "women could only find positions at women's colleges." Unfortunately, this is all she had to stay about Story, but we do know more. On 20 June 1878, he married Mary D. Harrison of Baltimore, and they had one son, William E. Story, Jr., who was an undergraduate at Harvard and then earned a Ph.D. in physics from Clark in 1907. Fabian Franklin wrote that the elder Story "was happy in his marriage as in his work" [*American Academy of Arts and Sciences. Proceedings* **70** (1935–1936), 580]. Story was "noted for his skill as a raconteur and his force in discussing scientific matters" [Worcester *Gazette*, 27 April 1921]. In addition, he was an excellent teacher, but with a fiery disposition:

> In the mathematics department the most picturesque figure was Story, who could be daily observed lecturing with the enthusiasm of a Bryan delivering his "Cross of Gold and Crown of Thorns"



Professor Story's office in 1914. The card catalog contains his bibliography of mathematical literature. It is not known whether it survives. (Clark University Archives)

> speech, to one student in infinitesmal [sic] geometry or the theory of hyperspace, and whose expostulations announced to passers-by in Main street trolley cars that a faculty meeting was being held in the opposite side of the university building. [Barnes 1925a, p. 275]

Besides the scientific honors mentioned earlier, Story was a member of the National Academy of Sciences (elected 1908), fellow and former vice president of the American Association for the Advancement of Science, and a member of the American Mathematical Society. After his retirement in 1921, Story served as president of the Omar Khayyam Club of America from 1924 to 1927. Earlier, he had written an interesting little pamphlet, *Omar Khayyam as a Mathematician* (1918), that reflects his long held interest in the history of mathematics. He died of pneumonia, after a very brief illness, on 10 April 1930.

> The ablest member of the department was Henry Taber—than whom no finer type of American scholar and gentleman has yet been produced, who lectured in polished and dignified English upon the theory of functions, and read the *Nation*, the *New Republic* and *Freeman* unabashed. [Barnes 1925a, p. 275]

In 1891, Taber was elected to the American Academy of Arts and Sciences. In a biographical memoir to their *Proceedings*, Archibald concurred that Taber was "ever ready to champion the cause of one whom he felt wronged" [Vol. 75, p. 176]. He belonged to the Worcester boat club and was an excellent tennis player. Taber had a wide range of interests, including chemistry, history, literature, music, and dancing. Pearson commented on his teaching:

> Tall, thin, reddish or sandy haired, and a vigorous lecturer. Also a classically absent-minded professor, illustrated by two stories: (1) One day Taber walked into the mathematics classroom and began lecturing, and lectured for twenty minutes before noticing that no one was there, and discovering that he was an hour late. (2) The Tabers lived on the second floor of their house, and one day Dr. Taber discovered he was locked out. So he borrowed a ladder from a neighbor, climbed through a second story window, came back down, and returned the ladder, subsequently discovering that he was still locked out.

Taber married Fanny Lawrence of New York in 1886, and they had three daughters. Sadly, his wife died in 1892, so he had to raise the girls alone. Henry Taber died 6 January 1936.

Naturally, it was Joseph de Perott who consumed the bulk of the interview, for "There were always numerous stories circulating about him." Because

Kovalevskaya would not divorce her husband to marry him, so the story goes, he

> vowed he would never again attempt to make himself attractive to women. This accounted for the mass of tangled hair down to his shoulders, which he never combed, keeping it in place with a derby hat jammed tightly over the top of his head.

This report contrasts with his obituary in the Worcester *Telegram*, on 23 May 1924 which described him as a friendly and happy person with a love of nature, a vast knowledge of languages, a knowledge of Shakespeare that Harvard coveted, and that "His long flowing gray hair and his neat but somewhat threadbare clothing, seemed to attract rather than repel the children" (23 May 1924). It was his mane of hair that earned him the nickname "Johnny the Lion" [Koelsch 1987a, p. 62]. For additional information see [Cooke and Rickey 199?a].

In 1919, President Hall, who had served for thirty-two years, asked to be relieved of his responsibilities at Clark University. Simultaneously, President Stanford, of Clark College, resigned so that a common successor could be found who would merge the two units. The Trustees chose the geographer Wallace W. Atwood as successor. Although the graduate departments of psychology and education had achieved an international reputation, there had been almost no new money in twenty years, and so the other departments were stagnating. The trustees decided that Clark could compete with the now larger graduate institutions only if it had something distinctive to offer, and so founded a department of geography. "The Department of Mathematics, which had very few students, discontinued graduate work, and the members of the staff, who had been in the University practically from the beginning, retired on pensions" [Atwood 1937a, p. 16]. It is sad to realize that such a glorious department had come to an end.

The various decisions that Atwood made sparked a report by the American Association of University Professors, their first comprehensive report of administrative practices. In it we read that Story "retired on account of age in 1921" and Taber "on account of health." In fact, the retirements were forced. Story expressed an interest in continuing in active service (Atwood to Story, 10 March 1921) and Taber was working with a master's student at the time (Taber to Atwood, 12 March 1921). At the time, Story was 71, Taber 61, and Perott 67. We have not forgotten them.

In conclusion, it is probably worthwhile to reflect on why mathematics at Clark University was only a brief success. The most important ingredient was there: well-trained mathematicians, with considerable research potential, and a will to excel. There was a good mix of experience and youth, a diversity of backgrounds, yet many shared interests. The library was excellent and, at first, the salaries were adequate. But it was a lack of money and poor administration that led to the internal strife and subsequent loss of faculty. William Story, and his colleagues Henry Taber and Joseph Perott, contributed to the development of mathematics by being carriers of our mathematical culture. Their careers illustrate the importance of dedicated "minor" mathematicians without whose work—learning, teaching, and doing mathematics—the community of research mathematicians would not grow.

11. Graduate Degrees in Mathematics, Clark University, 1889–1921

This is a complete list of the students in mathematics at Clark University who received either a master's degree or a doctor's degree between 1889 and 1921 (but not those after the Ph.D. program was reinstated in 1965). If known, we have given the call number, accession number, and date of accession of each dissertation in the Clark University Archives.

Allen, Reginald Bryant (1872–1938). At Clark 1901–1903 and 1904–1905. Ph.D. under Taber defended May 25, 1905: "On hypercomplex number systems belonging to an arbitrary domain of rationality," *Transactions of the American Mathematical Society* **9** (1908), 203–218. Clark Library: 49105, May 1909.

[**Boyce, James W.** Fellow at Clark 1896–1899. *Science* **10**, 132 lists him as receiving a Ph.D. at Clark in 1899 for a dissertation entitled "On the Steinerian Curve," but there is no reference to this in the Clark records.]

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Lie, Olaf Kristofer (d. 1914). At Clark 1905–1908. M.A. under Story received June 18, 1908: "On the reduction to the canonical forms of the equations of transformation groups with continuous parameters." Holographic copy in Clark Library: L716, 46112, September 1908.

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McCormick, Clarence (1888–??). At Clark 1913–1916. Fellow in 1915. M.A. under Taber received June 18, 1914: "On the theory of finite continuous groups." Clark Library: M131, 70489, December 1914. Ph.D. Columbia 1928.

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Michalopoulos, Aristotle D. (1899–1953). He later abbreviated his surname to Michal. A.B. 1920, assistant in mathematics and physics 1918–1920, fellow 1920–1921, A.M. 1921, all at Clark. M.A. under Taber: "Theory of analytic functions of a single complex variable." He was the first student of Clark College to enter the graduate mathematics program (letter of Story, December 15, 1920). Ph.D. at Rice, 1924. Has star in *American Men of Science*, edition 6. Classnotes in Clark Archives.

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Van der Vries, John Nicholas (1876–1936). At Clark 1897–1901. Ph.D. under Story defended June 14, 1901: "On the multiple points of twisted curves," *Proceedings of the American Academy of Arts and Sciences* **38** (1903), 473–532. He thanks Taber "for his careful supervision of my work in the first two years, and Professor A. G. Webster and M. Joseph Perott for frequent assistance throughout this work."

Waits, Benjamin Lewis (??–1967). Scholar in 1915. M.A. under Story received June 15, 1916: "Fourier's method for the separation of the roots of an algebraic equation." Clark Library: W1456, 78027, August 1916.

Williams, Frank Blair (1871–1933). At Clark 1897–December 1901 as a student; scholar 1897–1898, fellow 1898–December 1901. Became instructor in mathematics at Clark University in 1910; assistant professor, Clark College 1907, professor 1920. Ph.D. under Story defended June 4, 1900: "Geometry of ruled quartic surfaces," *Proceedings of the American Academy of Arts and Sciences* **36** (1900), 85–109.

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The Best Method. American Calculus Textbooks of the Nineteenth Century

GEORGE M. ROSENSTEIN, JR.

INTRODUCTION

The need for calculus textbooks in the United States was met by American authors from the 1840s onward and by 1870 the industry was well established. Between 1828 and 1920, editions of calculus books by about seventy different authors or sets of authors were published in the United States. A bibliography of these texts is appended to this paper. The books range from vanity pieces, privately printed, to the long-lived books of Davies (1836–1901), Loomis (1851–1902) and Granville (1904–1946).

I will examine books which appeared before 1910, which appeared in more than one edition, and which were not vanity pieces.[1] I refer to those books as "commercial texts." By examining these books, I can show that the last quarter of the nineteenth century was a period of experimentation at the end of which various features of contemporary texts became standard.

The first calculus texts published in the United States were American editions of British books and, notably, Farrar's translation of Bézout.[2] However, in 1828 James Ryan published the first calculus text written by an American. By 1850, several "native" books were available. By 1875, nearly a dozen

[1] Only one author, William Batchelder Greene, met the first two criteria but not the last.

[2] See Cajori, pp. 395 ff. A list of references will be found at the end of the paper. American texts to which I refer are in the bibliography.

books had appeared. These books and those that followed up to the end of the century display a remarkable diversity.

Most notable is the diversity of approaches to the derivative. Texts used limits, infinitesimals or rates (fluxions in a new dress) as fundamental notions. During this same period, European authors were also experimenting with appropriate pedagogical schemes for introducing the derivative. For example, the second edition of Serret's text, published in 1879,[3] displays an eclectic approach using limits and infinitesimals that American contemporaries would immediately recognize. In the United States, limits would not become the preferred approach to the derivative until late in the century. As we shall see, there were other differences also.

THE BEGINNING OF THE LINE

When American calculus teachers were presenting fluxions to their students, British texts could serve their needs. An American edition of Vince's *The Principles of Fluxions* appeared in 1812 and editions of Charles Hutton's *Course of Mathematics* appeared between 1812 and 1831. Both of these books were used in American colleges. Texts were not so easy to find, however, when teachers turned to the style of the French.[4]

Once American teachers became convinced that the continental style, represented by the French in the early nineteenth century, was preferable, they were without texts to help them. Few students could read French although Greek, Latin, and often Hebrew were standard parts of their education.[5] These teachers solved their problem by translating, perhaps with modifications, texts that they found exemplary.

The first translation was John Farrar's of a text by Bézout. Farrar was a graduate of Harvard who, after studying theology at Andover, returned to his alma mater in 1805 as tutor in Greek before assuming the chair in mathematics and natural philosophy in 1807. Farrar translated and edited for use at Harvard a large number of classic French texts. These texts formed the series known as Farrar's Cambridge Mathematics. Included were algebras taken from the works of both Euler and Lacroix, Legendre's geometry, Lacroix's trigonometry and, in 1824, *First Principles of the Differential and Integral Calculus, or The Doctrine of Fluxions, intended as an introduction to the physico-mathematical sciences; taken chiefly from the mathematics of*

[3]Serret, Joseph Alfred, *Cours de calcul différentiel et intégral*, Paris: Gauthier-Villars, 1879. The first edition was in 1868. In his advertisement, Serret says that the book covers the "substance of the lessons" he teaches each year at the Sorbonne. He begins by introducing limits in a fairly careful way, but quickly shifts to infinitesimals a page later.

[4]Cajori believes (p. 82) that fluxions or calculus was offered in "the better colleges" in the early part of the century. Also see Cajori's "Bibliography of fluxions and the calculus" (p. 395ff) for information on early texts.

