11th Chinese Girls' Mathematics Olympiad

Guangzhou, China

Day I 8:00 AM—12:00 PM

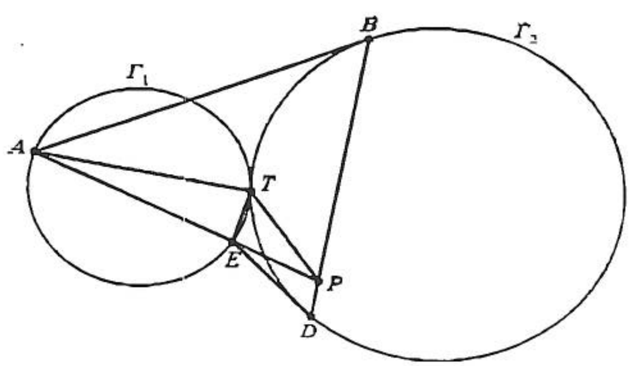
August 10, 2012

CGMO 2012

1. Let n be a positive integer, and $a_1, a_2, \cdots, a_n$ be non-negative real numbers. Show that

$$\frac{1}{1+a_1}+\frac{a_1}{(1+a_1)(1+a_2)}+\cdots+\frac{a_1 a_2 \cdots a_{n-1}}{(1+a_1)(1+a_2)\cdots(1+a_n)} \le 1 .$$

2. As shown in the figure, circles Γ_1 and Γ_2 are tangent to each other externally at point T. Points A and E are on the circle Γ_1, lines AB and DE are tangent to the circle Γ_2 at points B and D respectively, lines AE and BD meet at point P. Prove that (1) $\frac{AB}{AT}=\frac{ED}{ET}$; (2) $\angle ATP+\angle ETP=180°$.



3. Find all the pairs (a,b) of integers satisfying the following condition: there exists an integer $d \ge 2$ such that a^n+b^n+1 is divisible by d for all positive integers n.

4. There is a stone at each vertex of a given regular 13-gon, and the color of each stone is black or white. Prove that we may exchange the position of two stones such that the coloring of these stones are symmetric respect to some symmetric axis of the 13-gon.

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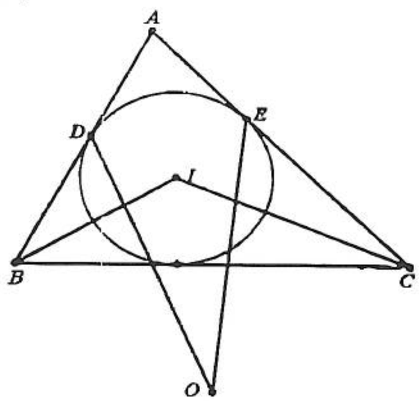
5. As shown in the figure below, the in-circle $\odot I$ of $\triangle ABC$ is tangent to sides AB and AC at points D and E respectively, and O is the circum-center of $\triangle BCI$. Prove that $\angle ODB = \angle OEC$.



6. There are n cities ($n \geq 3$) and two airline companies in a country. Between any two cities, there is exactly one 2-way flight connecting them which is operated by one of the two companies. A female mathematician plans a travel route, so that it starts and ends at the same city, passes through at least two other cities, and each city in the route is visited once. She finds out that wherever she starts and whatever route she chooses, she must take flights of both companies. Find the maximum value of n.

7. Let $a_1 \leq a_2 \leq \cdots$ be a sequence of positive integers such that $\dfrac{r}{a_r} = k+1$ for some positive integers k and r. Prove that there exists a positive integer s such that $\dfrac{s}{a_s} = k$.

8. Find the number of integers k in the set $\{0, 1, 2, \ldots, 2012\}$ such that the combination number $\dbinom{2012}{k} = \dfrac{2012!}{k!(2012-k)!}$ is a multiple of 2012.