[5]Rudolph, p. 25; see also Chapters 3–5.

Bézout, and translated from the French for the use of the students of the university at Cambridge, New England.[6]

Farrar had pedagogical reasons for choosing Bézout. In the Advertisement to the first edition, he writes

> [Bézout's book] was selected on account of the plain and perspicuous manner for which the author is so well known, as also on account of its brevity and adaptation in other respects to the wants of those who have but little time to devote to such studies. The easier and more important parts are distinguished from those which are more difficult or of less frequent use, by being printed in a larger character.[7]

As an introduction to the book, Farrar appended an essay by Carnot[8] that explains the "truth of the infinitesimal method."

Notice Farrar's desire to choose a book accessible to "those who have but little time to devote to such studies." Although calculus was part of the curriculum in a number of American colleges during the first third of the nineteenth century, very little time was devoted to it. For example, at Harvard in 1830, sophomores studied trigonometry and its applications, topography and calculus. Furthermore, this third of a year was the only calculus they studied.[9]

Also notice that Farrar has distinguished the "easier and more important parts" typographically from the more difficult. Although Harvard was one of the first colleges to experiment with electives, most of the curriculum in most American colleges was required until after the civil war.[10] Consequently, Farrar also needed a text that was accessible to those with little talent for mathematics.

The book opens by explaining that the object of calculus is "to decompose quantities into the elements of which they are composed, and to ascend or go back again from the elements to the quantities themselves. This is, strictly speaking, rather an application of the methods, and even a simplification of the rules of the former branches of analysis, than a new branch" (p. 7). Bézout–Farrar develops calculus in a manner similar to that used by Benjamin Peirce sixteen years later and by Peck and Bowser after that.

Farrar and those who followed him approached calculus from the perspective of infinitesimals. Despite the foundational improvements of Cauchy,

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[6]See Cajori, pp. 127–130.

[7]John Farrar, *First principles of the differential and integral calculus...*, Boston: Hilliard, Gray, & Co., 1836. I have considered this book a translation, as apparently Farrar did, and have not included it in my bibliography. Page references are from this edition.

[8]Carnot, Lazare N. M., *Réflexions sur la métaphysique du calcul infinitésimal*, Paris, 1797.

[9]Cajori, p. 132.

[10]Rudolph, Chapter 5.

this approach persisted into the twentieth century. Its best features, however, were adopted by the authors who favored limits. In this way, infinitesimal language outlived the line of texts that championed it. At the beginning of the line of limits authors is Charles Davies of the United States Military Academy at West Point.

The Military Academy rose to prominence after the arrival of Sylvanus Thayer in 1817. Thayer had been sent to Europe to study the systems military education used there. When he returned, he not only reorganized West Point along the lines of the French system, he also introduced French texts. In 1823, the chair of mathematics was assumed by Davies, a member of the class of 1815.[11]

Davies published over his career a series of books so widely used in the United States that Cajori refers to him thirteen years after his death as "one whose name is known to nearly every schoolboy in our land."[12] In addition to serving for 21 years at West Point, Davies spent four years as professor of mathematics at Trinity College in Hartford, another year at the University of New York and eight years as Professor of Higher Mathematics at Columbia College.[13]

The Davies series in mathematics eventually ran from arithmetic through calculus and included books on surveying and navigation, descriptive geometry, and "Shades, Shadows, and Perspective." Some of those books were translations, but others were more original. We, of course, are interested in his calculus.

Davies' Calculus

Davies' calculus books appeared between 1836 and 1901, with new editions every year or two between 1836 and 1860.[14] In the preface to the "Improved Edition" of 1843, he asserts that he is not writing an exhaustive book on the calculus, but only an "elementary treatise" as a textbook. He also acknowledges his sources:

> The works of Boucharlat and Lacroix have been freely used, although the general method of arranging the subjects is quite different from that adopted by either of those distinguished authors.[15]

[11]For background on West Point, see Ambrose.

[12]Cajori, p. 118.

[13]For biographical data on Davies, see Cullum. For assessments of the importance of the Military Academy in the development of American mathematics, see Cajori, p. 114ff, and Grabiner, "Mathematics".

[14]Although many editions appeared, I do not know how many copies were printed and/or sold of each one. Thus we can only conclude that there was a continuing market of an unknown size in this period.

[15]This and other references come from the 1843 edition of *Elements*.

Both Boucharlat and Lacroix were authors of popular French texts. Boucharlat's was published between 1815 and 1858, with a ninth edition appearing in 1926. (Boucharlat died in 1848, between the fifth and sixth editions.) His text was translated into English by Blakelock in 1828.[16] Lacroix was certainly the better mathematician and the better known.[17] His treatises on calculus appeared in long and short forms and in many editions between 1797 and 1881. His calculus books were extremely influential in Europe, as well as in the United States. In particular, his elementary treatise was translated into English by Babbage, Peacock and Herschel as part of their campaign to bring continental mathematics to England.[18] As we have seen, a number of his other texts were translated and used here.

Davies' book follows Boucharlat quite closely, both in the words he chooses and the examples he uses. Davies begins by noting that "if two variable quantities are so connected to each other that any change in the value of one necessarily produces a change in the value of the other, they are said to be functions of each other." This symmetric view of the functional relationship will prove very handy, for the text emphasizes the calculus of curves, as opposed to that of functions. He next examines, using the specific examples $u = ax^2$ and $u = x^3$, what happens when the independent variable is incremented by h. Looking at the quotient $(u' - u)/h$ where u' is the incremented value of the function, he declares:

> If we examine the second members of these equations, we find a term in each which does not contain the increment h.... If now, we suppose h to diminish, it is evident that the terms $2ax$ and $3x^2$, which do not contain h, will remain unchanged, while all the terms which contain h will diminish. Hence, the ratio
>
> $$\frac{u' - u}{h}$$
>
> in either equation, will change with h, so long as h remains in the second number of the equation; but of all the ratios which can subsist between
>
> $$\frac{u' - u}{h}$$

[16]For biographical data on Boucharlat, see *Nouvelle Biographie Générale...*, Paris, 1862, vol. 6, p. 855f.

[17]See Kline for references to Lacroix's work.

[18]See references in Grabiner and Kline.

> is there one which does not depend on the value of h? We have seen that as h diminishes, the ratio in the first equation approaches $2ax$, and in the second to $3x^2$; hence, $2ax$ and $3x^2$ are the limits toward which the ratios approach in proportion *a*[*s*] h is diminished; and hence, each expresses that particular ratio which is independent of the value of h. This ratio is called the limiting ratio of the increment of the variable to the corresponding increment of the function (pp. 17, 18).

Davies is a teacher, and not an extensively educated one. His concern is pedagogical, not mathematical. Yet even Davies is concerned that his readers may not understand this explanation. He tries another.

Davies goes on to say that "the limiting ratio of the increment of the variable to that of the function ... is called the differential coefficient of u regarded as a function of x" (p. 19). He immediately introduces an infinitesimal argument for defining the differential of x, telling students to "represent by dx the last value of h, that is, the value of h, which cannot be diminished, according to the law of change to which h or x is subjected, without becoming zero ...."

After explaining that du is the "corresponding difference between u' and u," Davies attempts once more to help his struggling students:

> It may be difficult to understand why the value which h assumes in passing to the limiting ratio, is represented by dx in the first member and made equal to 0 in the second. We have represented by dx the *last* value of h, and this value forms no appreciable part of h or x. For, if it did, it might be diminished without becoming 0, and therefore would not be the *last* value of h. By designating this last value by dx, we preserve a trace of the letter x, and express at the same time the last change which takes place in h, as it becomes equal to 0 (p. 18).

Notice also that Davies has not established any notation for calculating the derivative. Thus, when he wants to prove a theorem, he must go back to first principles. Davies gets around this stumbling block by introducing a property of the derivative used by Lagrange,[19] namely that

$$u' - u = Ph + P'h^2$$

where P is the differential coefficient and P' will in general be a function of h, as well as of x. He explicitly assumes this result on the basis of his previous examples (p. 21).

[19]See Grabiner, *Origins*, p. 118ff, for a discussion of the importance of this result.

Using this tool, Davies goes on to derive the rules of differentiation, including the chain rule. He also develops Taylor's Theorem and Maclaurin's and pays some attention to "cases in which [they do] not apply;" that is, to cases in which one of the derivatives is undefined. Again, this latter material shows the influence of Lagrange. All of the above is achieved in under fifty pages.

Davies' text is "Lagrangian" throughout. For example, he finds the derivative of an exponential function by using the binomial theorem to expand $(1 + b)^h$. Less than eighty pages into the book, he is developing the Taylor series for functions of two variables. His proofs of l'Hospital's rule and of what we sometimes call the first and second derivative tests for extrema are based on the Taylor series expansions.

The only application of the derivative in Davies is curve sketching. However, this subject is treated exhaustively. Indeed, as much space is devoted to this topic as to all of the "theory" of the derivative, including Taylor series. Davies discusses cusps, multiple points, involutes and evolutes, osculating curves, and transcendental curves, such as spirals and the cycloid. Then he turns to integrals.

For Davies, the integral is the antiderivative, "the method of finding the function which corresponds to a given differential." He does note that the integral sign denotes a sum and "was employed by those who first used the differential and integral calculus, and who regarded the integral of $x^m\,dx$ as the *sum* of all products which arise by multiplying the mth power of x, for all values of x, by the constant dx" (pp. 189, 190). Now Davies spends fifty pages on techniques of integration organized in several broad categories, including "Integration by series" (pp. 201–206). The book closes with 40 pages of geometric applications of integration: Rectification of curves, quadrature of curves and curved surfaces and cubature of solids, including double integrals.

Davies' text was the first commercially successful calculus text written by an American. Of course, we must be careful, for, as we have noted, it was largely derived from French work. As we shall see, authors continued to acknowledge sources of inspiration for many years.

The Antebellum Books and Their Authors

Before the civil war, American colleges generally had a fixed curriculum which included mathematics at an elementary level and a smattering of science. As new colleges spread through the West and South, they replicated the style and form of the earlier colleges from which their founders came. By the war, there were about 200 colleges in the United States, the majority of them founded after 1840 and many of them on the frontier.[20] The philosophy of the frontier, as well as other changes in the mood of the country, would affect the nature of higher education in the decades following the war. Until that time, with the exception of West Point and Rensselaer Polytechnic Institute, America needed only brief calculus books to strengthen and decorate the mind.

Beginning with Davies, these books were supplied by eight commercial authors who began publication before 1870. Six of them used limits as the foundation for the derivative; the other two, Peirce and Smyth, used infinitesimals. One (Loomis) had studied abroad for a year, but the remainder had received domestic educations, half of them at the Military Academy.

TABLE 1: Commercial Authors Who Began Publication Before 1860

Name	**Dates**	**Edit**	**Education**	**Positions**
Davies	1836–1868	many	USMA*	USMA, Columbia
Peirce	1841–1862	3	Harvard	Harvard
Church	1842–1872	many	USMA	USMA
M'Cartney	1844–1848	2	Jefferson	Lafayette
Loomis	1851–1902	many	Yale, Paris	Yale, Western Reserve, U. City of NY
Smyth	1854–1859	2	Bowdoin	Bowdoin
Courtenay**	1855–1876	8	USMA	USMA, U.PA, U.VA
Quinby	1856–1879	6	USMA	USMA, U. Rochester

Dates = span of frequent publication;
Edit = number of editions
*United States Military Academy
**Courtenay died in 1853, leaving a manuscript.

That West Point was so well represented is not surprising. This was the paragon of scientific education in the United States during that period. In addition to supplying engineers for the country's expansion, it was providing educators. Cajori (p. 127) reports that the Academy had provided 192 educators to American colleges, including 119 teachers, numbers that the Board of Visitors of the Academy, at least, found praiseworthy.

The only author on the list who might be called a professional mathematician is Benjamin Peirce, perhaps the outstanding American mathematician of his time and a founding member of the National Academy of Sciences.

[20]See Hofstadter, pp. 11–13, and Tewksbury, Chapter 1.

Peirce was a true nineteenth-century mathematician. His *Linear Associative Algebra* was the first major American contribution to pure mathematics.[21]

Peirce's book, apparently based on Farrar's, is mathematically intriguing but pedagogically painful. Before beginning his discussion of calculus, Peirce devotes a chapter to theorems on infinitesimals, proving for instance that "any power of an infinitesimal is infinitely smaller than any inferior power of the same infinitesimal." Although he doesn't provide a definition for an infinitesimal, he carefully lays out a program of definitions, theorems and corollaries which would delight the mathematician but horrify the sophomore.

The standard defense of the infinitesimal approach, however, is that it is more accessible to students and easier to apply to problems. Peirce's contemporary, William Smyth, justifies his choice of the "method of Leibnitz" on these grounds.

> The recent textbooks, both English and French, are in general based on the method of Newton [i.e., limits]. The expediency of this may well be questioned. The artifice which lies at the basis of the Calculus, consists in the employment of certain special auxiliary quantities adapted to facilitate the formation of the equations of a problem. The limit, or differential coefficient, the auxiliary employed in the method of Newton, is not easily represented to the mind, and being composed of two parts which cannot be separately considered [*dx* and *dy*?], it is with more difficulty applied to the solution of problems. On the other hand the differential ... is simple in itself, is very readily conceived, and adapts itself with wonderful facility to all the different classes of questions which require for their solution the aid of the calculus.[22]

As if to prove his point, Smyth includes in his text sections on applications to mechanics and astronomy in addition to those included in Davies. He considers the problem of a body falling through a hollow tube to the center of the earth, as well as center of gravity and fluid pressure problems. His book runs 240 pages and concludes with a section on the theory of limits.

Growth of the University: 1870–1895

After the Civil War, publication of calculus books from both the limit and infinitesimal lines continued. A new line, the method of rates, appeared, flourished briefly, and failed with the growing awareness in the United States

[21] For biographical information on Peirce, see Carolyn Eisele, "Benjamin Peirce," *Dictionary of Scientific Biography*, vol. 10, pp. 478–481. See also J. Grabiner, "Mathematics," p. 18, Cajori, p. 136, and Thwing, p. 304, for comments on Peirce as a teacher.

[22] Smyth, preface to the first edition.

of Cauchy's foundational work. Until the end of the century, however, authors continued to discuss the best method of presentation.

The educational environment in which these authors worked was different in several ways from that of their predecessors. The elective system replaced the required curriculum as the standard mode of education. The public began to demand a more practical education, a desire that was given support by the Morrill Act. Finally, American industrialists were funding their visions of higher education. All of these changes had begun in the decades before the war, but their impact came later.

Colleges had experimented with elective systems since 1824, when the University of Virginia adopted a completely elective curriculum. That experiment ended in 1831. Another attempt at Harvard about the same time also failed. However, the elective system established itself when the demands of the public for a more practical education and the intellectual demand of the sciences for a larger piece of the curricular pie had to be met.

Science, and mathematics with it, bloomed in the new land-grant colleges designed to encourage the study of agriculture and the mechanic arts, and authorized by Congress in The Morrill Act of 1862. It also flourished in the "Scientific Schools" formed at established colleges. The Lawrence School, established at Harvard in 1847 and the Sheffield School, established at Yale in 1854, both enriched by the gifts of wealthy patrons, are two examples. Finally, science and mathematics benefited through the creation of universities, such as Cornell, in 1869, and Johns Hopkins, in 1874, both named to honor their wealthy industrialist benefactors. In them, research and graduate education assumed a greater role than they had played in the colleges. Thus in the final quarter of the century, the German-style university began to replace the classical college as the model of American higher education.[23]

Scientific education for more, but better motivated, students demanded more advanced mathematics texts. Students needed calculus to study modern science. Ready to meet the demand were not only the earlier texts of Church and Loomis, for example, but also those of a more modern set of authors.

The Authors

Many of these authors (see chart) followed much the same career path as their predecessors. Others went different ways. Buckingham, a Military Academy graduate, was the president of the Chicago Steel Works when his books appeared. Byerly, the first Ph.D. on our list, was one of Harvard's first as well, with a dissertation on the heat of the sun. Along with Byerly, Johnson was one of the early members (the first from outside the New York

[23]See Rudolph on the rise of the American university; see also Grabiner, "Mathematics," pp. 17–23.

area) of the New York Mathematical Society, which soon became the American Mathematical Society. He wrote the first article in the first issue of the *Bulletin*.[24] One of the authors, Newcomb, deserves special mention.

Simon Newcomb was the fourth president of the American Mathematical Society. In Archibald's *Semicentennial History*..., the biographical sketches of the first three presidents take a total of fourteen pages; Newcomb's takes fifteen. Newcomb was America's foremost astronomer and was recognized internationally for his work. He was also a "scientific statesman," as his membership in many academies of science, his honorary degrees and his editorship of the *American Journal of Mathematics* show.[25]

TABLE 2: Commercial Authors who Began Publication 1870–1895

Name	Dates	Edit	Education	Positions
Olney	1870–1885	4	no formal	Kalamazoo, U. Mich
Peck	1870–1877	5**	USMA*	USMA, U. Mich, Columbia
Johnson##	1873–1909	many	Yale	USNA#, Kenyon, St. John's
Buckingham	1875–1885	3	USMA	Kenyon
Byerly	1879–1902	many	Ph.D. Harvard	Cornell, Harvard
Bowser	1880–1907	many	Santa Clara, Rutgers	Rutgers
Osborne	1889–1910	many	Harvard	USNA#, MIT
Taylor	1884–1902	9	Colgate	Colgate
Bass	1887–1905	6	USMA	USMA*
Newcomb	1887–1889	2	Harvard	Naval Observatory, Nautical Almanac, Johns Hopkins U.

Dates = span of frequent publication;
Edit = number of editions;
*United States Military Academy;
#United States Naval Academy;
**includes an edition published after 1877;
##includes the books written jointly with John Minot Rice.
Authors in *italics* used infinitesimals; in **boldface**, rates.

[24]For further information on Byerly and Johnson, see Archibald.
[25]Archibald, p. 124ff.

During this period, some authors cite other works in their prefaces, but some of the cited works are American. Some authors thank professional colleagues. Some do neither. Among the foreign works cited, Bertrand appears on the lists of Bowser, Byerly and Johnson. Bertrand's *Traité de calcul*[26] appeared in 1864 and nominally uses limits as its approach to calculus. However, very early in the book, Cauchy's definition of an infinitesimal, which we discuss below, appears and an extended discussion of orders of infinitesimals of the sort that Peck gives (see below) follows. Other European texts cited include the British books of Price, Todhunter and Williamson and Duhamel's book from France.[27] Generally, outside sources receive less attention than they do in the earlier period: Books are allowed to stand on their own.

Of the ten commercial authors who began publication in this period, only four based their books on limits. Three based their presentation on infinitesimals and three based theirs on the method of rates. (See Table 2.) In this period, we begin to see the merger of the infinitesimal approach into that of limits.

Limits

In the preface to his text, Osborne asserts that he has based his text "on the method of limits, as the most rigorous and most intelligible form of presenting the first principles of the subject." He goes on to state that many students have been introduced to limits in earlier courses and "may be assumed to be fully conversant with it on beginning the Differential Calculus."[28] Byerly says that a feature of his book is "the rigorous use of the Doctrine of Limits as a foundation of the subject," but adds that it's "preliminary to the adoption of the more direct and practically convenient infinitesimal notation and nomenclature ...."[29] Bass, writing later, believes that "the more rigorous and comprehensive method of infinitesimals is suitable only for a treatise, and not for a textbook intended for beginners."[30]

Whatever their belief, they all define and use infinitesimals. Bass introduces them on the page on which he defines the limit of a variable (p. 25). Byerly's introduction is much later (Chapter X). Osborne in his earlier work

[26]Joseph Louis Bertrand, *Traité de calcul différentiel et de calcul intégral*, Paris, 1864–1870. His *Cours...* appeared in 1875 and was republished until the end of the century.

[27]Bartholomew Price, *A treatise on the differential calculus...*, 1848 and later, Oxford. Price used the method of infinitesimals. Isaac Todhunter, *A treatise on the differential calculus and the elements of the integral calculus...*, 1852. Editions appeared until 1923. Benjamin Williamson, *An elementary treatise on the Differential Calculus...*, 1872, London. Jean Duhamel, *Cours d'analyse...*, 1840, Paris.

[28]Osborne, *Elementary treatise*, 1903, p. iii. The preface is dated 1891; the copyright date is 1891.

[29]Byerly, *Differential calculus*, 1879, p. iii. All Byerly references are to this edition.

[30]Bass, *Elements*, 1901, p. iii. The quotation is from the preface to the 1895 edition.

describes dx as an infinitely small Δx and lets it go at that, but in 1908 he talks about infinitesimals in a standard way.

The "standard way" is Cauchy's formulation: An infinitesimal is a variable with limit zero. With this convention, limits authors are able to utilize the advantages of infinitesimal techniques without becoming mathematically suspect, and its use continued well into the twentieth century.[31]

What is gained formally is the legerdemain of replacing limit talk with algebra, as the following proof from Bass illustrates. We wish to prove that if U and V are variables which under their laws of change are always equal, then their limits are equal. If C is the limit of U, then $C = U + e$ where e is an infinitesimal, or $U = C - e$. Since $U = V$, we have $V = C - e$ or $C = V + e$. Hence C is also the limit of V.[32]

We, of course, use this theorem regularly in our calculus courses. If $U = [(x + h)^2 - x^2]/h$ and $V = 2x + h$, then U and V, "under their laws of change," namely h is not zero, are always equal. Since V differs from $2x$ by an infinitesimal, its limit is $2x$ and, by the theorem, so is that of U. Interestingly, Bass gives roughly this example before he states his theorem. Indeed, the role of theory in Bass' book is uncertain.

Although much of the talk in these books seems to be about variables, in fact the authors clearly have functions in mind. For example, Bass defines the limit of a variable as

> a fixed quantity or expression which the variable, in accordance with a law of change, continually approaches but never equals; and from which it may be made to differ by a quantity less numerically than any assumed quantity however small (p. 22).

He makes clear, however, in a footnote, that he means the term "variable", to include all functions. For Bass and for Byerly, a function is a quantity the value of which depends upon another quantity. Indeed, when Bass gets around to defining the differential coefficient and differentials, he is quite explicit in his use of functional notation (although his explanation is quite obscure) (pp. 47, 55).

Notice how modern Bass' definitions of function and limit appear when compared to Davies'. A "modern" application also appears. There are related rates problems, including ships sailing on perpendicular courses and men walking to or from lamp posts (p. 84ff). However, there are no extremum problems[33] and, like Davies, he belabors the geometry of curves. Again, he includes all of the topics in Davies, and adds a number of exercises.

[31] See, for example, Granville et al., 1904 and later.

[32] Bass, *Introduction*, p. 25. Other citations from Bass are from this book.

[33] His 1901 book does include extremum problems.

Bass does not treat integration. Both Byerly and Osborne do, and, again, in a modern way. Integrals and definite integrals are identified as different entities. That requires a Fundamental Theorem of Calculus. Quinby was the only earlier author to prove this. Byerly[34] uses an infinitesimal argument (as did Quinby); Osborne provides an example that shows that the limit of the sum yields the difference of the two values of the antiderivative (my term).

In addition to the applications of the integral to the geometric problems of arc length, area and volume, Byerly and Osborne apply it to physical problems. Byerly treats centers of gravity, mean distances and probability. Osborne discusses moments of inertia.

Finally, these authors used series much more carefully than their antebellum colleagues. All of them, at least by their later editions, worried about convergence. All of them stated and proved—more or less—Taylor's theorem with the Lagrange remainder.[35]

INFINITESIMALS

By contrast, the infinitesimal authors all develop series in a manner essentially identical to that of the earlier group. An expansion of $f(x + h)$ in the form $A + Bh + Ch^2 + \cdots$ is assumed and the necessary values of the coefficients are calculated. Some attention is paid to values for which the development fails, as before.

While their treatment of series reflects a dated foundational view, all of the authors are clearly aware of the limit approach and have deliberately rejected it for pedagogical reasons. Olney chooses infinitesimals because it's simpler and because it facilitates the application of calculus to "practical problems." He goes on to criticize the "general use" of limits in textbooks for "preventing the common study" of calculus. His complaint is the same as Smyth's fifteen years earlier.

> This method is not only exceedingly cumbrous, but it has the misfortune that its element is a ratio. The abstract nature of a ratio, and the fact that it is a compound concept, peculiarly unfit it for elementary purposes. The beginner will never use it with satisfaction, for it does not give him simple, direct and clearly defined conceptions.[36]

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[34]There are differences between the 1881 and 1889 editions of Byerly's *Integral calculus*. In the former, he speaks of the computation of the definite integral as the limit of a sum; in the latter, of its definition that way.

[35]Byerly, whose text is much more modern than those of his contemporaries, uses Rolle's Theorem in 1879 to get the remainder.

[36]Olney, 1871, p. v. All references to Olney are from this edition.

While not as outspoken as Olney on the disadvantages of limits, both Bowser and Peck agree with him on the advantages of infinitesimals. It's the easiest method to understand and apply.

But just as the limits authors did not neglect infinitesimals, neither do these authors neglect limits. Bowser devotes his third chapter to "limits and derived functions" and, using limit techniques, rederives the basic formulas. Olney claims (p. 5) that, in fairness he will introduce and use the theory of limits when he wants to. Peck's "Note on the Method of Limits" appears as an appendix, is not listed in the table of contents and is credited to E. H. Courtenay whose publisher was also A. S. Barnes.[37]

Peck, son-in-law of Charles Davies, developed infinitesimals in much the same manner as Bowser and in only a slightly different way from Peirce.[38] He defined a quantity to be infinitely small with respect to another if the quotient was "less than any assignable number." A number that was infinitely small when compared to a "finite number," for instance 1, was called an infinitesimal. He followed these definitions with a discussion of orders of infinitesimals that was less formal than Peirce's, but had the same objectives. Peck concluded that "an infinitesimal may be disregarded in comparison with a finite quantity, or with an infinitesimal of lower order" when added or subtracted (pp. 13, 14).

Now Peck is ready to teach the student to find differentials. He has a very simple algorithm.

> In order to find the differential of a function, we give to the independent variable its infinitely small increment, and find the corresponding value of the function; from this we subtract the preceding value and reduce the result to its simplest form; we then suppress all infinitesimals which are added to, or subtracted from, those of a lower order, and the result is the differential required (p. 14).

He then notes wisely that the method is too long for general use and will be employed only to derive some general rules.

Consider Peck's proof of the quotient rule. If s and t are functions of x, we are required to find $d(s/t)$. Augment x by its infinitely small increment dx. Then s is augmented by ds; t by dt; and s/t by $d(s/t)$. Thus we have

$$\frac{s+ds}{t+dt} = \frac{s}{t} + d\left(\frac{s}{t}\right).$$

[37]Courtenay, *Treatise*.

[38]Peck, *Practical treatise*, 1870. Page references that follow are from this edition.

Subtracting s/t from both sides, finding a common denominator on the left and simplifying, we now have

$$\frac{t\,ds - s\,dt}{t^2 + t\,dt} = d\left(\frac{s}{t}\right).$$

But we can suppress the $t\,dt$ term in the denominator because it is of lower order (p. 17).

Infinitesimals work particularly well for applications. For example, since a curve is made up of infinitesimal elements, the slope of a tangent line to a curve is dy/dx practically by definition, or, as Peck says, "an element of the curve ... does not differ from a straight line. Hence, the slope of a curve, at any point, is measured by the first differential coefficient at that point" (p. 53). Similar arguments provide the motivations for arc length and area.

In the same manner, suppose the object of integration to be, as Bowser says, finding "the relations between finite values of variables from given relations between the infinitesimal elements of those variables, or ... the process of finding the function from which any given differential may have been obtained" (p. 238). Then the area under $y = f(x)$ is, naturally, given by the integral of $f(x)\,dx$, the differential of the area, and the length of the curve is the integral of ds, the differential of the arc length. This intuition also leads to direct solutions of many physical problems. Interestingly, only Peck, Professor of Mechanics in the School of Mines as well as Professor of Mathematics and Astronomy at Columbia College, among the infinitesimal authors provides applications other than geometric ones.

The infinitesimal books are clearly dated when compared to the books using limits in this period. The books using rates, however, were truly from another age. They represent a return to fluxions.

THE METHOD OF RATES

While infinitesimal techniques remain part of our heuristics in teaching calculus, the method of rates has been completely discarded. In this approach, calculus is regarded directly as the mathematics of change. The fundamental questions of calculus, proponents of rates argue, are not about tangent lines and areas, but about how one quantity changes in response to changes in another. This point of view is, of course, that of Newton's fluxions, but, except for one British book of 1845[39], it had completely disappeared even in England when the books we are considering were written.

Nevertheless, two very important texts used this method into the first decade of this century: Rice and Johnson and James M. Taylor. As I noted above, Johnson was an important member of the American mathematical

[39] Connell, James, *The elements of the differential and integral calculus*, London: Longman & Co., 1845. This was the only edition of the book.

scene. He served as one of only five elected members of the Council of the American Mathematical Society for the 1892–1893 term.[40] Thus, while his choice may have been eccentric, it was not an ignorant one. The last rates text, that of Edward Nichols, appeared in 1900 with a second edition in 1918.[41]

All of the authors eventually introduced limits. Taylor and Nichols did this in early chapters and then proceeded to use whatever seemed handiest. Rice and Johnson gave in somewhat later. Of course, our interest here is not in the limit portion of the texts, but the more unusual part.

The authors begin by describing uniform change of a variable as occurring when "its value changes equal amounts in equal arbitrary portions of time." Now turning to non-uniform change, Taylor says (Rice and Johnson is quite similar),

> If a variable changes non-uniformly with respect to x, the measure of its rate is what its increment corresponding to the increment 1 of x would be if at the value considered its change became uniform (p. 7).

Now the differential of a variable is its rate of change.

In this system, the differential triangle with legs dx and dy and hypotenuse ds is not simply the representation of an infinitesimal figment by a fine Euclidean object, but the realization of what would have happened had the rates become uniform. Similarly, if $A(x)$ is the area under the curve $y = f(x)$, then dA is $y\,dx$ simply from the definition of the differentials involved. This system thus had advantages for certain applications.

However, the method of rates exacted a terrible cost when the authors tried to prove something as basic as the product rule for derivatives. Rice and Johnson derive this from the rule for differentiating x^2, since $xy = (1/2)(x+y)^2 - (1/2)x^2 - (1/2)y^2$. That rule is not easy to derive. First, they set $z = mx$. Then it follows that $dz = m\,dx$ and that $d(z^2) = m^2 d(x^2)$. Now dividing the second of these equations by the first, substituting z/x for m and separating the variables, they get

$$\frac{1}{z}\frac{d(z^2)}{dz} = \frac{1}{x}\frac{d(x^2)}{dx}$$

At this point the authors invoke their Fundamental Theorem: The value of dy/dx does not depend upon dx, but is a function of x alone (p. 17). Thus, denoting x^2 by $f(x)$, the equation becomes $f'(z)/z = f'(x)/x$ and this is true for all values of x and z, since the constant m was arbitrary. Now we

[40] Archibald, p. 97.

[41] Rice and Johnson, *The elements.* The argument used to derive the product rule is from the first part of the 1874 book. Taylor, *Elements.* Citations are from the 1894 edition. Nichols, *Differential.*

can conclude that $f'(x)/x$ is some constant c, or that $d(x^2) = cx\,dx$. The remaining problem is to find c.

To find c, we apply this last result to the identity

$$(x + h)^2 = x^2 + 2xh + h^2.$$

Since we know that the differential of a constant is zero and that the differential of a sum is the sum of the differentials, we have

$$c(x + h)\,dx = cx\,dx + 2h\,dx \text{ or } (c - 2)h\,dx = 0.$$

"Since h and dx are arbitrary quantities, we have $c = 2$, which gives ... $d(x^2) = 2x\,dx$" (pp. 21–23).

Despite some painful developments such as this, the method of rates authors believed that their approach was, if not the best, a satisfactory one. Rice and Johnson were blunt about their choice.

> The difficulties usually encountered on beginning the study of the Differential Calculus, when the fundamental idea employed is that of infinitesimals or that of limits, together with the objectionable use of infinite series involved in Lagrange's method of derived functions, have induced several writers to return to the employment of Newton's conception of rates or fluxions (1877, p. iii).

Taylor was more circumspect by 1898.

> ... [A]n attempt has been made to present in their unity the three methods commonly used in the Calculus. The concept of Rates is essential to a statement of the problems of the Calculus; the principles of Limits make possible general solutions of these problems, and the laws of Infinitesimals greatly abridge these solutions (1898, p. iii).

Taylor goes to considerable pains to defend rates against the charge that it invokes a "foreign element," namely time. However, these late nineteenth century books would be the last gasp of fluxions in America.

The Professional Climate, 1870–1895

One of the factors in the coming demise of the nineteenth century texts was the growth of a mathematical community. In 1888, the New York Mathematical Society was founded and by 1895 (as the American Mathematical Society), it had 268 members. It also had a new journal that published reviews of books, including calculus texts, and discussions on the teaching of calculus.

Reviews of calculus texts had appeared in earlier journals, but they tended to be complimentary rather than analytic. Issues of the *Mathematical Visitor* regularly contained notices of new books. In January 1880, the editors describe Rice and Johnson's revised edition as "the most extensive work on the Differential Calculus yet published in this country" and "heartily commend it to all who want a good textbook on the subject." In the same issue, they describe Byerly's book as "a good work" and a year later call Bowser's "a work of rare excellence." The *Analyst* published similar reviews.[42]

By contrast, in the first volume (1891–1892) of the *Bulletin* of the New York Mathematical Society, Charlotte Scott[43] wrote a scathing seven-page review of a British text by Joseph Edwards. After noticing how well written it was and how nicely printed, Scott asserted that the book had "many defects" and she proceeded to point them out. Among other faults, she observed that Edwards was not aware of Weierstrass's example of a continuous, nowhere-differentiable function.

Other articles also reflected the intellectual growth of the mathematics community. The opening paper of the October, 1893 issue of the *Bulletin* was a reprint of Felix Klein's inaugural address to the Chicago congress and the following paper in the same issue was a report by T. H. Safford on "Instruction in Mathematics in the United States." The December issue contained a translation of a circular describing the program in mathematics at Göttingen. A new age in mathematics was beginning.

It is not clear, however, that teachers of calculus were ready for the new day. In his 1889 survey of American colleges and schools[44], Cajori discovered that about half of those teaching calculus favored limits. Almost 30% favored infinitesimals. (140 of the 160 colleges and universities responding to the survey apparently taught calculus.) Since the vast majority of the respondents taught mainly from textbooks, the disappearance of the infinitesimal books near the beginning of the new century should have been the cause of some concern.

A New Century

The "old fashioned" books disappeared as new standards for authors emerged by the beginning of the twentieth century. Among those (see Table 3) who began publication in the period between 1900 and 1910, at least 30% had studied in Europe and 45% had doctoral degrees. About half of the degrees were from U. S. universities. This phenomenon, the ascendancy of the Ph.D., is as marked as any textbook feature we have examined.

[42]See Cajori, p. 277ff, for comments on journals of the nineteenth century.

[43]Charlotte A. Scott, "Edwards' Differential Calculus," *Bulletin of the New York Mathematical Society*, 1 (1891–92), p. 217ff.

[44]Cajori, pp. 296–360.

TABLE 3: Commercial Authors who Began Publication 1895–1910

Name	Dates	Edit	Education	Positions
Gould	1896–1907	4	E. des Mines	engineer
Fisher	1897–1909	8[1]	Ph.D. Yale, European study	Yale
Hall	1897–1905	7[2]	Lafayette	Lafayette
Lambert	1898–1907	2	Lehigh, European study	Lehigh
Love	1898–1899	2	U. No. Carolina Harvard, Hopkins	UNC, Harvard
Murray	1898–1908	4	Ph.D. J.Hopkins	Dalhousie, NYU, Cornell, McGill
Hardy	1900–1912	2	Lafayette	Lafayette
Nichols	1900–1918	3	VMI	VMI
Echols	1902–1908	2	U. Virginia	Mo. School of Mines, U. Virginia
Osgood	1902–1938	8[3]	Ph.D. Erlangen	Harvard
Granville	1902–1957	many	Ph.D. Yale	Yale, Gettysburg, insurance
Smith			Ph.D. Yale, European study	Yale
Longley			Ph.D. Chicago	Yale, Colgate
Snyder[4]	1902–1912	2	Ph.D. Göttingen	Cornell
Hutchinson			Ph.D. Chicago, European study	Cornell
Campbell	1904–1919	5	Ph.D. Harvard	Harvard, IIT, actuary
Cain	1905–1911	4	NC Mil.Inst.	U. N. Carolina
Keller	1907–1908	2	*	
Knox			*	
Woods	1907–1954	many	Ph.D. Göttingen	Wesleyan, MIT
Bailey			*	
Townsend	1908–1911	5[5]	Ph.D. Göttingen	U. Ill
Goodenough			Mich. Ag. C.	Mich. Ag. C., U. Ill
Brown	1909–1912	2	Cornell	Naval Academy
Capron			Harvard	Naval Academy
Ransom	1909–1949	5	Tufts, Harvard	Harvard, Tufts

Dates = span of frequent publication; Edit = number of editions *Biographical data is missing

[1]Includes two editions published after 1909.

[2]Includes two editions published after 1905.

[3]Includes one edition after 1938; does not include his *Advanced calculus*.

[4]Snyder also published a book with James McMahon in 1898.

[5]Includes 1925 edition.

Authors in *italics* used infinitesimals; in **boldface**, rates

The Ph.D.'s with their uniform approach to the calculus dominated textbook production despite the fact that most calculus teachers did not have doctorates and did not learn their calculus from books like the new ones. In 1899, fewer than 180 Americans held doctorates in mathematics and most of them were located at the universities.[45] However, their number was growing and their professional organization, the American Mathematical Society, was growing too.

Between 1895 and 1907, the number of members of the AMS doubled to 568 and a single section had become four. Moreover, the presidents of the organization were young. Of the first ten (through 1910), only Van Amringe, McClintock and indefatigable Newcomb were over fifty when they presided. Half of them had studied in Germany.[46] In an age that cherished "progress," traditionalists would have been hard pressed to stop the rush of these enthusiastic students of brilliant German teachers to reform the teaching of the calculus.

The Bulletin of the Society continued to be filled with reports on teaching mathematics at all levels and on teaching calculus in particular. Osgood's presidential address in 1907 was called "The Calculus in Our Colleges and Technical Schools."[47] Importantly, calculus books were reviewed critically in the Bulletin.[48] Old publications were pushed out and new "modern" books took their place.

The Publication Record

Davies' books were published regularly for 44 years and editions appeared regularly over 65 years. Loomis' books appeared regularly for 36 years and the last edition was published 51 years after the first. Rice and Johnson was published for over 35 years and the staying power of Granville, Smith and Longley is legendary. Davies, Loomis, and Rice and Johnson, however, were separated from Granville, et al. by a barrier between the old and the new. Books published before the barrier did not get far beyond it. On the other side were the new books, the books of the new profession.

Looking at Table 4, we can see the abruptness of this change. No commercial author who began publication before 1897 and only three authors who began publication before 1902 had a calculus book published after 1912. This

[45]See Richardson for data on Ph.D.s. Richardson's data show that, in 1935, less than 30% of those teaching mathematics in colleges and universities had doctorates. Also see Kevles, Table 7 on the distribution of employment for productive Ph.D.s in the period up to 1915. Richardson's data and Kevles' are difficult to reconcile. The orders of magnitudes seem to agree, however.

[46]For the early history of the AMS, see Archibald.

[47]*The Bulletin of the American Mathematical Society*, 2nd series, vol. 13, June 1907, pp. 449–467.

[48]I have found reviews of about fifteen calculus texts in the *Bulletin* between 1900 and 1910.

TABLE 4: Publication Dates of Commercial Texts, 1885–1930, of Authors who Began Publication Before 1910

Author	85	90	00	10	20
Davies			*		
Loomis	**	*	*		
Olney	*				
Peck		*			
Rice & ...	****	* * * *	* * * *		
Buckingham	*				
Byerly	*	*** * **	**		
Bowser	***	*** * *	* * * *		
Osborne	*	** * *	*** ****	*	
Taylor	** *	** **	*		
Bass	* *	*	* * *		
Newcomb	* *				
Gould		* *	*		
Fisher		*	* * * *		*
Hall		***	* *	*	*
Lambert		*	*		
Love		**			
Murray		*	** *		
Hardy			*	*	
Nichols			* *	*	
Echols			* *		
Osgood			* * *		** * **
Granville, et al.			***	**	*
Snyder & ...			*	*	
Campbell			**	* * *	
Cain			* * *	*	*
Keller/Knox			**		
Woods/Bailey			* *	*	* * *
Townsend & ...			*	**	*
Brown/Capron			*	*	
Ransom			*	*	

Authors are listed in order of first publication date.
Authors in italics used infinitesimals; in boldface, rates.
Authors above first dashed line began publication before 1897; those between the lines, between 1897 and 1902. (See text)

was in spite of the fact that twelve commercial authors began publication between 1880 and 1901. Of the three, one was William Shaffer Hall who had two stray editions in 1915 and 1922; one was the fluxions author, Edward West Nichols; and the last was an economist, Irving Fisher. (Virgil Snyder, a Göttingen Ph.D., published with James McMahon in 1898 and with John Hutchinson in 1902 and 1912.)[49] The standard for calculus texts was changing so markedly and rapidly in the final decade of the nineteenth century

[49]Hall, *Elements*, 1897–1922; Nichols, *Differential*, 1900–1918; Fisher, *A brief*, 1897–1937; McMahon and Snyder, *Elements*, 1898; Snyder and Hutchinson, *Differential*, 1902 and *Elementary*, 1912.

that earlier authors, no matter how virtuous their product might have been, simply lost their market to the newcomers.

A "MODERN" BOOK

Just before the end of the century, the last two infinitesimal books appeared.[50] They were books small enough to fit in a coat pocket and cost less than a dollar each, according to the brief review by Thomas Fiske in the *Bulletin*. Although Fiske's tone is quite mild, he is clearly unhappy with the books. Gould's "gives rules without pretense of demonstration and almost without explanation." Fisher's is quoted to show his inappropriate handling of infinitesimals.[51]

A decade later, by contrast, the initial edition of Granville received an extended and generally positive review from Edward Van Vleck.[52] He asserts at the end of the review that he knows "of no work which has greater promise of success in our college classes." His assessment was accurate. Granville, Smith and Longley, successor to Granville's 1904 book, was the standard calculus text, the book against which others were measured in the United States for nearly five decades. The "modern textbook," as its authors described it, had arrived.[53]

Granville, like the authors of earlier books, has explicit pedagogical goals for his text. In the preface, he describes his book as a "drill book" and certainly one feature of the text is its large number of exercises. For example, compared with Snyder and Hutchinson's 1902 edition, Granville's book has about twice as many exercises on calculating the derivative and three times as many extremum problems. However, Granville has more to offer than exercises.

Granville believes that the results in the text "should be made intuitionally as well as analytically evident to the student." He chooses to introduce ideas and results intuitively first, then supply the analytic argument that proves the result (p. iii). His discussions of extrema dramatically illustrate his approach. In the ninth chapter, Granville produces the now-standard derivative tests for extrema by encouraging his readers to examine the graphs; forty pages and six chapters later, he proves the results using the Mean Value Theorem.

However to our ears, over eighty years later, some of his explanations sound forced and artificial. Partially, this is the result of dated ideas, such as subtangents and subnormals, that appear in the book and of language that

[50] Gould, *A primer*, 1896, and Fisher, *A brief*, 1897.

[51] Thomas S. Fiske, "Recent textbooks in calculus," *Bulletin of the American Mathematical Society*, Series 2, 4 (Feb. 1898), p. 237f.

[52] Edward B. Van Vleck, "Granville's Differential and Integral Calculus," *Bulletin of the American Mathematical Society*, Series 2, 11 (Jan. 1906), p. 181ff.

[53] Granville, *Differential*, 1904, p. iii. All references are to this edition.

we find stilted, but partially this is the result of trying to deal with difficult technical notions without using precise mathematical language.

The book has been denigrated for its lack of rigor, and, from our vantage point, there is plenty of material that is open to question.[54] For example, he explicitly assumes that all functions are continuous and continuously differentiable, except possibly at a finite number of points. Some of his proofs, such as that for the chain rule, are faulty, and he seems to prefer quick, if not quite correct, proofs to more complete and careful ones. Indeed, many of Granville's "lapses", such as the chain rule proof, appear deliberate, as if, knowing better, he has chosen not to write a rigorous book. We should not confuse, however, Granville's intent with that of the more rigorous contemporary texts of, for example, Hardy, or with treatises on the theory of functions of a real variable, such as Jordan.[55] Granville is writing an introductory text for American students. In fact, he does avoid the most egregious errors of his predecessors. However, Granville is still talking about limits of variables and infinitesimals.

Granville invokes a new standard formula for the limit of a variable.

> If a variable v takes on successively a series of values that approach nearer and nearer to a constant value l in such a manner that $|v - l|$ *becomes and remains less* [my emphasis] than any arbitrarily small positive quantity, then v is said to approach the limit l, or to converge to the limit l (p. 19).

The phrase "becomes and remains less," or a similar one that reflects Weierstrassian mathematics, appeared in many of the new books, including Snyder and Hutchinson and Echols, and in later books.

Granville's fourth chapter is devoted to the theory of limits and infinitesimals in Cauchy's sense. He proves the theorems on the algebra of limits by first using a mixture of intuition and epsilons to validate—"prove" is too strong—the corresponding results for infinitesimals. He introduces the derivative as a limit in Chapter V. Finally, in the sixth chapter, using the limit theorems, he derives the algebra of derivatives and rules for differentiating all of the elementary functions. Then he turns to applications.

Some of the old geometry remains. In addition to subtangents and subnormals, radius of curvature, evolutes and envelopes are there. However, related rates, curve sketching and extremum problems appear in modern

[54]See Halmos for a nostalgic look at a later edition. I remember discussions about 1957 in which Granville, Smith and Longley, finally out of print, was still being flailed.

[55]Hardy, Godfrey Harold, *A course of pure mathematics*, Cambridge: The University Press, 1908. Hardy's well-known book went through ten editions plus two American editions and several reprintings of the ninth, which first appeared in 1944. Also, Jordan, Camille, *Cours d'analyse...*, Paris, 1882–87. The third edition of this three volume work appeared between 1909 and 1915.

form. The Mean Value Theorem rates a chapter heading and Van Vleck praises Granville's intuitive handling of the theory there. The section on series comes late, rather than early, in the about 280 pages devoted to the differential calculus.

The first seventy pages on integration are devoted to the indefinite integral, and include techniques of integration. The chapter on the definite integral begins with a section on the differential of an area. This, of course, is exactly the same argument that infinitesimal authors had used many years earlier. The infinitesimal line has now been completely absorbed into the limit books. However, Granville does go on to treat integrals as sums in his next chapter. His applications are to areas and volumes and to moments of inertia.

We have seen the features of this book emerging in the nineteenth century. In Granville, they are collected and refined. Nevertheless, Granville does not look like a text from 1985 or even like one from 1955. We have noted that Granville has not utilized the mathematics of his period fully and that some of his presentations are suspect. However, both the author and the reviewer thought of this as a modern book.

These "modern" authors and their contemporaries thanked their friends for help with their text. Some authors mentioned other books to which they had referred. For example, Murray and Echols each mention several, but Granville and Snyder mention none. There is no question, however, that these authors are presenting their own product, a product for the students of the American university.

CONCLUSION

During the nineteenth century, American calculus textbooks evolved from being translations and copies of European sources to mathematically "modern" resources. As the mathematical community became better educated, texts that were less acceptable to professional mathematicians began to disappear, to be replaced by ones that reflected more closely the foundations of the subject as developed by Cauchy and then Weierstrass.

In this paper, I have examined this evolution. While I have been primarily concerned with the analysis of the changing contents of the books, I have also tried to tie those changes to changes in the professional and educational contexts in which the authors of the books labored and in which the books found their market.

We know that the mathematical community was maturing during the second half of the century. We have seen that this development affected these texts both directly through the increased mathematical sophistication of the authors and indirectly through more stringent critical standards. By the early part of the twentieth century, books differed little from each other in content and approach, although thirty years earlier variety was more normal than similarity.

But the mathematical community itself was a subset of a larger academic environment which had changed markedly during the post-Civil War period. The classical college that had dominated American higher education from the founding of Harvard to that war was replaced by the "practical" colleges encouraged and supported by the Morrill Act. These, in turn, and others endowed by wealthy industrialists began to evolve by the end of the century into the great research universities.

These colleges and universities provided both an expanding market for texts and an academic home for well-trained mathematicians. Although few in number, these mathematicians, in turn, created a demand for better texts; that is, for texts that met the standards of the emerging mathematics profession.

References

Ambrose, Stephen E., *Duty, Honor, Country: A History of West Point*, Baltimore, 1966.

Archibald, Raymond Clare, *A Semicentennial History of the American Mathematical Society 1888–1938*, Amer. Math. Soc., 1938.

Cajori, Florian, *The Teaching and History of Mathematics in the United States*, U.S. Government Printing Office, 1890.

Cullum, George W., ...*Biographical Register of Officers and Graduates of the U.S. Military Academy...*, New York, 1879.

Grabiner, Judith V., "Mathematics in America, The First Hundred Years," in *Bicentennial Tribute to American Mathematics*, 1776–1976, edited by J. Dalton Tarwater, Math. Assoc. of America, 1976.

Grabiner, Judith V., *The Origins of Cauchy's Rigorous Calculus*, Cambridge, MA, 1981.

Halmos, P. R., "Some Books of Auld Lang Syne," *A Century of Mathematics in America, Part I*, Amer. Math. Soc., 1988, pp. 131–174.

Hofstadter, Richard, "The Development of Higher Education in America," in Richard Hofstadter and C. DeWitt Hardy, *The Development and Scope of Higher Education in the United States*, New York, 1952.

Kevles, Daniel, "The Physics, Mathematics, and Chemistry Communities: A Comparative Analysis," from *The Organization of Knowledge in Modern America, 1860–1920*, edited by Alexandra Oleson and John Voss, Baltimore, 1979.

Kline, Morris, *Mathematical Thought from Ancient to Modern Times*, New York, 1972.

Richardson, R. G. D., "The Ph.D. Degree and Mathematical Research," *American Mathematical Monthly*, vol. 43, 1936, pp. 199–215. Reprinted in *A Century of Mathematics in America, Part II*, Amer. Math. Soc., 1989.

Rudolph, Frederick, *The American College and University: A History*, New York, 1968.

Tewksbury, Donald G., *The Founding of American Colleges and Universities before the Civil War*, New York, 1965.

Thwing, Charles Franklin, *A History of Higher Education in America*, New York, 1906.

Appendix: Calculus Textbooks by American Authors, 1828–1920

This list attempts to cite all Calculus texts written by residents of the United States or Canada and published between the first such book (Ryan, 1828) and 1920. The closing date is arbitrary. Books that were published after 1920 are included in the list if the author has other texts that appeared before 1920 (e.g., Granville, Osgood). Furthermore, "American author" has been liberally interpreted to include natives of other regions who were in the United States or Canada when their books were published (e.g., Bonnycastle).

This list was compiled primarily by searching the shelves of the Library of Congress and a variety of other libraries for books from the appropriate period. When a book was discovered, the National Union Catalog, Pre-1956 Imprints was searched for additional information. Although I have examined many of the books listed, I cannot claim to have seen them all. While I am confident that the list is off by probably no more than ten authors, I have no confidence that it is complete or entirely correct. I invite (indeed, beg) additions and corrections.

This list differs from Cajori's "Bibliography of Fluxions and the Calculus" not only in the period covered but in the criteria for inclusion. Cajori was interested in textbooks printed in the United States. He therefore included books written by non-American authors (e.g., Hutton, Vince) but published in the United States. He also included books that were translations by Americans of foreign texts (e.g., Farrar's translation of Bézout). As we have seen, there is a fine line between translation and inspiration in some early texts.

Finally, Cajori has annotated his list extensively, while I have restricted my notes to a few technical details, such as a change of publisher.

The list is arranged alphabetically by primary author. The name of the primary author is followed by the name(s) of any collaborators. Beneath this is a chronological list of all calculus books written by the author with the first-listed book being the earliest. With each book is the date of first publication, followed by the date of last publication. The name of the publisher and the place of publication follow the dates. Occasionally, the publisher and/or the place of publication is missing.

Bass, Edgar Wales, *Introduction to the differential calculus* ... 1887, USMA Press, West Point, NY.

——, *Differential calculus* ..., 1889–1892, USMA Press and Bindery, West Point, NY.

____, *Elements of differential calculus*, 1896–1905, John Wiley & Sons, NY and London.

Bayma, Joseph (SJ), *Elements of infinitesimal calculus*, 1889, A. Waldteufel, San Francisco.

Bonnycastle, Charles, *Syllabus of a course of lectures, upon the differential and integral calculus*, 1838 C. P. M'Kennie, Charlottesville.

Bowser, Edward Albert, *An elementary treatise on the differential and integral calculus with numerous examples*, 1880–1907, D. Van Nostrand, NY.

Brown, Stimson Joseph (Capron, Paul), *The calculus, an elementary treatise on the differential and integral calculus, with applications, prepared for the use of the midshipmen of the United States Naval Academy*, 1909–1912, The Lord Baltimore Press, Baltimore.

Buchanan, Roberdeau, *An introduction to the differential calculus by means of finite differences*, 1905, Washington, DC, Reprinted from Popular Astronomy, vol. XIII, nos. 5, 6.

Buckingham, Catharinus Putnam, *Elements of the differential and integral calculus, by a new method, founded on the true system of Sir Issac Newton, without the use of infinitesimals or limits*, 1875–1885, S. C. Griggs & Co., Chicago.

____, *The method of final ratios commonly called the method of limits*, 1879, S. C. Griggs & Co., Chicago.

Byerly, William Elwood, *Elements of the differential calculus, with examples and applications*, 1879–1901 Ginn & Heath, Boston.

____, *Elements of the integral calculus, with a key to the solution of differential equations*, 1881–1902, Ginn, Heath & Co., Boston.

____, *Problems in differential calculus*, Supplementary to a treatise on differential calculus, 1895–1904, Ginn & Co., Boston.

A short table of integrals by B. O. Peirce, added starting in '89. An 1888 edition(?) was reprinted by G. E. Stechart & Co. (NY) in 1941.

Cain, William, *A brief course in the calculus*, 1905–1911, D. VanNostrand & Co., NY. Third edition apparently reprinted in London in 1930. Also paper, "On the fundamental principles of the differential calculus," J. Elisha Mitchell Scientific Soc., 1892.

Campbell, Donald Francis, *The elements of the differential and integral calculus, with numerous examples*, 1904–1919, The Macmillan Co., NY.

Chandler, George Henry, *Elements of the infinitesimal calculus*, 1907, Wiley, NY. I don't have the early publishing history of this book; the 1907 edition is the "3rd ed; rewritten."

Church, Albert Ensign, *Elements of the differential and integral calculus*, 1842–1872, Wiley and Putnam (notes) NY. Publishers: '55, Barnes: '60, '63, '64, Barnes & Burr.

Clark, James Gregory, *Elements of the infinitesimal calculus, with numerous examples and applications to analysis and geometry*, 1875, Wilson, Hinkle & Co., Cincinnati, NY.

Cook, Hiram, *An elementary treatise on variable quantities*, in two parts, the direct and inverse, 1921, Privately printed, Berkeley.

Book published after Cook's death in 1917. Preface dated 1916.

Courtenay, Edward Henry, *Treatise on the differential and integral calculus and on the calculus of variations*, 1855–1876, A. S. Barnes & Co., NY. Courtenay died in 1853.

Davies, Charles, *Elements of the differential and integral calculus*, 1836–1889, Wiley & Long (see note), NY.

——, *Elements of analytical geometry and of the differential and integral calculus*, 1859–1901, A. S. Barnes & Burr, NY.

——, *Differential and integral calculus, designed for elementary instruction*, 1860, A. S. Barnes & Burr, NY.

——, *Differential and integral calculus on the basis of continuous quantity and consecutive differences, designed for elementary instruction*, 1873–1901, A. S. Barnes & Co. (notes), NY & Chicago, Barnes.

Davies' books changed publishers: A. S. Barnes (or Barnes & Burr) took over "Elements ..." in '38; American Book Co. (Cincinnati) published '01 edition of "... elementary ...".

Davis, Ellery Williams (Wm. Chas. Brenke, E. R. Hedrick), *The calculus*, 1912–1930, The Macmillan Co., NY. This book is "edited by Earle Raymond Hedrick." Brenke is sometimes coauthor, sometimes assistant.

Docharty, Gerardus Beekman, *Elements of analytical geometry and of the differential and integral calculus*, 1865, Harper & Brothers, NY.

Echols, William Harding, *An elementary textbook on the differential and integral calculus*, 1902–1908, Henry Holt & Co., NY.

Fisher, Irving, *A brief introduction to the infinitesimal calculus; designed especially to aid in reading mathematical economics and statistics*, 1897–1937, Macmillan & Co., NY.

Franklin, William Suddards (Barry MacNutt & R. Charles), *An elementary treatise on calculus; a textbook for colleges and technical schools*, 1913, published by the authors, S. Bethleham. Barry MacNutt is coauthor with Franklin of a number of physics/engineering books; Rollin Charles authors no other books.

Gould, E(dward) Sherman, *A primer of the calculus*, 1896–1907, D. Van Nostrand, NY.

Granville, William Anthony (Percey F. Smith, Wm. R. Longley), *Elements of the differential and differential calculus*, 1904–1957, Ginn & Co., Boston.

——, *Elements of calculus*, 1946, Ginn & Co., Boston.

See also entries for Smith, Longley. The roles of Smith and Longley on the title page change over time.

Greene, William Batchelder, *An expository sketch of a new theory of the calculus*, 1859, printed for the author, Paris.

——, *The theory of the calculus*, 1870, Lee & Shepard, Boston.

——, *Explanation of "The Theory of the Calculus"*, 1870, Lee & Shepard, Boston.

Groat, Benjamin Feland, *An introduction to the summation of differences of a function; an elementary exposition of the nature of the algebraic processes replaced by the abbreviations of the infinitesimal calculus*, 1902, H. W. Wilson, Minneapolis.

Hackley, Charles William, *Differential calculus, for the use of the senior class of Columbia College ...*, 1856, Baker & Godwin, printers, NY.

Hall, William Shaffer, *Elements of the differential and integral calculus with applications*, 1897–1922, D. Van Nostrand, NY.

Hardy, Joseph Jonston, *Infinitesimals and limits*, 1900–1912, Chemical Publishing Co., Easton, PA.

Hathaway, Arthur Stafford, *A primer of calculus*, 1901, Macmillan and Co., NY, London.

Hayes, Ellen, *Calculus, with applications; an introduction to the mathematical treatment of science*, 1900, Allyn & Bacon, Boston.

Hayward, Harrison Washburn, *Notes on calculus; for the use of students of the Lowell Institute school for industrial foremen*, Massachusetts Institute of Technology, 1915, The Taylor Press, Boston.

Hedrick, Earle Raymond (O. D. Kellogg), *Applications of the calculus to mechanics*, 1909, Ginn and Co., Boston. Translated Goursat's *Mathematical Analysis*; also see Ellery W. Davis.

Hulburt, Lorrain Sherman, *Differential and integral calculus, an introductory course for colleges and engineering schools*, 1912–1943, Longmans, Green and Co., NY. 1943 edition published by Barnes and Noble, New York.

Johnson, William Woolsey (John Minot Rice), *An elementary treatise on the integral calculus founded on the method of rates or fluxions*, 1881–1909, John Wiley & Sons, NY.

——, *An elementary treatise on the differential calculus, founded on the method of rates*, 1904–1908, John Wiley & Sons, NY.

——, *A treatise on the integral calculus founded on the method of rates*, 1907, John Wiley & Sons, NY.

For earlier versions written with Rice, see Rice. "A treatise on the integral ..." is an enlargement of "An elementary treatise ...."

Keller, Samuel Smith (W. F. Knox), *Mathematics for engineering students; analytical geometry and calculus*, 1907–1908, D. Van Nostrand Co., NY.

Lambert, Preston Albert, *Differential and integral calculus for technical schools and colleges*, 1898–1907, The Macmillan Co., NY, London.

Longley, William Raymond (W. A. Wilson, P. F. Smith, {Granville}), *An introduction to the calculus*, 1924.

——, *Analytic geometry and calculus*, 1951.

Wallace Alvin Wilson was coauthor of the first book; Percey Smith (see) and Wilson were coauthors of the second.

Loomis, Elias, *Elements of analytical geometry and of the differential and integral calculus*, 1851–1872, Harper, NY.

——, *Elements of the differential and integral calculus*, 1874–1902, Harper, NY.

The 1902 edition of the later book was published by the American Book Co., NY. This book is unchanged between 1874 and 1902.

Love, James Lee, *An introductory course in the differential and integral calculus; for students in engineering in the Lawrence Scientific School*, 1898–1899, Harvard University, Cambridge.

M'Cartney, Washington, *The principles of the differential and integral calculus; and their applications to geometry*, 1844–1848, E. C. Biddle, Philadelphia. The 1848 edition was published by E. H. Butler & Co., Philadelphia.

March, Herman William (Henry C. Wolff), *Calculus*, 1917–1937, McGraw Hill, NY.

McMahon, James (Virgil Snyder), *Elements of the differential calculus*, 1898, American Book Co., NY, Cincinnati. See Snyder for additional books.

Murray, Daniel Alexander, *An elementary course in the integral calculus*, 1898, American Book Co., NY, Cincinnati.

——, *A first course in infinitesimal calculus*, 1903–1904, Longmans, Green & Co., NY.

——, *Differential and integral calculus*, 1908, Longmans, Green & Co., NY. Murray taught in Canada.

Newcomb, Simon, *Elements of the differential and integral calculus*, 1887–1889, Holt, NY.

Nichols, Edward West, *Differential and integral calculus with applications; for colleges, universities, and technical schools*, 1900–1918, D. C. Heath & Co., Boston.

Nicholson, James William, *Elements of the differential and integral calculus, with examples and practical applications*, 1896, University Publishing Co., NY & New Orleans. There may have been an 1894 edition of this book also.

Olney, Edward, *A general geometry and calculus.* Including part I of the general geometry, treating loci in a plane; and an elementary course in the differential and integral calculus, 1870–1885, Sheldon & Co., NY. The 1870 version contains just the first three chapters (geometry) of the 1871 book.

Osborne, George Abbott, *Notes on differentiation of functions. With examples....*, 1884, J. S. Cushing & Co., Boston.

——, *The differential calculus applied to plane curves and maxima and minima*, 1889–1890, J. S. Cushing & Co., Boston.

——, *The integral calculus applied to plane curves. Successive integration*, 1889, J. S. Cushing & Co., Boston.

——, *Differential and integral calculus, with examples and applications*, 1891–1910, Heath, Boston.

——, *An elementary treatise on the differential and integral calculus, with examples and applications*, 1891–1906, Leach, Boston & NY.

With the 1899 edition, Heath became the publisher of "An elementary treatise ...." "Notes ..." is a 40 page booklet, perhaps originally paperback, marked "Printed, not Published" on title page.

Osgood, William Fogg, *A modern English calculus*, 1902, The Macmillan Co., NY.

——, *A first course in the differential and integral calculus*, 1907–1929, The Macmillan Co., NY.

——, *Elementary calculus*, 1921, The Macmillon Co., NY.

——, *Introduction to the calculus*, 1922–1954, The Macmillan Co., NY.

——, *Advanced calculus*, 1945, The Macmillan Co., NY.

"Introduction ..." is called "a revision of ... 'A first Course...'."

Peck, William Guy, *Practical treatise on the differential and integral calculus, with some applications to mechanics and astronomy*, 1870–1898, A. S. Barnes & Co., NY, Chicago. 1892, 1898 editions are identical to 1870 edition; 1898 edition published by American Book Co., NY, Cincinnati, Chicago.

Peirce, Benjamin, *An elementary treatise on curves, functions, and forces.* 2 volumes, 1841–1862, James Munroe and Co., Boston & Cambridge.

Phillips, Henry Bayard, *Differential calculus ...*, 1916, John Wiley & Sons, NY.

——, *Integral calculus*, 1917, John Wiley & Sons, NY.

——, *Calculus*, 1927–1940, Wiley, NY.

——, *Analytical geometry and calculus*, 1942–1946, Addison Wesley, Cambridge, MA.

The 1940 edition of "Calculus" was published by Cummings, Cambrige, MA; the 1946 edition of "Analytical geometry ..." was published by Wiley.

Ransom, William Richard, *Freshman calculus; a presentation of fundamental conceptions and methods for students of science and engineering*, 1909, lithographed, Boston.

——, *Early calculus*, 1915, Tufts College, Medford, MA.

——, *A working calculus*, 1936, planograph, Boston.

——, *The calculus, according to a new plan*, 1947–1949, Tufts College, Medford, MA. "The calculus ..." is a revision of "A working calculus."

Rice, John Minot (William Woolsey Johnson), *On a new method of obtaining the differentials of functions, with especial reference to the Newtonian conception of rates or velocities*, 1873–1875, John Wiley & Sons, NY.

——, *The elements of the differential calculus founded on the method of rates or fluxions*, 1874, John Wiley & Sons, NY.

——, *An elementary treatise on the differential calculus founded on the method of rates or fluxions*, 1877–1904, John Wiley & Sons, NY. See Johnson for the continuation of this series, authored by Johnson alone.

Robinson, Horation Nelson (Issac Ferdinand Quinby), *Elements of analytical geometry and the differential and integral calculus*, 1856–1859, J. Ernst, Cincinnati.

——, *A new treatise on the elements of the differential and integral calculus*, 1868–1879, Ivison, Phinney, Blakeman, NY. The 1858 and 1859 editions of "Elements ... " were published by Ivison in NY.

Quinby is called the editor of "A new treatise ... ;" all editions appear after Robinson's death in 1867.

Ryan, James, *The differential and integral calculus*, 1828, White, Gallaher & White, NY.

Sestini, Benedict, *Manual of geometrical and infinitesimal analysis*, 1871, John Murphy & Co., Baltimore.

Smith, Percey Franklyn (W.A. Granville, W. A. Longley), *Elementary calculus; a textbook for the use of students in general science*, 1902–1903, American Book Co., NY, Cincinnati.

——, *Elementary analysis*, 1910, Ginn and Co., Boston, NY.

——, *Intermediate calculus*, 1931, Ginn and Co., Boston, NY.

Coauthors: "Elementary ... " with Granville, "Intermediate ... " with Longley. See also Granville, Longley. The latter's independent publications began after 1920.

Smith, William Benjamin, *Infinitesimal analysis ...*, 1898, Macmillan Co., NY.

Smyth, William, *Elements of the differential and integral calculus*, 1854–1859, Sanborn & Carter, Portland.

——, *Elements of calculus*, 1859, Sanborn & Carter, Portland.

Snyder, Virgil (John I. Hutchinson, J. McMahon), *Differential and integral calculus*, 1902, American Book Co., NY, Cincinnati.

——, *Elementary textbook on the calculus*, 1912, American Book Co., NY, Cincinnati.

John Irwin Hutchinson is coauthor of these books. See also James McMahon.

Spare, John, *The differential calculus: with unusual and particular analysis of its elementary principles and copious illustrations of its practical applications*, 1865, Bradley Dayton & Co., Boston.

Strong, Theodore, *A treatise on the differential and integral calculus*, 1869, C. A. Alvord, NY.

Taylor, James Morford, *Elements of the differential and integral calculus with examples and applications*, 1884–1902, Ginn, Heath, & Co., Boston.

Thomas, Robert Gibbes, *Applied calculus; principles and applications, essentials for students and engineers*, 1919–1924. D. Van Nostrand Co., NY. 1924 edition is "an abridged and rev. ed. of *Applied calculus*, with additional exercises and formulas."

Townsend, Edgar Jerome (George Alfred Goodenough), *First course in calculus*, 1908–1910, Holt, NY.

——, *Essentials of calculus*, 1910–1925, Holt, NY.

Goodenough coauthored both books.

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Editors' note: The following review of a popular calculus textbook is reprinted as an indication of American mathematicians' sophistication and growing insistence on precision and rigor during the late nineteenth century. It illustrates a point made by George Rosenstein in the preceding article.

EDWARDS' DIFFERENTIAL CALCULUS.

An Elementary Treatise on the Differential Calculus, with applications and numerous examples. By JOSEPH EDWARDS, M.A., formerly Fellow of Sidney Sussex College, Cambridge. Second edition, revised and enlarged. London and New York, Macmillan & Co. 1892. 8vo, pp. xiii + 521.

WHEN a mathematical text book reaches a second edition, so much enlarged as this, we know at once that the book has been received with some favour, and we are prepared to find that it has many merits. We are at once struck by Mr. Edwards' lucid and incisive style; his expositions are singularly clear, his words well chosen, his sentences well balanced. In the text of the book we meet with various useful results, notably in the chapter on "some well known curves," and moreover the arrangement is such that these results are easy to find; and in addition to these, numbers of theorems are given among the examples, and, this being a feature for which we are specially grateful, in nearly every case the authority is cited. Recognizing these merits, however, we notice that the book has many defects, some proper to itself, some characteristic of its species; and just because it is so attractive in appearance, it seems worth while examining it in detail, and pointing out certain specially vicious features.

A book of this size may fairly be required to serve as a preparation for the function theory; at all events, the influence of recent Continental researches should be evident to the eyes of the discerning. Mr. Edwards' preface strengthens this reasonable expectation, for he promises us "as succinct an account as possible of the most important results and methods which are up to the present time known." But we soon find that the "important results and methods" are those of the Mathematical Tripos; and in our disappointment we utter a fervent wish that instead of the "large number of university and college examination papers, set in Oxford, Cambridge, London, and elsewhere," Mr. Edwards had consulted an equally large number of mathematical memoirs published, principally, elsewhere. The Mathematical Tripos for any given year is not intended for a *Jahrbuch* of the progress of mathematics during the past year; and as long as so many will insist on regarding it in that light, text books of this type will continue to be published.

Nothing in this book indicates that Mr. Edwards is familiar with such works as Stolz's *Allgemeine Arithmetik*, Dini's *Fondamenti per la teorica delle funzioni di variabili reali*, or Tannery's *Théorie des fonctions d'une variable.* In support of our contention we may instance the definitions of function,

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limit, continuity, etc. On page 2, Lejeune Dirichlet's definition of a function is adopted. According to this very general definition, there need be no analytical connection between y and x; for y is a function of x even when the values of y are arbitrarily assigned, as in a table. That Mr. Edwards does not adhere to this definition is evident from his tacit assumption that *every* function $\varphi(x)$ can be represented by a succession of continuous arcs of curves. Whatever definition is adopted for a continuous function y of x, it is evident that to small increments of x must correspond small increments of y; but Weierstrass has proved that there exist functions which have this property, but which have nowhere differential coefficients. The well known example of such a function is

$$f(x) = \sum_{n=0}^{\infty} b^n \cos (a^n x\pi),$$

where a is an odd integer, b a positive constant less than 1, and ab greater than $1 + 3\pi/2$. According to the accepted definition, this function of x is continuous; according to Mr. Edwards' definition, it is not continuous, inasmuch as it cannot be represented by a curve $y = f(x)$ with a tangent at every point.

We acknowledge that Mr. Edwards displays a considerable degree of consistency in his view of the meaning of a continuous function, but we insist that after the adoption of the curve definition he should have been at some pains to prove that the numerous series of the type $\sum_1^{\infty} f_n(x)$ scattered throughout the book give rise to curves with tangents, whereas he never even takes the trouble to prove that they are continuous functions of x in any sense of the term. No more damaging charge can be brought against any treatise laying claim to thoroughness than that of recklessness in the use of infinite series; and yet Mr. Edwards has everywhere laid himself open to this charge. One of the most difficult things to teach the beginner in mathematics is to give proper attention to the convergency of the series dealt with. All the more need, then, that a text book of this nature should set an example of consistent, even *aggressive* carefulness in this respect. We do, it is true, find an occasional mention of convergence (pp. 9, 81, 454, etc.), but as a rule it is ignored. Mr. Edwards rearranges the terms of infinite and doubly infinite series, applying the law of commutation without pointing out that his series are unconditionally convergent; he differentiates $f(x) = \sum_1^{\infty} f_n(x)$ term by term, and gets $f'(x) = \sum_1^{\infty} f'_n(x)$, implying that the process is universally valid (*e.g.* p. 84); or, at all events, giving no hint that there are cases in which the differential coefficient of the sum of a convergent series is different from the sum of the differential coefficients of the individual terms. We find no formal recognition of the importance of uniform convergence in modern analysis, nothing even to suggest that he has ever heard of the distinction between uniform and non-uniform convergence. We begin to suspect that he has never looked into Chrystal's Algebra.

The unreasoning mechanical facility thus acquired in performing operations unhampered by any doubts as to their legitimacy, naturally leads Mr. Edwards to view with favour "the analytical house of cards, composed of complicated and curious formulæ, which the academic tyro builds with such zest upon a slippery foundation," *—and to build up many a one. A curious and interesting specimen is

$$f(x) = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$$

to be continued to infinity. This expression has been examined by Seidel,† who points out that Eisenstein's paper in *Crelle*, vol. 28, requires correction. Before such an expression can be differentiated, a definite meaning must be assigned to it; but Seidel's conclusion is that, denoting x^x by x_1, x^{x_1} by x_2, x^{x_2} by x_3, and so on, then as x varies from 0 to $1/e^e$, $\underset{n=\infty}{L}\ x_{2n}$ increases from 0 to $1/e$, while $\underset{n=\infty}{L}\ x_{2n+1}$ decreases from 1 to $1/e$; beyond these limits for x, the case is different. In particular when $x > e^{1/e}$, the expression diverges. Our objection is not to the non-acceptance of Seidel's conclusions, but to the unnecessary use of a function of this doubtful character. Examples can be found to illustrate every point that ought to be brought up in an elementary treatise on the differential calculus without ranging over examination papers in search of striking novelties.

Feeling now somewhat familiar with Mr. Edwards' point of view, we examine his proofs of the ordinary expansions with a tolerably clear idea of what we are to expect. We find, of course, "the time-honoured short proof of the existence of the exponential limit, which proof is half the real proof plus a *suggestio falsi*"; we find in the chapter on expansions a general disregard of convergency considerations; we find throughout the book the assumption that

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* Professor CHRYSTAL, in *Nature*, June 25, 1891.

† *Abhandlungen der k. Ak. d. Wiss.* Bd. xi.

$\phi(a) = \underset{x=a}{L}\,\phi(x)$, and that $\phi(0, 0) = \underset{x=0,\,y=0}{L}\,\phi(x, y)$ *; we find the usual assumptions as to expansibility in series proceeding by integral powers, with disastrous results further on. We find the usual dread of the complex variable, though Mr. Edwards has given one or two examples involving it, without however explaining what is meant by $f(x + iy)$. We can hardly regard these examples, even with § 190, as a sufficient recognition of the complex variable in a treatise of this size. We must notice also the thoroughly faulty treatment of the inverse functions. For example, no explanation is given of the signs in $\frac{dy}{dx}$, when $y = \cos^{-1}x$ or $\sin^{-1}x$. Mr. Edwards' attitude towards many valued functions is simple enough; as a rule, he ignores the inconvenient superfluity of values. He does, it is true, give in § 54 a note, clear and correct, on this point; but he is very careful to confine this within the limits of the single section, and to indicate, by choice of type, that it is quite unimportant.

We pass on now to the second part, applications to plane curves; and here we must object emphatically to the introduction of so many detached and disconnected propositions relating to the theory of higher plane curves. From Mr. Edwards' point of view this is doubtless justified; we are quite ready to acknowledge that we know of no book that would enable a candidate to answer more questions on subjects of whose theory he is totally ignorant. The deficiency of a curve, *e.g.*, is a conception entirely independent of the differential calculus; but probably this single page will obtain many marks for candidates in the Mathematical Tripos; these we should not grudge if we thought an equivalent would be lost by a reproduction of Mr. Edwards' treatment of cusps. Our spirits rose when we remarked the italicised phrase on p. 224, that there is "*in general a cusp*" when the tangents are coincident. But three pages further on we find that the exception here indicated is simply our old friend, the conjugate point, whose special exclusion from the class in which it appears must be a perpetual puzzle to a thoughtful student with no better guidance than a book of this kind. Such a student, probably already familiar with projection, knows that the real can be projected into the imaginary, and the imaginary into the real. If then the acnode, appearing as a cusp, has to be specially excluded, why not the crunode? But here Mr. Edwards reproduces the now well established

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* See *e.g.* p. 122; and on this page note also the assumption that the relation between h, k, while $x + h$, $y + k$, tend to the limits x, y exerts no influence on the result.

error, calling tacnodes, formed by the contact of real branches, double cusps of the first and second species, and excluding those formed by the contact of imaginary branches; he even goes further astray, introducing Cramer's osculinflexion as a cusp that changes its species.

This matter of double cusps is a fundamentally serious one, and not a mere question of nomenclature. This persistent misnaming effectually disguises the essential characteristic of the cusp. It is *not the coincidence of the tangents* that makes a cusp. From the geometrical point of view it is the turning back of the (real) tracing point, expressed by the French and German names, {*point de rebroussement, Rückkehrpunkt*}; from the point of view of algebraical expansions (of y in terms of x, $y = 0$ being the tangent) the essential characteristic of a single cusp is that at some stage in the expansion there shall be a fractional exponent with an even denominator, so that the branch changes from real to imaginary *along its tangent;* from the point of view of the function theory, which is really equivalent to the last, the simple cusp is characterised by the presence of a *Verzweigungspunkt* combined with a double point. The simple cusp, that is, presents itself as an evanescent loop. A double cusp, then, in the sense in which Mr. Edwards uses the term, does not exist. There cannot be two consecutive cusps, vertex to vertex; for the branch if supposed continued through the cusp, changes from real to imaginary; and two *distinct* cusps, brought together to give a point of this appearance, produce a quadruple point.

While on this subject, we must mention Mr. Edwards' rule for finding the nature of a cusp. Find the two values of $\frac{d^2y}{dx^2}$; these by their signs determine the direction of convexity (§ 296). How does this apply *e.g.* to $y^2 = x^3$?

This confusion regarding cusps is made worse by the assumption already noticed that when $f(x, y) = 0$ is the equation of the curve, y can be expanded in a series of integral powers of x. This error is repeated on p. 258, where to obtain the branches at the origin, this being a double point, we are told to expand y by means of the assumption $y = px + \frac{qx^2}{2!} +$ etc. The whole exposition of this theory of expansion is most inadequate. In § 382 there is no hint that the terms obtained are the beginning of an infinite series, giving the expansion of (say) y in powers, not necessarily integral, of x; there is no hint what to do when the first terms of the expansion are found; there is no suggestion of the interpretation of the result when two expansions begin with the same terms. A thoughtful student *may* by a happy comparison of scattered examples (p. 200, and ex. 3, p. 230) arrive at the correct theory; but he surely deserves better guidance.

One or two more points must be noticed. The theory of asymptotes, when two directions to infinity coincide, cannot be satisfactorily developed without assuming a knowledge of double points; and the only way of giving the true geometrical significance is to introduce the conception of the line infinity, and to consider the nature of the intersections of the curve by this line. A tangent lying entirely at infinity does *not* "count as one of the n theoretical asymptotes"; if counted among the asymptotes at all, it has to be counted as the equivalent of two out of the n. This is one of the strongest arguments against including the line infinity in enumerating the asymptotes. The various expressions for the radius of curvature involve an ambiguity in sign; what is the meaning of this? The omission of this explanation causes obscurity, notably in § 330. The equation of a curve, referred to oblique axes, being $\varphi(x, y) = 0$, what is the condition for an inflexion? As a matter of fact it is the same as in the case of rectangular axes, given on p. 264; but as this is obtained from a formula for the radius of curvature, the investigation is not applicable. Throughout Mr. Edwards displays an almost exclusive preference for rectangular axes, and seems to regard the metric properties so obtained as of equal importance with descriptive properties. For instance, in the case of an ordinary double point (p. 224) instead of the *three* cases usually distinguished, we have *four*, the additional one being that of perpendicular tangents.

In the third part we notice that in the chapter on "undetermined forms" there is no discussion of the case of two variables, though it is on this that we have to rely for a rigorous proof of the theorem $\frac{\partial^2\varphi}{\partial x\partial y} = \frac{\partial^2\varphi}{\partial y\partial x}$. We recognize an old friend, the discussion of the limit of ∞/∞, in which it is first assumed, and then proved, that the limit exists. The statement of ex. 17, p. 457, is somewhat misleading; the formula there given for the expansion of $(x + a)^m$ is true when m is a positive integer; but when $m = -1$, it is evidently not true for $x = -b, -2b$, etc.* The treatment of maxima and minima of functions of two variables (§§ 497–501) is incomplete and incorrect. The geometrical illustration, as given on p. 424, omits the case of a section with a cusp, which is the simplest case that can occur when $rt = s^2$; of the more complicated cases Mr. Edwards attempts no discrimination; he does not even state correctly the principles that must guide us in this discrimination. The inexactness of the ordinary

* LAURENT, *Traité d'Analyse*, iii., 386.

criteria (given in § 498) appears at once from the example $u = (y^2 - 2px)(y^2 - 2qx)$ [Peano]. The origin is a point satisfying the preliminary conditions; taking then for x, y, small quantities h, k, the terms of the second degree are positive for all values except $h = 0$; when $h = 0$, the terms of the third degree vanish, and the terms of the fourth degree are positive; nevertheless the point does not give a minimum, which it should do by the test of § 498. For we can travel away from O in between the two parabolas, so coming to an adjacent point at which u has a small negative value, while for points inside or outside both parabolas the value of u is positive. The truth is, the nature of the value a of the function u at a point (x_0, y_0) at which $\frac{\delta\varphi}{\delta x}$ and $\frac{\delta\varphi}{\delta y}$ vanish, depends on the nature of the singularity of the curve $u = a$ at this point. If this curve has at (x_0, y_0) an isolated point of any degree of multiplicity, we have a true maximum or minimum of u; but if through (x_0, y_0) pass any number of real non-repeated branches of the curve, we have not a maximum or minimum; in Peano's example the branches coincide in the immediate neighbourhood of the origin, but then they separate, and therefore we have not a minimum value for u.

We object, then, to Mr. Edwards' treatise on the Differential Calculus because in it, notwithstanding a specious show of rigour, he repeats old errors and faulty methods of proof, and introduces new errors; and because its tendency is to encourage the practice of cramming "short proofs" and detached propositions for examination purposes.

CHARLOTTE ANGAS SCOTT.

BRYN MAWR, PA., *May* 18, 1892.